Notes on Intuitionistic Fuzzy Sets
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Extended intuitionistic fuzzy graphs

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Abstract: On the basis of the definition of the concept of the extended intuitionistic fuzzy index matrix, in the paper the concept of an extended intuitionistic fuzzy graph is proposed. Some properties are discussed and examples are given.

Keywords: Extended intuitionistic fuzzy index matrix, Extended intuitionistic fuzzy graph, Index matrix, Intuitionistic fuzzy index matrix.
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1 Introduction

The Intuitionistic Fuzzy Sets (IFSs, see [2, 5]) were defined as extensions of the ordinary fuzzy sets, but during the last 25 years they have also been object of extensions and applications.

Exactly 20 years ago, Anthony Shannon and the author introduced the concept of an Intuitionistic Fuzzy Graph (IFG) [9]. The definition from [2, 5] is the following.

Let V be a set of vertices. For a fixed Cartesian product $\circ$ over IFSs (see [2, 5]), the set

$$G^* = \{\langle\langle x, y\rangle, \mu_G(x, y), \nu_G(x, y)\rangle\ \langle x, y\rangle \in V \times V\}$$

is called $\circ$-IFG (or briefly, an IFG) if the functions $\mu_G : V \times V \to [0, 1]$ and $\nu_G : V \times V \to [0, 1]$ define the degree of membership and the degree of non-membership, respectively, of the arc $\langle x, y\rangle \in V \times V$ to the set $G \subset V \times V$, where $\times$ is a standard set-theoretical Cartesian product and the functions μ and ν have the forms of the corresponding components of the o-Cartesian product over IFSs and for all $\langle x, y\rangle \in V \times V$,

$$0 \le \mu_G(x,y) + \nu_G(x,y) \le 1.$$

For simplicity, we write G instead of G^*.

2 Short remarks on index matrix, intuitionistic fuzzy index matrix and extended intuitionistic fuzzy index matrix

The concept of Index Matrix (IM) was introduced in [1] and discusssed in more details in [3, 4]. Here, following [3], the basic definitions and properties related to IMs are given.

Let I be a fixed set of indices and $\mathcal{R}$ be the set of all real numbers. By IM with index sets K and L $(K, L \subset I)$, we mean the object,

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array},$$

where $K = \{k_1, k_2, ..., k_m\},\ L = \{l_1, l_2, ..., l_n\}$, and for $1 \le i \le m$, and $1 \le j \le n : a_{k_i,l_j} \in \mathcal{R}$.

On the basis of the above definition, in [4] the new object – the Intuitionistic Fuzzy IM (IFIM) – was introduced in the form

$$[K, L, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$$

$$\equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle & \langle \mu_{k_1,l_2}, \nu_{k_1,l_2} \rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle \\ k_2 & \langle \mu_{k_2,l_1}, \nu_{k_2,l_1} \rangle & \langle \mu_{k_2,l_2}, \nu_{k_2,l_2} \rangle & \dots & \langle \mu_{k_2,l_n}, \nu_{k_2,l_n} \rangle \\ \vdots & & & & \\ k_m & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle & \langle \mu_{k_m,l_2}, \nu_{k_m,l_2} \rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle \end{array},$$

where for every $1 \le i \le m, 1 \le j \le n$: $0 \le \mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \le 1$.

The Extended IFIM (EIFIM) has defined in [6] by:

$$[K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$$

$$\equiv \begin{array}{c|ccccc} & l_1, \langle \alpha_1^l, \beta_1^l \rangle & \dots & l_j, \langle \alpha_j^l, \beta_j^l \rangle & \dots & l_n, \langle \alpha_n^l, \beta_n^l \rangle \\ \hline k_1, \langle \alpha_1^k, \beta_1^k \rangle & \langle \mu_{k_1,l_1}, \nu_{k_1,l_1} \rangle & \dots & \langle \mu_{k_1,l_j}, \nu_{k_1,l_j} \rangle & \dots & \langle \mu_{k_1,l_n}, \nu_{k_1,l_n} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_i, \langle \alpha_i^k, \beta_i^k \rangle & \langle \mu_{k_i,l_1}, \nu_{k_i,l_1} \rangle & \dots & \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle & \dots & \langle \mu_{k_i,l_n}, \nu_{k_i,l_n} \rangle \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ k_m, \langle \alpha_m^k, \beta_m^k \rangle & \langle \mu_{k_m,l_1}, \nu_{k_m,l_1} \rangle & \dots & \langle \mu_{k_m,l_j}, \nu_{k_m,l_j} \rangle & \dots & \langle \mu_{k_m,l_n}, \nu_{k_m,l_n} \rangle \end{array},$$

where for every $1 \leq i \leq m, 1 \leq j \leq n$: $\mu_{k_i,l_j}, \nu_{k_i,l_j}, \mu_{k_i,l_j} + \nu_{k_i,l_j} \in [0,1]$,

$$\alpha_1^k, \beta_1^k, \alpha_1^k + \beta_1^k \in [0,1],$$

$$\alpha_1^l, \beta_1^l, \alpha_1^l + \beta_1^l \in [0,1]$$

and here and below,

$$K^* = \{\langle k_i, \alpha_i^k, \beta_i^k \rangle | k_i \in K\} = \{\langle k_i, \alpha_i^k, \beta_i^k \rangle | 1 \leq i \leq m\},$$

$$L^* = \{\langle l_j, \alpha_j^l, \beta_j^l \rangle | l_j \in L\} = \{\langle l_j, \alpha_j^l, \beta_j^l \rangle | 1 \leq j \leq n\}.$$

Let

$$K^* \subset P^* \text{ iff } (K \subset P) \; \& \; (\forall k_i = p_i \in K)((\alpha_i^k < \alpha_i^p) \; \& \; (\beta_i^k > \beta_i^p)).$$

$$K^* \subseteq P^* \text{ iff } (K \subseteq P) \; \& \; (\forall k_i = p_i \in K)((\alpha_i^k \leq \alpha_i^p) \; \& \; (\beta_i^k \geq \beta_i^p)).$$

All operations and relations over EIFIM must be re-defined, because they have different forms from the above ones. Obviously, the hierarchical operators are not applicable now.

