Notes on Intuitionistic Fuzzy Sets
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Intuitionistic fuzzy sets in group decision making – A novel approach

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Abstract: We use the natural properties of intuitionistic fuzzy sets (IFSs for short) to represent the pros, cons, and lack of knowledge concerning different options/alternatives, aiding in decision making, particularly in group decision making. We present a novel approach. We do not compare options/alternatives in pairs, we do not use distances. The approach is transparent and easily understandable for decision makers. The novel method points out the best option by ranking them.

Keywords: Intuitionistic fuzzy sets, Decision making, Group decision making, Ranking.
2020 Mathematics Subject Classification: 03E72.

1 Introduction

Decision making and group decision making are inherent, everyday human activities. To make a decision, we should consider possible alternatives / options and pick up the best of them. We have previously considered group decision making in [12, 20, 21, 24–27, 29, 30, 32, 37]. Here, we propose a novel approach. We describe the alternatives / options using intuitionistic fuzzy sets (IFSs, for short) – the apparatus proposed by Atanassov (see [1–4]). To be more precise, we use intuitionistic fuzzy elements representing the alternatives/options. We will call the alternatives / options *intuitionistic fuzzy alternatives/options*. It is a very natural approach as an intuitionistic fuzzy element is described by three terms: membership value, non-membership value, and hesitation margin (cf. Section 2). The three terms mean respectively: advantages, disadvantages, and lack of knowledge concerning an option. It is worth noting that it is a very natural way of describing options – it is the same approach what human beings do when making decisions. An important advantage of the IFSs is an inherent possibility to take a lack of knowledge into account by using the so-called hesitation margin or intuitionistic fuzzy index.

Thus we adhere to the highly convenient tool known as intuitionistic fuzzy sets (IFSs) for decision making. However, we no longer employ the method of comparing options in pairs as outlined in some previous papers. Consider we have 10 options, each option is described by three terms. It would not be practical for a decision maker to undertake pairwise comparisons of all options. It is better to evaluate pros, cons, and lack of knowledge of each option individually, especially when an option is characterized by multiple attributes.

We also do not calculate distances between options, as the measure is not bijective (distances to different options can be the same). Instead, we rank the options, pointing out the best one.

The approach is transparent, easy to understand and explain, and aligned with human thinking.

2 A brief introduction to IFSs

One of the possible generalizations of a fuzzy set in X (Zadeh, [54]) given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle | x \in X\}, \tag{1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A^{'}$, is an IFS (Atanassov [1,3,4]) A is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}, \tag{2}$$

where: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x), \nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively. (See Szmidt and Baldwin [17] for assigning memberships and non-memberships for IFSs from data.)

Obviously, each fuzzy set may be represented by the following IFS:

$$A = \{\langle x, \mu_{A'}(x), 1 - \mu_{A'}(x)\rangle | x \in X\}. \tag{4}$$

An additional concept for each IFS in X, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanassov [3])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{5}$$

a *hesitation margin* of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov, [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [18, 19, 22, 29, 31], entropy (Szmidt and Kacprzyk [23, 28, 33]), similarity (Szmidt and Kacprzyk [34, 38, 49]) for the IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks (Szmidt [14]).

The hesitation margin turns out to be relevant for applications – in image processing (cf. Bustince et al. [7]), the classification of imbalanced and overlapping classes (cf. Szmidt and Kukier [51–53]), the classification applying intuitionistic fuzzy trees (cf. Bujnowski [6]), attribute selection [46, 47], ranking of alternatives [48], multiagent decisions, negotiations, voting, group decision making, etc. (cf. [5,9,12,20,21,24–26,29,30,32,37]), genetic algorithms [13]. Sometimes the concept of the hesitation margin is just indispensable, for example, for a proper definition of the Hausdorff distance [41], when ranking the alternatives [48], [50], calculating Pearson's, Spearman, Kendall rank correlations [36, 39, 40, 42], and seeing IFSs like different ones from interval-valued fuzzy sets [44, 45].

