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OPTIMAL SELECTION OF THE MOST SUITABLE METHOD OUT OF n ALTERNATIVES : AN INTUITIONISTIC FUZZY APPROACH

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Abstract

We consider a problem of selecting the best out of n alternative actions on the basis of performance values or ratings corresponding to m factors (or criteria). Our assumption is that for each alternative the performance values correspond to an intuitionistic fuzzy set of the set F of all factors. An algorithm for the method is presented and a hypothetical case-study is made.

keywords : Intuitionistic fuzzy set (IFS), performance value, performance, performance matrix, super performance value, worst performance value, dominating matrix, relative score.

1 INTRODUCTION

We are aware of the potential applications of fuzzy set theory in wide varities of fields. In real life situations, data available are not always crisp. Rather, they are imprecise or vague data or sometimes linguistic data on the basis of which we have to find the solutions of many problems. Even data available are not fuzzy always but intuitionistic fuzzy [8]. There are some situations where instead of fuzzy set theory, intuitionistic fuzzy set theory introduced by Atanassov [1,2,3,4,5,6] is more appropriate to deal with. One such situation has been studied in [8]. Mathematically too, fuzzy sets are intuitionistic fuzzy sets but converse is not

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necessarily true [1]. The problems which are dealt with fuzzy sets can be well dealt with intuitionistic fuzzy sets(IFSs). In this sense the IFS theory is another major tool to deal with vagueness, and it is a generalization of fuzzy set theory[17]. It has been cultured in [9] that vague sets [12] are IFSs. In the present paper we consider a multi-criteria based decision making problem where data are intuitionistic fuzzy, and suggest a method for the solution. Decision making problems have been already studied by Szmidt and Kacprzyk [16] using IFSs (besides a number of works of Kacprzyk in this area using fuzzy set theory). Our method here is named by "Distance Method".

2 PRELIMINARIES

We recollect some basic preliminaries, mainly the work of Atanassov [1].

Definition 2.1

Let a set E is fixed. An intuitionistic fuzzy set or IFS A of E is an object having the form

$A = \{ <x, \mu_A(x), \nu_A(x)> \mid x \in E \}$

where the function $\mu_A : E \rightarrow [0,1]$ and $\nu_A : E \rightarrow [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2

If A and B are two IFSs of the set E, then

$A \subset B$ iff $\forall x \in E$, $[\ \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)\ ]$

$A \subset B$ iff $B \supset A$

$A = B$ iff $\forall\ x \in E$, $[\ \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)\ ]$

$\bar{A} = \{ <x, \nu_A(x), \mu_A(x)> \mid x \in E \}$

$A \cap B = \{ <x\ ,\ \min(\mu_A(x)\ ,\ \mu_B(x))\ ,\ \max(\nu_A(x), \nu_B(x))> \mid x \in E \}$

$A \cup B = \{ <x\ ,\ \max(\mu_A(x)\ ,\ \mu_B(x))\ ,\ \min(\nu_A(x), \nu_B(x))> \mid x \in E \}$

$A + B = \{ <x\ ,\ \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)\ ,\ \nu_A(x).\nu_B(x)> \mid x \in E \}$

$A\ .\ B = \{ <x\ ,\ \mu_A(x).\mu_B(x)\ ,\ \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)> \mid x \in E \}$

$\Box A = \{ <x, \mu_A(x), 1 - \mu_A(x)> \mid x \in E \}$

$\Diamond A = \{ <x, 1 - \nu_A(x), \nu_A(x)> \mid x \in E \}$

$C(A) = \{\langle x,K,L\rangle \mid x\in E \}$
where $K= \max\limits_{x\in E} \mu_A(x)$, $L= \min\limits_{x\in E} \nu_A(x)$
$I(A) = \{\langle x,k,l\rangle \mid x\in E \}$
where $k= \min\limits_{x\in E} \mu_A(x)$, $l= \max\limits_{x\in E} \nu_A(x)$
Obviously every fuzzy set has the form
$\{\langle x , \mu_A(x) , \mu_{A^c}(x) \rangle \mid x\in E \}$

Definition 2.3[10]

Let $X=\{ x_1,x_2,\cdots,x_n \}$. The Hamming distance between two IFSs A and B is defined by

$$d_H(A,B) = \sum_{i=1}^{n} \frac{|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|}{2}$$

Clearly, $0\leq d_H(A,B) \leq N$.