3 Standard operations over EIFIMs

For the EIFIMs $A = [K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}], B = [P^*, Q^*, \{\langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle\}]$, operations that are analogous to the usual matrix operations of addition and multiplication are defined, as well as other specific ones.

(1.a) Alddition-(max,min)

$$A \oplus_{(\max,\min)} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$T^* = K^* \cup P^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K \cup P\},$$

$$V^* = L^* \cup Q^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in L \cup Q\},$$

$$\alpha_u^t = \begin{cases} \alpha_i^k, & \text{if } t_u \in K - P \\ \alpha_r^p, & \text{if } t_u \in P - K \\ \max(\alpha_i^k, \alpha_r^p), & \text{if } t_u \in K \cap P \end{cases},$$

$$\beta_w^v = \begin{cases} \beta_j^l, & \text{if } v_w \in L - Q \\ \beta_s^q, & \text{if } t_w \in Q - L \\ \min(\beta_j^l, \beta_s^q), & \text{if } t_w \in L \cap Q \end{cases},$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = l_j \in L - Q \\ & \quad \text{or } t_u = k_i \in K - P \\ & \quad \text{and } v_w = l_j \in L; \\ \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \\ & \quad \text{and } v_w = q_s \in Q - L \\ & \quad \text{or } t_u = p_r \in P - K \\ & \quad \text{and } v_w = q_s \in Q; \\ \\ \langle \max(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ \quad \min(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \quad \text{and } v_w = l_j = q_s \in L \cap Q \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

(1.b) Addition-(min,max)

$$A \oplus_{(\text{min,max})} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where $T^*, V^*, \alpha_u^t, \beta_w^v$, have the forms from (1.a), but

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = l_j \in L - Q \\ & \quad \text{or } t_u = k_i \in K - P \\ & \quad \text{and } v_w = l_j \in L; \\ \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P \\ & \quad \text{and } v_w = q_s \in Q - L \\ & \quad \text{or } t_u = p_r \in P - K \\ & \quad \text{and } v_w = q_s \in Q; \\ \\ \langle \min(\mu_{k_i,l_j}, \rho_{p_r,q_s}), & \text{if } t_u = k_i = p_r \in K \cap P \\ \quad \max(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle, & \quad \text{and } v_w = l_j = q_s \in L \cap Q \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

(2.a) Termwise multiplication-(max,min)

$$A \otimes_{(\text{max,min})} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$T^* = K^* \cap P^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K \cap P\},$$

$$V^* = L^* \cap Q^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in L \cap Q\},$$

$$\alpha_u^t = \min(\alpha_i^k, \alpha_r^p), \text{ for } t_u = k_i = p_r \in K \cap P,$$

$$\beta_w^v = \min(\beta_j^l, \beta_s^q), \text{ for } v_w = l_j = q_s \in L \cap Q$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \max(\mu_{k_i,l_j}, \rho_{p_r,q_s}), \min(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle.$$

(4.2.b) Termwise multiplication-(min,max)

$$A \otimes_{(\min,\max)} B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where $T^*, V^*, \alpha_u^t, \beta_w^v$, have the forms from (2.a), but

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \min(\mu_{k_i,l_j}, \rho_{p_r,q_s}), \max(\nu_{k_i,l_j}, \sigma_{p_r,q_s}) \rangle.$$

(3.a) Multiplication-(max,min)

$$A \odot_{(\max,\min)} B = [T^*, V^*, \langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle\}],$$

where

$$T^* = (K \cup (P - L))^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K \cup (P - L)\},$$

$$V^* = (Q \cup (L - P))^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in Q \cup (L - P)\},$$

$$\alpha_u^t = \begin{cases} \alpha_i^k, & \text{if } t_u = k_i \in K \\ \\ \alpha_r^p, & \text{if } t_u = p_r \in P - L \end{cases},$$

$$\beta_w^v = \begin{cases} \beta_j^l, & \text{if } v_w = l_j \in L - P \\ \\ \beta_s^q, & \text{if } t_w = q_s \in Q \end{cases},$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$= \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = l_j \in L - P - Q \\ \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P - L - K \\ & \quad \text{and } v_w = q_s \in Q \\ \\ \langle \max\limits_{l_j = p_r \in L \cap P} (\min(\mu_{k_i,l_j}, \rho_{p_r,q_s})), & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = q_s \in Q \\ \min\limits_{l_j = p_r \in L \cap P} (\max(\nu_{k_i,l_j}, \sigma_{p_r,q_s})) \rangle, & \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

(3.b) Multiplication-(min,max)

$$A \odot_{(\min,\max)} B = [T^*, V^*, \langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle \}],$$

where $T^*, V^*, \alpha_u^t, \beta_w^v$, have the forms from (3.a), but

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle =$$

$$= \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = l_j \in L - P - Q \\ \langle \rho_{p_r,q_s}, \sigma_{p_r,q_s} \rangle, & \text{if } t_u = p_r \in P - L - K \\ & \quad \text{and } v_w = q_s \in Q \\ \langle \min\limits_{l_j = p_r \in L \cap P} (\max(\mu_{k_i,l_j}, \rho_{p_r,q_s})), & \text{if } t_u = k_i \in K \\ & \quad \text{and } v_w = q_s \in Q \\ \quad \max\limits_{l_j = p_r \in L \cap P} (\min(\nu_{k_i,l_j}, \sigma_{p_r,q_s})) \rangle, & \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases}$$

(4) Structural subtraction

$$A \ominus B = [T^*, V^*, \{\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle \}],$$

where

$$T^* = (K - P)^* = \{\langle t_u, \alpha_u^t, \beta_u^t \rangle | t_u \in K - P\},$$
$$V^* = (L - Q)^* = \{\langle v_w, \alpha_w^v, \beta_w^v \rangle | v_w \in L - Q\},$$

for the set–theoretic subtraction operation and

$$\alpha_u^t = \alpha_i^k, \text{ for } t_u = k_i \in K - P,$$
$$\beta_w^v = \beta_j^l, \text{ for } v_w = l_j \in L - Q$$

and

$$\langle \varphi_{t_u,v_w}, \psi_{t_u,v_w} \rangle = \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, \text{ for } t_u = k_i \in K - P \text{ and } v_w = l_j \in L - Q.$$

(5) Negation of an EIFIM

$$\neg A = [T^*, V^*, \{\neg \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle \}],$$

where $\neg$ is one of the intuitionistic fuzzy negations given in [5], or another possible negation.