3 Two term, and three term representation of the IFSs as a basis for calculating distances

Almost all important measures used in the area of IFSs, as well as in other areas, are non-linear. For example distances which play a decisive role in many models, and are a basis for many other measures, are usually non-linear. So it seems natural to expect that quite different qualitative results will be obtained while using the two term representation of the IFSs than the results obtained while using the three term representation of the IFSs. Szmidt and Kacprzyk [22, 31], Szmidt and Baldwin [15, 16] discuss the results obtained for the most often used distances when the two and the three term representations of the IFS are used. Examples of the distances between any two IFSs A and B in $X = \{x_{1,} x_2, \ldots, x_n\}$ while using the three term representation (Szmidt and Kacprzyk [22], Szmidt and Baldwin [15, 16]) may be as follows:

- the normalized Hamming distance:

$$l_{IFS}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \tag{6}$$

- the normalized Euclidean distance:

$$e_{IFS}(A, B) = (\frac{1}{2n} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2)^{\frac{1}{2}} \tag{7}$$

The values of both distances are from the interval $[0, 1]$.

The corresponding distances to the above ones while using the two term representation of the IFSs are:

- the normalized Hamming distance:

$$l^{'}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|) \tag{8}$$

- the normalized Euclidean distance:

$$q^{'}(A,B) = (\frac{1}{2n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2)^{\frac{1}{2}} \tag{9}$$

Both the two term distances and three term distances [14,22] are correct from the mathematical point of view, i.e., all the needed properties are fulfilled, but practical problem solutions are more intuitively appealing while using the three term representations (cf. e.g., Szmidt and Kacprzyk [37,43]).

Even a more convincing argument for using the three term representation of the IFSs occurs in the case of the Hausdorff distance based on the Hamming metric [41].

Consider in detail the meaning of the terms in the formulas (6)–(9) in the context of the distances. When we speak of a distance in real life we often think about a distance between two towns or two objects. We express the distance, e.g., in kilometers/metes and are satisfied when knowing that a distance is, e.g., 100 km. When asking about a distance we are less interested in the direction. If we assume that a goal of our trip should be not more farther away than 100 km, we consider a circle with radius of 100 km. Any direction is acceptable.

However, when considering distances in different mathematical models, the distance components usually have their specific meaning.

Example 1. Imagine an element $x_0(0.5, 0.2, 0.3)$ and two other elements: $x_1(0.5, 0.1, 0.4)$ and $x_2(0.5, 0.3, 0.2)$. It is easy to verify that from (7) we obtain "the same" results, namely: *the normalized Euclidean distance:*

$$e_{IFS}(x_0, x_1) = 0.5((0.5 - 0.5)^2 + (0.2 - 0.1)^2 + (0.3 - 0.4)^2)^{\frac{1}{2}} = 0.5(0.02)^{\frac{1}{2}} \tag{10}$$

and

$$e_{IFS}(x_0, x_2) = 0.5((0.5 - 0.5)^2 + (0.2 - 0.3)^2 + (0.3 - 0.2)^2 = 0.5(0.02)^{\frac{1}{2}}. \tag{11}$$

The same situation occurs when we use (6), (8) or (9). Namely, we are not able to give a difference between a distance x_0 and x_1 (10) on the one hand and between x_0 and x_2 (11) on the other hand although x_1 and x_2 are different. So having different options, a distance is not always able to show it.

It is easy to notice that not only the results of (10) and (11) are the same but also all the components in the brackets are the same, i.e., the differences between the membership values, non-membership values, and hesitation margins are the same (i.e., $0, 0.1, 0.1$) in both cases. So even if we consider them as a triplet instead of a single value from (10) and/or (11), we are not able to make any differences. The same situation occurs for our examples when using the Hamming distance, the same is when we use the formulas with two parameters only (the drawbacks of the approach were stressed in our previous papers).

The problem with "the same distances", e.g., from the ideal element $(1, 0, 0)$ representing the best option) for quite different intuitionistic fuzzy elements representing different options, does not boil down to the fact that we get the same value from the formulas. It is a problem but not the only one.