Definition 2.4[10]

The euclidean distance between two intuitionistic fuzzy sets A and B in X is defined by

$$d_E(A,B) = \left[\sum_{i=1}^{n} \frac{[\mu_A(x_i)-\mu_B(x_i)]^2+[\nu_A(x_i)-\nu_B(x_i)]^2}{2}\right]^{\frac{1}{2}}$$

3 OPTIMAL SELECTION : AN IF APPROACH

Consider the problems of selecting most suitable method or action out of n number of alternatives on the basis of m number of factors (or criteria). Let the n methods are respectively $M_1,M_2,\cdots,M_n$ and each method depends upon all of the m factors $F_1,F_2,\cdots, F_m$. Our assumption is that corresponding to the factor F_j (j=1,2,.., m) the rating or performance value of the method M_i (i=1,2,..,n) is a paired value $p_{ij} = (\mu_{ij}, \nu_{ij})$ such that for a fixed i the values p_{ij} (j=1,2,..m) correspond to an intuitionistic fuzzy set of the set F of all factors. Thus, the performance values could be arranged in the form of a matrix P, called by Performance Matrix, as shown below:

$$P = \begin{array}{c|cccc|} & F_1 & F_2 & \cdots & F_m \\ \hline M_1 & p_{11} & p_{12} & \cdots & p_{1m} \\ M_2 & p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & & & & \\ M_n & p_{n1} & p_{n2} & \cdots & p_{nm} \\ \hline \end{array}$$

A performance value $p_{ij}= (\mu_{ij},\nu_{ij})$ consists of the membership value μ_{ij} and the non-membership value ν_{ij}. Clearly, the hesitation part is $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$. We use the following notations :

$\mu(p_{ij}) = \mu_{ij}$, $\nu(p_{ij}) = \nu_{ij}$, $\pi(p_{ij}) = \pi_{ij}$.

We assume that if the performance values are more, the efficiency of the corresponding method is also more.

Our problem is to select the most suitable method i.e., the method which dominates each of the rest methods. Since the data are not crisp but intuitionistic fuzzy, the selection is not straightforward. We present here a method, which we call by "Distance Method", based on two basic values:

(i) corresponding to how many factors, a method M_i dominates the others ?

and (ii) for how many factors, a method M_i is dominated by the others ?

For the concept of "dominance" used here, we need the following definitions.

Definition 3.1

A performance value or rating s is called the super performance value if $\mu(s)$=1. In this case, the non-membership value $\nu(s)$ is zero and there will be no hesitation i.e., $\pi(s) = 0$

Definition 3.2

A performance value or rating w is said to be the worst performance value if $\nu(w) = 1$. In this case, the membership value $\mu(w)$ is zero and hesitation is nil.

Definition 3.3

A performance value v_1 is said to dominate a performance value v_2 if

$$d(v_1,s) \leq d(v_2,s)$$

i.e., in another terminology, if

$$d(v_1,w) \geq d(v_2,w)$$

where d is some distance function, viz. Hamming distance [10], Eucledian distance [10].