(6) Termwise subtraction

$$A -_{\max,\min} B = A \oplus_{\max,\min} \neg B,$$

$$A -_{\min,\max} B = A \oplus_{\min,\max} \neg B.$$

Operations "reduction", "projection" and "substitution" over EIFIMs coincide with the respective operations defined in [3, 4, 6].

4 Operations "reduction" over an EIFIM

Here and below we use symbol "$\perp$" for lack of some component in the separate definitions. In some cases, it is suitable to change this symbol with "0".

Now, we introduce operations $(k, \perp)$- and $(\perp, l)$-reduction of a given EIFIM $A = [K, L, \{a_{k_i,l_j}\}]$:

$$A_{(k,\perp)} = [K - \{k\}, L, \{c_{t_u,v_w}\}]$$

where

$$c_{t_u,v_w} = a_{k_i,l_j} \text{ for } t_u = k_i \in K - \{k\} \text{ and } v_w = l_j \in L$$

and

$$A_{(\perp,l)} = [K, L - \{l\}, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = a_{k_i,l_j} \text{ for } t_u = k_i \in K \text{ and } v_w = l_j \in L - \{l\}.$$

Second, we define

$$A_{(k,l)} = (A_{(k,\perp)})_{(\perp,l)} = (A_{(\perp,l)})_{(k,\perp)},$$

i.e.,

$$A_{(k,l)} = [K - \{k\}, L - \{l\}, \{c_{t_u,v_w}\}],$$

where $c_{t_u,v_w} = a_{k_i,l_j}$ for $t_u = k_i \in K - \{k\}$ and $v_w = l_j \in L - \{l\}$.

Third, let $P = \{k_1, k_2, ..., k_s\} \subseteq K$ and $Q = \{q_1, q_2, ..., q_t\} \subseteq L$. Then, we define the following three operations:

$$A_{(P,l)} = (...((A_{(k_1,l)})_{(k_2,l)})...)_{(k_s,l)},$$

$$A_{(k,Q)} = (...((A_{(k,l_1)})_{(k,l_2)})...)_{(k,l_t)},$$

$$A_{(P,Q)} = (...((A_{(p_1,Q)})_{(p_2,Q)})...)_{(p_s,Q)} = (...((A_{(P,q_1)})_{(P,q_2)})...)_{(P,q_t)}.$$

Obviously,

$$A_{(K,L)} = I_\emptyset \quad \text{and} \quad A_{(\emptyset,\emptyset)} = A.$$

5 Operation "substitution" over an EIFIM

Let EIFIM $A = [K, L, \{a_{k,l}\}]$ be given. First, local substitution over the EIFIM is defined for the couples of indices (p, k) and/or (q, l), respectively, by

$$\left[\frac{p}{k}; \perp\right] A = [(K - \{k\}) \cup \{p\}, L, \{a_{k,l}\}],$$

$$\left[\perp; \frac{q}{l}\right] A = [K, (L - \{l\}) \cup \{q\}, \{a_{k,l}\}],$$

Second,

$$\left[\frac{p}{k}; \frac{q}{l}\right] A = \left[\frac{p}{k}; \perp\right] \left[\perp; \frac{q}{l}\right] A,$$

i.e.

$$\left[\frac{p}{k};\frac{q}{l}\right] A = [(K - \{k\}) \cup \{p\}, (L - \{l\}) \cup \{q\}, \{a_{k,l}\}] .$$

Let the sets of indices $P = \{p_1, p_2, ..., p_u\}$, $Q = \{q_1, q_2, ..., q_v\}$ be given. Third, for them we define sequentially:

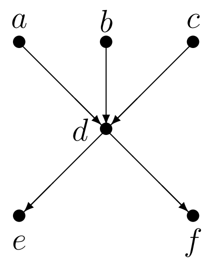
$$\left[\frac{P}{K};\perp\right] A = \left[\frac{p_1}{k_1}\frac{p_2}{k_2}...\frac{p_u}{k_u};\perp\right] A,$$

$$\left[\perp;\frac{Q}{L}\right] A = (\left[\perp;\frac{q_1}{l_1}\frac{q_2}{l_2}...\frac{q_v}{l_v}\right] A),$$

$$\left[\frac{P}{K};\frac{Q}{L}\right] A = \left[\frac{P}{K};\perp\right] (\left[\perp;\frac{Q}{L}\right] A).$$

6 Short remarks on index matrix representation of ordinary graphs

Let us have the following oriented graph C



For it, we can construct the $(0, 1)$-IM, i.e., IM with elements from set $\{0, 1\}$, which is an adjacency matrix of the graph

$$C = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 0 & 0 & 0 & 1 & 0 & 0 \\ b & 0 & 0 & 0 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & 0 & 0 & 1 & 1 \\ e & 0 & 0 & 0 & 0 & 0 & 0 \\ f & 0 & 0 & 0 & 0 & 0 & 0 \end{array} .$$

Shortly, we denote this matrix by “Adjacency IM” (AdIM).