When we pay attention to the components of the formulas (6), (7), or to be more precise, to the meaning of the components, we see that we aggregate quite different quantities. The membership values hold a meaning that contrasts significantly with that of the non-membership values and the hesitation margins, each bearing its own distinct significance in decision-making. Nevertheless, we aggregate all of these factors. An argument often presented is that the approach satisfies the properties of a distance. However, is this justification valid when considering decision-making? What implications does it hold for the interpretation of the results obtained?

Consider negotiations. Let the first opinion of an expert is like in the Example 1, i.e., $x_0(0.5, 0.2, 0.3)$, and two other elements representing two possible change of opinion are: $x_1(0.5, 0.1, 0.4)$ and $x_2(0.5, 0.3, 0.2)$. The question is about which opinion is closer to the opinion expressed by x_0.

As it was shown by (10) and (11), distances do not give us a satisfactory answer. Also the components of (10) and (11) examined separately, do not do it, too (they are the same). However, opinions $x_1(0.5, 0.1, 0.4)$ and $x_2(0.5, 0.3, 0.2)$ are different.

To be more precise,

$$\begin{aligned}\Delta\mu_{0,1} &= \mu(x_0) - \mu(x_1) = 0.5 - 0.5 = 0\\ \Delta\nu_{0,1} &= \nu(x_0) - \nu(x_1) = 0.2 - 0.1 = 0.1\\ \Delta\pi_{0,1} &= \pi(x_0) - \pi(x_1) = 0.3 - 0.4 = -0.1\end{aligned}$$

The above results show that the change of the membership did not occur ($\Delta\mu_{0,1} = 0$), the non-membership value decreased ($\Delta\nu_{0,1} > 0$), and the hesitation margin increased ($\Delta\pi_{0,1} < 0$).

In the case of the second change (from x_0 to x_2) we have

$$\begin{aligned}\Delta\mu_{0,2} &= \mu(x_0) - \mu(x_2) = 0.5 - 0.5 = 0\\ \Delta\nu_{0,2} &= \nu(x_0) - \nu(x_2) = 0.2 - 0.3 = -0.1\\ \Delta\pi_{0,2} &= \pi(x_0) - \pi(x_2) = 0.3 - 0.2 = 0.1\end{aligned}$$

From the above equations we can see that in the case of x_2 the change of the membership did not occur ($\Delta\mu_{0,2} = 0$), non-membership value increased ($\Delta\nu_{0,1} < 0$), and the hesitation margin decreased ($\Delta\pi_{0,1} > 0$).

In other words, the changes went into different directions as far as the non-membership values, and the hesitation margins are concerned. This fact is not visible when considering distances.

Given the aforementioned points, we chose not to evaluate the options using a method that relies on distance principles, such as, e.g., the TOPSIS method, which aims to identify the optimal solution that is closest to the positive-ideal solution while being farthest from the negative-ideal one.

Instead, we employ a different ranking method, elaborated upon in detail in Szmidt and Kacprzyk [35, 48, 50]. Although the method has primarily been utilized successfully for attribute selection in [48], it appears to serve as a convenient and transparent tool for decision-making as well.

Having in mind the distinct interpretation of the three terms describing an element belonging to an IFS we can characterize the options which are described by different attributes. As the values of each attribute $A_k(x_i)$, $k = 1, \ldots, K$ for different options (x_i) may be different, an option can be described by average values of memberships (12), non-memberships (13), and hesitation margins (14), that are obtained by the weight operator W (cf. [4]), i.e.,

$$\overline{\mu}(x_i) = \frac{1}{K} \sum_{l=1}^{K} \mu_{A_l}(x_i), \tag{12}$$

$$\overline{\nu}(x_i) = \frac{1}{K} \sum_{l=1}^{K} \nu_{A_l}(x_i), \tag{13}$$

$$\overline{\pi}(x_i) = \frac{1}{K} \sum_{l=1}^{K} \pi_{A_l}(x_i), \tag{14}$$

where K is a number of the attributes.

Description of the options by (12)– (14) makes it possible to indicate the best option. To do it, we apply the procedure of ranking the options (cf. [35, 48, 50]) which is reminded briefly below.