Definition 3.4

For a given performance matrix P, the dominating matrix D = (d_{ij}) is a square (n×n) matrix such that an element d_{ij} indicates the number of factors for which the method M_i dominates the method M_j on the basis of performance values. Thus the dominating matrix D is given by:

$$D = \begin{array}{c|cccc|c} & M_1 & M_2 & \cdots & M_n & \\ \hline M_1 & d_{11} & d_{12} & \cdots & d_{1n} & r_1 \\ M_2 & d_{21} & d_{22} & \cdots & d_{2n} & r_2 \\ \vdots & & & & & \\ M_n & d_{n1} & d_{n2} & \cdots & d_{nn} & r_n \\ \hline & c_1 & c_2 & \cdots & c_n & \end{array}$$

where r_i and c_i are the row-sum and column-sum respectively $\forall$i = 1,2,..,n. Clearly $r_i \geq$m , $c_i \geq$m because d_{ii}=m $\forall$i. The dominating matrix reveals that r_i is the total number of times (on the basis of all factors) M_i dominates all the methods and c_i is the total number of times M_i is dominated by all the methods.

The Method "Distance Method" :

From the information available in the performance matrix P, we construct the dominating matrix D. Now, calculate the relative-score s_i for each M_i using the relation

$$s_i = r_i - c_i \text{ , i=1,2,3,..,n.}$$

The optimal selection will be that method for which relative score is maximum i.e., which dominates most and is dominated least. Thus, if $s_k = \max_i s_i$, M_K is to be chosen as most suitable method. In case a tie occurs, select M_k for which the total hesitation $\pi_k(= \sum_{j=1}^{m} \pi_{kj})$ is greatest.

Algorithm :

1. Compute the performance matrix from the available information whose elements are intuitionistic fuzzy.
2. Construct the dominating matrix.
3. Calculate relative-scores $s_i = r_i - c_i$, $\forall$i=1,2,..n.
4. Find k for which $s_k = \max_i s_i$.
5. If k has more than one value, choose that one corresponding to which $\pi_k (= \sum_{j=1}^{m} \pi_{kj})$ is greatest.
6. The optimal selection is M_k.
7. Stop.

A case-study :

We make a case-study with hypothetical data. There are three alternatives M_1,M_2,M_3 and there are three factors F_1,F_2 and F_3. The performance matrix is

$P =$

	F_1	F_2	F_3
M_1	(0.6,0.2)	(0.7,0.3)	(0.5,0.2)
M_2	(0.4,0.3)	(0.8,0.1)	(0.4,0.3)
M_3	(0.6,0.4)	(0.7,0.1)	(0.4,0.4)

Using Hamming distance [10], we see that d(p_{11},s) =0.3, d(p_{12},s) =0.3, d(p_{13},s) =0.35
d(p_{21},s) =0.45, d(p_{22},s) =0.15, d(p_{23},s) =0.45
d(p_{31},s) =0.4, d(p_{32},s) =0.2, d(p_{33},s) =0.5
Therefore, the dominating matrix is

$D =$

	M_1	M_2	M_3	
M_1	3	2	2	$r_1 = 7$
M_2	1	3	2	$r_2 = 6$
M_3	1	2	3	$r_3 = 6$
	$c_1 = 5$	$c_2 = 7$	$c_3 = 7$	

Clearly, relative-score for M_1 is $s_1 = 2$

relative-score for M_2 is $s_2 = -1$

relative-score for M_3 is $s_3 = -1$

and $s_1 = \max_i s_i$.

$\Rightarrow$ The optimal selection is M_1.

To find the second best, we see that there is a tie between M_2 and M_3. Now, total hesitation in the information on M_2 is $\pi_2 = 0.7$, and in the information on M_3 is $\pi_3 = 0.4$. Therefore M_2 is the second best.

4 CONCLUSIONS

In this paper we have studied a problem of selecting most suitable method out of n alternatives on the basis of m criteria, when information are not crisp but intuitionistic fuzzy. The method is named to be "Distance Method". The method not only select the most suitable, but make a panel too, revealing the second most suitable, the third and so on. In case of a tie, we select that one for which the global hesitation is the maximum value. Justification is that besides a huge part still remaining indeterministic, it nevertheless carries equal merit.

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