Obviously, the columns indexed by a, b, c and the rows, indexed by e, f contain only zeros and do not give any information. So, we can transform the AdIM to the form

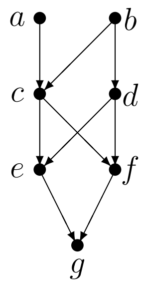
$$D = \begin{array}{c|ccc} & d & e & f \\ \hline a & 1 & 0 & 0 \\ b & 1 & 0 & 0 \\ c & 1 & 0 & 0 \\ d & 0 & 1 & 1 \end{array} ,$$

in which the isolated vertices are omitted. This new $(0,1)$-IM can be called “reduced AIM”.

An important question is whether this modification is a correct one. Really, we see that the connections between the immediate neighbouring vertices of the graph are seen, but we must check the basic property of the standard adjacency matrix X, that the elements of the multiplication $X^2 = X \odot_{(\times,+)} X$ represent which vertices are adjacent (see, e.g., [8]). Using the operation $\odot_{(\max,\min)}$, we obtain for the $(0,1)$-IM D

$$D \odot_{(\max,\min)} D = \begin{array}{c|ccc} & d & e & f \\ \hline a & 1 & 0 & 0 \\ b & 1 & 0 & 0 \\ c & 1 & 0 & 0 \\ d & 0 & 1 & 1 \end{array} \odot_{(\max,\min)} \begin{array}{c|ccc} & d & e & f \\ \hline a & 1 & 0 & 0 \\ b & 1 & 0 & 0 \\ c & 1 & 0 & 0 \\ d & 0 & 1 & 1 \end{array} = \begin{array}{c|ccc} & d & e & f \\ \hline a & 0 & 1 & 1 \\ b & 0 & 1 & 1 \\ c & 0 & 1 & 1 \\ d & 0 & 0 & 0 \end{array}.$$

Now, we illustrate the results of the applications of different operations over $(0,1)$-IMs, that represent some oriented graphs. Let us have the graph E



Its $(0,1)$-IM (in the reduced form, i.e., with omission of the row indexed by g) is

$$E = \begin{array}{c|ccccc} & c & d & e & f & g \\ \hline a & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 1 & 1 & 0 \\ d & 0 & 0 & 1 & 1 & 0 \\ e & 0 & 0 & 0 & 0 & 1 \\ f & 0 & 0 & 0 & 0 & 1 \end{array}.$$

Then, we calculate:

$$E^2 = E \odot_{\min} E = \begin{array}{c|ccccc} & c & d & e & f & g \\ \hline a & 0 & 0 & 1 & 1 & 0 \\ b & 0 & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 & 1 \\ e & 0 & 0 & 0 & 0 & 0 \\ f & 0 & 0 & 0 & 0 & 0 \end{array}.$$

As above, we can reduce the $(0,1)$-IM E^2 to the form

	e	f	g
a	1	1	0
b	1	1	0
c	0	0	1
d	0	0	1

.

It is interesting to see that

$E \odot_{\min} E^2 =$

	c	d	e	f	g
a	0	0	0	0	1
b	0	0	0	0	1
c	0	0	0	0	0
d	0	0	0	0	0
e	0	0	0	0	0
f	0	0	0	0	0

$= E^2 \odot_{\min} E.$

Therefore, $E \odot_{\min} E^2 = E^3 = E^2 \odot_{\min} E$. We see that the resultant $(0,1)$-IM can be reduced to

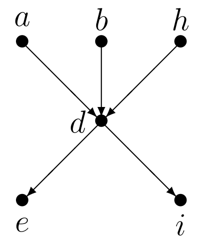
	g
a	1
b	1

.

Let us construct a graph, that is a result of operation substitution $\left[\frac{h}{c}\frac{i}{f};\frac{h}{c}\frac{i}{f}\right]$ over graph C, then we obtain the graph F with IM:

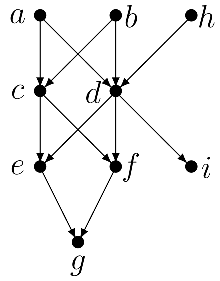
	a	b	h	d	e	i
a	0	0	0	1	0	0
b	0	0	0	1	0	0
h	0	0	0	1	0	0
d	0	0	0	0	1	1
e	0	0	0	0	0	0
i	0	0	0	0	0	0

and with the form:



In the present case, when both index sets coincide, it is suitable to use notation $\left[\frac{h}{c}\frac{i}{f}\right]$.

If we like to unite the graphs E and F, we obtain the following graph with the form



and with $(0,1)$-IM

$E \oplus_{\max} F =$

	c	d	e	f	g	i
a	1	1	0	0	0	0
b	1	1	0	0	0	0
c	0	0	1	1	0	0
d	0	0	1	1	0	1
e	0	0	0	0	1	0
f	0	0	0	0	1	0
h	0	1	0	0	0	0

.

The $(0,1)$-IM that is a result of operation termwise multiplication with sub-operation "$\max$" over $(0,1)$-IMs E and C is

$E \otimes_{\max} F =$

	d	e
a	1	0
b	1	0
d	0	1

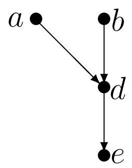
,

with sub-operation "$\min$" is

$E \otimes_{\min} F =$

	d	e
a	0	0
b	1	0
d	0	1

and it has, respectively, the graph-forms



and

In addition, we mention that the graph-representation of the $(0,1)$-IM

$E \oplus_{\min} F =$

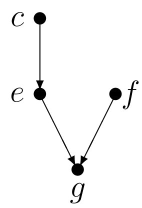
	c	d	e	f	g	i
a	0	0	0	0	0	0
b	0	1	0	0	0	0
c	0	0	0	0	0	0
d	0	0	1	0	0	0
e	0	0	0	0	0	0
f	0	0	0	0	0	0
h	0	0	0	0	0	0

and in the reduced form

$E \ominus_{\min} F =$

	d	e
b	1	0
d	0	1

i.e., the graph form is



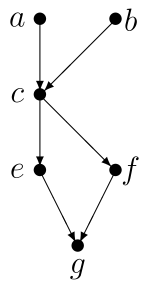
If we like to omit some vertex of a graph, we can do this, using operation "reduction". For example,

$E_{(d,d)} =$

	c	e	f	g
a	1	0	0	0
b	1	0	0	0
c	0	1	1	0
e	0	0	0	1
f	0	0	0	1

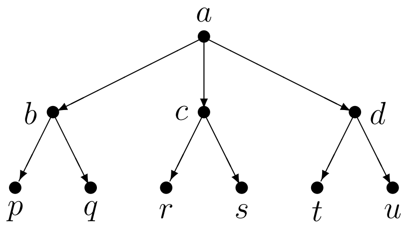
.