3.1 The procedure of ranking intuitionistic fuzzy options

The method of ranking *intuitionistic fuzzy alternatives* we use here, consists of two steps. In the first step, Definition 1 is used.

Definition 1 ([2, 3]). *For two intuitionistic fuzzy alternatives $x_1(\mu_1, \nu_1)$ and $x_2(\mu_2, \nu_2)$, $x_1 \leq x_2$ if*

$$\mu_1 \leq \mu_2 \quad \textit{and} \quad \nu_1 \geq \nu_2. \tag{15}$$

If the conditions of Definition 1 are fulfilled, then we obtain an order which is well justified and acceptable without any doubts.

When it is not possible to use Definition 1 because the assumptions are not fulfilled, an approach that ranks higher the alternatives with bigger membership values and lower hesitation values is applied. To be more precise, the following measure R (16) (cf. [35, 48, 50]) is applied:

$$R(x) = 0.5(1 + \pi_x) l_{IFS}(M, x), \tag{16}$$

where $l_{IFS}(M, x)$ is the Hamming distance (6) x from ideal positive alternative $M(1, 0, 0)$. In result we obtain:

$$l_{IFS}(M, x) = \frac{1}{2}(|1 - \mu_x| + |0 - \nu_x| + |0 - \pi_x|) = 1 - \mu_x. \tag{17}$$

Finally, (16) is given as:

$$R(x) = 0.5(1 + \pi_x)(1 - \mu_x). \tag{18}$$

Equation (18) expresses the "quality" of an alternative. The lower the value of $R(x)$, (18), the better the alternative. From (18) we conclude that for the best alternatives the amount of positive information included μ_x is as big as possible, and the hesitation margin π_x is as small as possible.

The above procedure was examined and compared with other approaches [48, 50]). Now we will demonstrate how the procedure can be applied in group decision making.

Example 2. Assume there is one expert considering three alternatives. If the alternatives are described by more than one attribute, we use (12)–(14). In effect, we obtain the alternatives described as $x_i(\mu, \nu, \pi)$, $i = 1, 2, 3$:

$x_1(0.5, 0.1, 0.4)$
$x_2(0.6, 0.2, 0.2)$
$x_3(0.5, 0.2, 0.3)$

Comparing x_1 and x_2 we can not use Definition 1 because the assumptions are not fulfilled ($\mu_{x_1} < \mu_{x_2}$ and $\nu_{x_1} < \nu_{x_2}$).

From (18) we obtain that x_2 is a better alternative as $\mu_{x_2} > \mu_{x_1}$ and $\pi_{x_2} < \pi_{x_1}$.

Next we compare x_2 and x_3. We see that the non-membership values of x_2 and x_3 are the same so we can use Definition 1. It means that x_2 is again a better option ($\mu_{x_2} > \mu_{x_3}$). In result the expert will point out x_2 as the best option.

When we have n experts, each of them points out an option which is the best in his/her opinion. Having n options to consider, obtained from n experts, we apply the approach described in Example 2.

If we wish to point out not only the best option but to order all of them, we remove the best option found as above, and look for the next best option. Repeating the algorithm we can order the options from the best one to the worst one.

An interesting area of research for extending the new model proposed in this paper would be, for instance, the use of some elements of rough sets theory as a foundation framework for a group decision making model of the class considered, along the lines of Nurmi and Kacprzyk [10] and Nurmi, Kacprzyk and Fedrizzi [11]. Moreover, a larger class of OWA based aggregation operators, cf. Kacprzyk, Yager and Merigo [8], would provide a deep insight and new solution concepts.

4 Conclusions

We have presented a novel method of decision making, i.e., pointing out the best option considered. The options are expressed via intuitionistic fuzzy elements. It means that we point out their pros, cons, and hesitation margins (lack of knowledge concerning pros, and cons). We do not compare the options in pairs. The options can be characterized by several attributes. The best decision (option) can be point out by one decision maker or by a group of them. The considered options can be ranked from the best one to the worst one.

The approach is transparent, and easy to understand by decision makers. It is simple yet powerful.

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