This $(0,1)$-IM has a graph-representation



Now, we illustrate operation $\odot_{(\circ,*)}$. Let us have graphs G and H

Let us like to add to each of the lower vertices of G new graphs with the form of H and let the vertices of these new graphs be the triples $(b,p,q),(c,r,s)$ and (d,t,u) that will replace vertices (x,y,z), respectively. Then, we obtain the graph



that has the $(0,1)$-IM-representation

$$G \odot_{(\circ,*)} \left(\left[\frac{b\,p\,q}{x\,y\,z} \right] H \oplus_{(\max)} \left[\frac{c\,r\,s}{x\,y\,z} \right] H \oplus_{(\max)} \left[\frac{d\,t\,q}{x\,y\,u} \right] H \right).$$

Of course, if, e.g., the graph C is not an oriented, then its AdIM has the form

	a	b	c	d	e	f
a	0	0	0	1	0	0
b	0	0	0	1	0	0
c	0	0	0	1	0	0
d	1	1	1	0	1	1
e	0	0	0	1	0	0
f	0	0	0	1	0	0

,

while, if we add to it, e.g., the arc (a,e), its AdIM obtains the form

	a	b	c	d	e	f
a	0	0	0	1	1	0
b	0	0	0	1	0	0
c	0	0	0	1	0	0
d	1	1	1	0	1	1
e	1	0	0	1	0	0
f	0	0	0	1	0	0

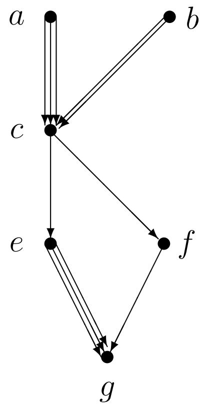
.

If the graph has a loop, e.g., (b, b), then its AdIM has the form

	a	b	c	d	e	f
a	0	0	0	1	1	0
b	0	1	0	1	0	0
c	0	0	0	1	0	0
d	1	1	1	0	1	1
e	1	0	0	1	0	0
f	0	0	0	1	0	0

.

Let us have, for example, the following multi-graph



Now, the AdIM has the form

$P =$

	c	e	f	g
a	3	0	0	0
b	2	0	0	0
c	0	1	1	0
e	0	0	0	3
f	0	0	0	1

.

By similar way we can represent a weighted graph. When we apply to AdIM P operations "Sum-row-aggregation" and "Sum-column-aggregation", we obtain the IMs (they are not $(0, 1)$-IM):

$\sigma_{sum}(P, k) =$

	c	e	f	g
k	5	1	1	4

and

$\rho_{sum}(P, l) =$

	l
a	3
b	2
c	2
e	3
f	1

that shows how many arcs enter and how many arcs leave the individual vertices, where, following [7] we define for the IM A:

Sum-row-aggregation operation

$$\rho_{sum}(A, k_0) = \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_0 & \sum_{i=1}^{m} a_{k_i,l_1} & \sum_{i=1}^{m} a_{k_i,l_2} & \dots & \sum_{i=1}^{m} a_{k_i,l_n} \end{array}$$

and

Sum-column-aggregation operation

$$\sigma_{sum}(A, l_0) = \begin{array}{c|c} & l_0 \\ \hline k_1 & \sum_{j=1}^{n} a_{k_1,l_j} \\ \vdots & \vdots \\ k_m & \sum_{j=1}^{n} a_{k_m,l_j} \end{array}.$$

7 Main results: Extended intuitionistic fuzzy graphs

Let $V = \{v_1, v_2, ..., v_n\}$ be a fixed set of vertices and let each vertex x to have a degree of existence $\alpha(x)$ and a degree of non-existence $\beta(x)$. Therefore, we can construct the Intuitionistic Fuzzy Set (IFS; see [2, 5]):

$$V^* = \{\langle x, \alpha(x), \beta(x)\rangle | x \in V\} = \{\langle v_i, \alpha(v_i), \beta(v_i)\rangle | 1 \leq i \leq n\},$$

where for each $x \in V$:

$$0 \leq \alpha(x), \beta(x), \alpha(x) + \beta(x) \leq 1.$$

Let H be a set of arcs between vertices from V. We again can juxtapose to each arc a degree of existence $\mu(x, y)$ and a degree of non-existence $\nu(x, y)$. Therefore, we can construct the new IFS

$$H^* = \{\langle\langle x, y\rangle, \mu(x, y), \nu(x, y)\rangle | x, y \in V\}$$
$$= \{\langle\langle v_i, v_j\rangle, \mu(v_i, v_j), \nu(v_i, v_j)\rangle | 1 \leq i, j \leq n\},$$

where for each $x, y \in V$:

$$0 \leq \mu(x, y), \nu(x, y), \mu(x, y) + \nu(x, y) \leq 1.$$

Now, for the graph $G = (V, H)$ we can construct the Extended Intuitionistic Fuzzy Graph (EIFG) $G^* = (V^*, H^*)$. It has the following IM-representation:

$$[V^*, V^*, \{\langle\mu(x, y), \nu(x, y)\rangle\}] = [V^*, V^*, \{\langle\mu(v_i, v_j), \nu(v_i, v_j)\rangle\}]$$

$$\equiv \begin{array}{l|ccc} & v_1, \langle\alpha(v_1), \beta(v_1)\rangle & \dots & v_n, \langle\alpha(v_n), \beta(v_n)\rangle \\ \hline v_1, \langle\alpha(v_1), \beta(v_1)\rangle & \langle\mu_{v_1,v_1}, \nu_{v_1,v_1}\rangle & \dots & \langle\mu_{v_1,v_n}, \nu_{v_1,v_n}\rangle \\ \vdots & \vdots & \dots & \vdots \\ v_i, \langle\alpha(v_i), \beta(v_i)\rangle & \langle\mu_{v_i,v_1}, \nu_{v_i,v_1}\rangle & \dots & \langle\mu_{v_i,v_n}, \nu_{v_i,v_n}\rangle \\ \vdots & \vdots & \dots & \vdots \\ v_n, \langle\alpha(v_n), \beta(v_n)\rangle & \langle\mu_{v_n,v_1}, \nu_{v_n,v_1}\rangle & \dots & \langle\mu_{v_n,v_n}, \nu_{v_n,v_n}\rangle \end{array},$$

where for every $1 \leq i, j \leq n$: $\mu_{v_i,v_j}, \nu_{v_i,v_j} \in [0,1], \mu_{v_i,v_j} + \nu_{v_i,v_j} \in [0,1]$, $\alpha(v_i), \beta(v_i) \in [0,1], \alpha(v_i) + \beta(v_i) \in [0,1]$.

Let us discuss here for simplicity only the case of oriented graph. Let us denote by $x \to y$ the fact that both vertices x and y are connected by an arc and x is higher that y. Let operation $\circ \in \{+, \max, @, \min, \times\}$.

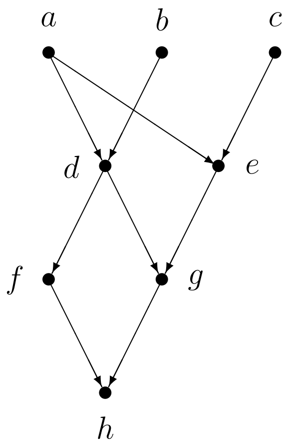
We call that the EIFG G^* is "well-top-down-(very strong, strong, middle, weak, very weak)-ordered", or shortly, "well-top-down-$\circ$-ordered", if for every two vertices v_i and v_j, such that $v_i \to v_j$, the following inequality holds:

$$\langle \alpha_i, \beta_i \rangle \circ \langle \mu_{v_i,v_j}, \nu_{v_i,v_j} \rangle \geq \langle \alpha_j, \beta_j \rangle.$$

Analogously, we call that the EIFG G^* is "well-bottom-up-(very strong, strong, middle, weak, very weak)-ordered", or shortly, "well-bottom-up-$\circ$-ordered", if for every two vertices v_i and v_j, such that $v_i \to v_j$, the following inequality holds:

$$\langle \alpha_i, \beta_i \rangle \circ \langle \mu_{v_i,v_j}, \nu_{v_i,v_j} \rangle \leq \langle \alpha_j, \beta_j \rangle.$$

We illustrate the way for IM-interpretation of the EIFGs by the following example. Let us have the EIFG G^* with the form



Its H^*-component has the following form (where, obviously, the information about the IFS V^* is included in it):

$$H^* = [\{\langle a, \frac{1}{2}, \frac{1}{3}\rangle, \langle b, \frac{1}{3}, \frac{1}{3}\rangle, \langle c, \frac{1}{3}, \frac{1}{2}\rangle, \langle d, \frac{2}{3}, \frac{1}{8}\rangle, \langle e, \frac{3}{4}, \frac{1}{4}\rangle, \langle f, \frac{1}{10}, \frac{7}{8}\rangle, \langle g, \frac{2}{5}, \frac{3}{5}\rangle,$$
$$\langle h, \frac{1}{5}, \frac{1}{5}\rangle\},\ \{\langle a, \frac{1}{2}, \frac{1}{3}\rangle, \langle b, \frac{1}{3}, \frac{1}{3}\rangle, \langle c, \frac{1}{3}, \frac{1}{2}\rangle, \langle d, \frac{2}{3}, \frac{1}{8}\rangle, \langle e, \frac{3}{4}, \frac{1}{4}\rangle, \langle f, \frac{1}{10}, \frac{7}{8}\rangle,$$
$$\langle g, \frac{2}{5}, \frac{3}{5}\rangle, \langle h, \frac{1}{5}, \frac{1}{5}\rangle\}, \{\mu_{x,y}, \nu_{x,y}\}].$$

Now, we can modify the IM to the form

$$H^* = [\{\langle a, \frac{1}{2}, \frac{1}{3}\rangle, \langle b, \frac{1}{3}, \frac{1}{3}\rangle, \langle c, \frac{1}{3}, \frac{1}{2}\rangle, \langle d, \frac{2}{3}, \frac{1}{8}\rangle, \langle e, \frac{3}{4}, \frac{1}{4}\rangle, \langle f, \frac{1}{10}, \frac{7}{8}\rangle, \langle g, \frac{2}{5}, \frac{3}{5}\rangle\},$$
$$\{\langle d, \frac{2}{3}, \frac{1}{8}\rangle, \langle e, \frac{3}{4}, \frac{1}{4}\rangle, \langle f, \frac{1}{10}, \frac{7}{8}\rangle, \langle g, \frac{2}{5}, \frac{3}{5}\rangle, \langle h, \frac{1}{5}, \frac{1}{5}\rangle\}, \{\mu_{x,y}, \nu_{x,y}\}].$$

The form of the new IM is

	$d, \langle \frac{2}{3}, \frac{1}{8} \rangle$	$e, \langle \frac{3}{4}, \frac{1}{4} \rangle$	$f, \langle \frac{1}{10}, \frac{7}{8} \rangle$	$g, \langle \frac{2}{5}, \frac{3}{5} \rangle$	$h, \langle \frac{1}{5}, \frac{1}{5} \rangle$
$a, \langle \frac{1}{2}, \frac{1}{3} \rangle$	$\langle \frac{3}{4}, \frac{1}{5} \rangle$	$\langle \frac{1}{2}, \frac{1}{4} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$b, \langle \frac{1}{3}, \frac{1}{3} \rangle$	$\langle \frac{2}{3}, 0 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$c, \langle \frac{1}{3}, \frac{1}{2} \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{1}{5}, \frac{2}{5} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$d, \langle \frac{2}{3}, \frac{1}{8} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{1}{5}, \frac{2}{5} \rangle$	$\langle \frac{3}{4}, \frac{1}{5} \rangle$	$\langle 0, 1 \rangle$
$e, \langle \frac{3}{4}, \frac{1}{4} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{2}{3}, \frac{1}{6} \rangle$	$\langle 0, 1 \rangle$
$f, \langle \frac{1}{10}, \frac{7}{8} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{1}{3}, \frac{1}{4} \rangle$
$g, \langle \frac{2}{5}, \frac{3}{5} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{3}{5}, \frac{1}{5} \rangle$

Let the EIFIM $A = [K^*, L^*, \{\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle\}]$ be given.

Let for $i = 1, 2, 3 : \rho_i, \sigma_i, \rho_i + \sigma_i \in [0, 1]$ be fixed numbers.

In [2, 5], several level operators were defined. One of them, for a given IFS

$$X = \{\langle x, \mu_X(x), \nu_X(x) \rangle | x \in E\}$$

is

$$N_{\alpha,\beta}(X) = \{\langle x, \mu_X(x), \nu_X(x) \rangle | x \in E \ \& \ \mu_X(x) \geq \alpha \ \& \ \nu_X(x) \leq \beta\},$$

where $\alpha, \beta \in [0, 1]$ are fixed and $\alpha + \beta \leq 1$.

Here, its analogues are intoduced. They are three: $N^1_{\rho_1,\sigma_1}, N^2_{\rho_2,\sigma_2}, N^3_{\rho_3,\sigma_3}$ and affect the K-, L-indices and $\langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle$-elements, respectively. The three operators can be applied over an EIFIM A either sequentially, or simultaneously. In the first case, their forms are

$$N^1_{\rho_1,\sigma_1}(A) = [N_{\rho_1,\sigma_1}(K^*), L^*, \{\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle\}],$$

where

$$\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle = \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle$$

only for $\langle k_i, \alpha^k_i, \beta^k_i \rangle \in N_{\rho_1,\sigma_1}(K^*)$ and for each $\langle l_j, \alpha^l_j, \beta^l_j \rangle \in L^*$;

$$N^2_{\rho_2,\sigma_2}(A) = [K^*, N_{\rho_2,\sigma_2}(L^*), \{\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle\}],$$

where

$$\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle = \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle$$

for each $\langle k_i, \alpha^k_i, \beta^k_i \rangle \in K^*$ and only for $\langle l_j, \alpha^l_j, \beta^l_j \rangle \in N_{\rho_2,\sigma_2}(L^*)$;

$$N^3_{\rho_3,\sigma_3}(A) = [K^*, L^*, \{\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle\}],$$

where

$$\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle = \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } \mu_{k_i,l_j} \geq \rho_3 \ \& \ \nu_{k_i,l_j} \leq \sigma_3 \\ \\ \langle 0, 1 \rangle, & \text{otherwise} \end{cases},$$

In the second case, their form is

$$(N^1_{\rho_1,\sigma_1}, N^2_{\rho_2,\sigma_2}, N^3_{\rho_3,\sigma_3})(A) = [N_{\rho_1,\sigma_1}(K^*), N_{\rho_2,\sigma_2}(L^*), \{\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle\}],$$

where

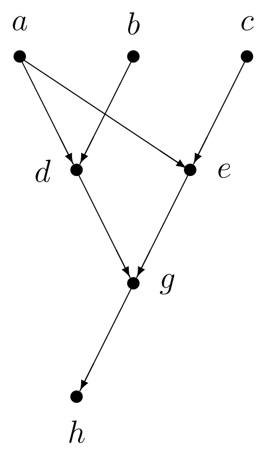
$$\langle \varphi_{k_i,l_j}, \psi_{k_i,l_j} \rangle = \begin{cases} \langle \mu_{k_i,l_j}, \nu_{k_i,l_j} \rangle, & \text{if } \langle k_i, \alpha_i^k, \beta_i^k \rangle \in N_{\rho_1,\sigma_1}(K^*) \\ & \text{and } \langle l_j, \alpha_j^l, \beta_j^l \rangle \in N_{\rho_2,\sigma_2}(L^*) \\ & \text{and } \mu_{k_i,l_j} \geq \rho_3 \;\&\; \nu_{k_i,l_j} \leq \sigma_3 \\ & \\ \langle 0, 1 \rangle, & \text{if } \langle k_i, \alpha_i^k, \beta_i^k \rangle \in N_{\rho_1,\sigma_1}(K^*) \\ & \text{and } \langle l_j, \alpha_j^l, \beta_j^l \rangle \in N_{\rho_2,\sigma_2}(L^*) \\ & \text{and } \mu_{k_i,l_j} < \rho_3 \vee \nu_{k_i,l_j} > \sigma_3 \end{cases},$$

Now, we can apply one or more of the level-operators $N^1_{\rho_1,\sigma_1}, N^2_{\rho_2,\sigma_2}, N^3_{\rho_3,\sigma_3}$ and in a result, the form of the graph will be changed. It is important to mention that in the present case (when the two index sets coincide), the first two level operators must have equal parameters and, therefore, if some vertex has to be omitted from one of both index sets, it will be omitted from the other index set, too. For example, if we apply operator $N^1_{\frac{1}{5},\frac{1}{4}}$ over G^*, we obtain

$N^1_{\frac{1}{5},\frac{1}{4}}(G^*) =$

	$d, \langle \frac{2}{3}, \frac{1}{8} \rangle$	$e, \langle \frac{3}{4}, \frac{1}{4} \rangle$	$g, \langle \frac{2}{5}, \frac{3}{5} \rangle$	$h, \langle \frac{1}{5}, \frac{1}{5} \rangle$
$a, \langle \frac{1}{2}, \frac{1}{3} \rangle$	$\langle \frac{3}{4}, \frac{1}{5} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$b, \langle \frac{1}{3}, \frac{1}{3} \rangle$	$\langle \frac{2}{3}, 0 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$c, \langle \frac{1}{3}, \frac{1}{2} \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{1}{5}, \frac{2}{5} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$d, \langle \frac{2}{3}, \frac{1}{8} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{3}{4}, \frac{1}{5} \rangle$	$\langle 0, 1 \rangle$
$e, \langle \frac{3}{4}, \frac{1}{4} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{2}{3}, \frac{1}{6} \rangle$	$\langle 0, 1 \rangle$
$g, \langle \frac{2}{5}, \frac{3}{5} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{3}{5}, \frac{1}{5} \rangle$

and the new graph has the form

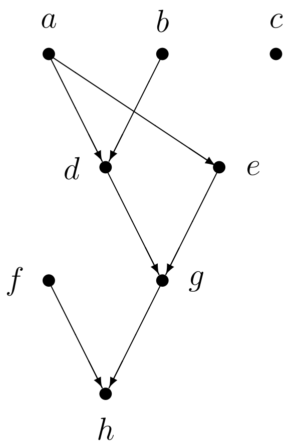


On the other hand, if we can apply, e.g., operator $N^3_{\frac{1}{4},\frac{1}{3}}$ over G^*, we obtain

$N^3_{\frac{1}{4},\frac{1}{3}}(G^*) =$

	$d, \langle \frac{2}{3}, \frac{1}{8} \rangle$	$e, \langle \frac{3}{4}, \frac{1}{4} \rangle$	$f, \langle \frac{1}{10}, \frac{7}{8} \rangle$	$g, \langle \frac{2}{5}, \frac{3}{5} \rangle$	$h, \langle \frac{1}{5}, \frac{1}{5} \rangle$
$a, \langle \frac{1}{2}, \frac{1}{3} \rangle$	$\langle \frac{3}{4}, \frac{1}{5} \rangle$	$\langle \frac{1}{2}, \frac{1}{4} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$b, \langle \frac{1}{3}, \frac{1}{3} \rangle$	$\langle \frac{2}{3}, 0 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$c, \langle \frac{1}{3}, \frac{1}{2} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$
$d, \langle \frac{2}{3}, \frac{1}{8} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{3}{4}, \frac{1}{5} \rangle$	$\langle 0, 1 \rangle$
$e, \langle \frac{3}{4}, \frac{1}{4} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{2}{3}, \frac{1}{6} \rangle$	$\langle 0, 1 \rangle$
$f, \langle \frac{1}{10}, \frac{7}{8} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{1}{3}, \frac{1}{4} \rangle$
$g, \langle \frac{2}{5}, \frac{3}{5} \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle 0, 1 \rangle$	$\langle \frac{3}{5}, \frac{1}{5} \rangle$

and the new graph has the form



and the IM G^* can be reduced to

	$d, \langle\frac{2}{3},\frac{1}{8}\rangle$	$e, \langle\frac{3}{4},\frac{1}{4}\rangle$	$g, \langle\frac{2}{5},\frac{3}{5}\rangle$	$h, \langle\frac{1}{5},\frac{1}{5}\rangle$
$a, \langle\frac{1}{2},\frac{1}{3}\rangle$	$\langle\frac{3}{4},\frac{1}{5}\rangle$	$\langle\frac{1}{2},\frac{1}{4}\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$b, \langle\frac{1}{3},\frac{1}{3}\rangle$	$\langle\frac{2}{3},0\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$c, \langle\frac{1}{3},\frac{1}{2}\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$d, \langle\frac{2}{3},\frac{1}{8}\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle\frac{3}{4},\frac{1}{5}\rangle$	$\langle 0,1\rangle$
$e, \langle\frac{3}{4},\frac{1}{4}\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle\frac{2}{3},\frac{1}{6}\rangle$	$\langle 0,1\rangle$
$f, \langle\frac{1}{10},\frac{7}{8}\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle\frac{1}{3},\frac{1}{4}\rangle$
$g, \langle\frac{2}{5},\frac{3}{5}\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle\frac{3}{5},\frac{1}{5}\rangle$

Obviously, vertex f remain in the first index set, because an arc goes out of it. On the other hand, vertex c here is an isolated one.

References

[1] Atanassov, K. Generalized index matrices, *Comptes rendus de l'Academie Bulgare des Sciences*, Vol. 40, 1987, No. 11, 15–18.

[2] Atanassov, K. *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer, Heidelberg, 1999.

[3] Atanassov, K. On index matrices, Part 1: Standard cases. *Advanced Studies in Contemporary Mathematics*, Vol. 20, 2010, No. 2, 291–302.

[4] Atanassov, K. On index matrices, Part 2: Intuitionistic fuzzy case. *Proceedings of the Jangjeon Mathematical Society*, Vol. 13, 2010, No. 2, 121–126.

[5] Atanassov, K. *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.

[6] Atanassov, K. Extended intuitionistic fuzzy index matrices, *Notes on Intuitionistic Fuzzy Sets*, Vol. 19, 2013, No. 4, 27–41.

[7] Atanassov, K., E. Sotirova, V. Bureva. On index matrices. Part 4: New operations over index matrices. *Advanced Studies in Contemporary Mathematics*, Vol. 23, 2013, No. 3, 547–552.

[8] Christofides, N. *Graph Theory*. Academic press, new York, 1975.

[9] Shannon, A., K. Atanassov. A first step to a theory of the intuitionistic fuzzy graphs, *Proc. of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.)*, Sofia, Sept. 28–30, 1994, 59–61.

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