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Properties of intuitionistic fuzzy line graphs

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Abstract: Concepts of graph theory have applications in many areas of computer science including data mining, image segmentation, clustering, image capturing, networking. An intuitionistic fuzzy set is a generalization of the notion of a fuzzy set. Intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compared to the fuzzy models. In this paper, we investigate some interesting properties of intuitionistic fuzzy line graphs.
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1 Introduction

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well known Königsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science.

In 1983, Atanassov [6] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [16]. Atanassov added a new component(which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [5].

In 1975, Rosenfeld [13] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [11] in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Bhattacharya [8] gave some remarks on fuzzy graphs. The complement of a fuzzy graph was defined by Mordeson [12]. Atanassov [5] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, and further studied in [4, 7, 9, 10, 14, 15]. Akram and Davvaz introduced the notion of intuitionistic fuzzy line graphs in [4]. In this paper, we investigate some interesting properties of intuitionistic fuzzy line graphs. We have used standard definitions and terminologies in this paper. For other notations, terminologies and applications not mentioned in the paper, the readers are referred to [1-3, 12].

2 Preliminaries

By graph, we mean a pair $G^* = (V, E)$, where V is the set and E is a relation on V. The elements of V are vertices of G^* and the elements of E are edges of G^*. We write $x, y \in E$ to mean $\{x, y\} \in E$, and if $e = xy \in E$, we say x and y are *adjacent*. Formally, given a graph $G^* = (V, E)$, two vertices $x, y \in V$ are said to be *neighbors, or adjacent nodes*, if $(x, y) \in E$. An *isomorphism* of graphs G_1^* and G_2^* is a bijection between the vertex sets of G_1^* and G_2^* such that any two vertices v_1 and v_2 of G_1^* are adjacent in G_1^* if and only if $f(v_1)$ and $f(v_2)$ are adjacent in G_2^*. Isomorphic graphs are denoted by $G_1^* \simeq G_2^*$.

By an *intersection graph* of a graph $G^* = (V, E)$, we mean, a pair $P(S) = (S, T)$ where $S = \{S_1, S_2, \ldots, S_n\}$ is a family of distinct nonempty subsets of V and $T = \{S_iS_j \,|\, S_i, S_j \in S,\ S_i \cap S_j \neq \emptyset, i \neq j\}$. It is well know that every graph is an intersection graph. By a *line graph* of a graph $G^* = (V, E)$, we mean, a pair $L(G^*) = (Z, W)$ where $Z = \{\{x\} \cup \{u_x, v_x\} \,|\, x \in E,\ u_x, v_x \in V,\ x = u_xv_x\}$ and $W = \{S_xS_y \,|\, S_x \cap S_y \neq \emptyset,\ x, y \in E,\ x \neq y\}$, and $S_x = \{x\} \cup \{u_x, v_x\}$, $x \in E$. It is reported in the literature that the line graph is an intersection graph.

Proposition 2.1. *If G is regular of degree k, then the line graph $L(G^*)$ is regular of degree $2k-2$.*

Definition 2.2. [16, 17] A *fuzzy subset* μ on a set X is a map $\mu : X \to [0, 1]$. A map $\nu : X \times X \to [0, 1]$ is called a *fuzzy relation* on X if $\nu(x, y) \leq \min(\mu(x), \mu(y))$ for all $x, y \in X$. A fuzzy relation ν is *symmetric* if $\nu(x, y) = \nu(y, x)$ for all $x, y \in X$.

Definition 2.3. [5] An *intuitionistic fuzzy set* $B = \{< x, \mu_B, \nu_B > \ | \ x \in X\}$ in a universe of discourse X is characterized by a membership function, μ_B, and a non-membership function, ν_B, as follows: $\mu_B : X \to [0, 1]$, $\nu_B : X \to [0, 1]$, and $\mu_B(x) + \nu_B(x) \leq 1$ for all $x \in X$.

3 Properties of intuitionistic fuzzy line graphs

Definition 3.1. [4] By an *intuitionistic fuzzy graph* is a pair $G = (A, B)$ where $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set on V and $B = (\mu_B, \nu_B)$ is an intuitionistic fuzzy relation on A such that

$$\mu_B(xy) \le \min(\mu_A(x), \mu_A(y)) \quad \text{and} \quad \nu_B(xy) \ge \max(\nu_A(x), \nu_A(y))$$

for all $xy \in E$. Note that B is a symmetric intuitionistic fuzzy relation on A.

Throughout this paper, we denote G^* a crisp graph, and G an intuitionistic fuzzy graph.

Definition 3.2. Consider an intersection graph $P(S) = (S, T)$ of a crisp graph $G^* = (V, E)$. Let $A_1 = (\mu_{A_1}, \nu_{A_1})$ and $B_1 = (\mu_{B_1}, \nu_{B_1})$ be intuitionistic fuzzy sets on V and E, $A_2 = (\mu_{A_2}, \nu_{A_2})$ and $B_2 = (\mu_{B_2}, \nu_{B_2})$ on S and T, respectively. Then an *intuitionistic fuzzy intersection graph* of the intuitionistic fuzzy graph $G = (A_1, B_1)$ is an intuitionistic fuzzy graph $P(G) = (A_2, B_2)$ such that

(a) $\mu_{A_2}(S_i) = \mu_{A_1}(v_i), \quad \nu_{A_2}(S_i) = \nu_{A_1}(v_i),$

(b) $\mu_{B_2}(S_iS_j) = \mu_{B_1}(v_iv_j), \quad \nu_{B_2}(S_iS_j) = \nu_{B_1}(v_iv_j)$

for all $S_i, S_j \in S$, $S_iS_j \in T$.

Example 3.3. Consider a graph $G^* = (V, E)$, where $V = \{v_1, v_2, v_3\}$ is the set of vertices, and $E = \{v_1v_2,\ v_2v_3,\ v_3v_1\}$ is the set of edges. Let A_1 and B_1 be intuitionistic fuzzy sets on V and E (respectively) defined by

	v_1	v_2	v_3
μ_{A_1}	0.2	0.3	0.4
ν_{A_1}	0.3	0.4	0.5

	v_1v_2	v_2v_3	v_3v_1
μ_{B_1}	0.1	0.2	0.1
ν_{B_1}	0.5	0.7	0.6

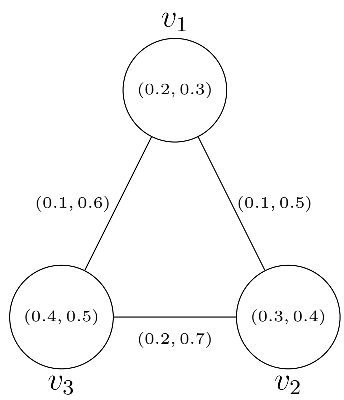

Figure 1: Intuitionistic fuzzy graph

By routine computations, it is easy to see that G is an intuitionistic fuzzy graph.

Consider an intersection graph $P(S) = (S, T)$ such that

$$S = \{S_1 = \{v_1, v_2\}, S_2 = \{v_2, v_3\}, S_3 = \{v_1, v_3\}\}$$

and

$$T = \{S_1S_2,\ S_2S_3,\ S_3S_1\}.$$

Let $A_2 = (\mu_{A_2}, \nu_{A_2})$ and $B_2 = (\mu_{B_2}, \nu_{B_2})$ be intuitionistic fuzzy sets on S and T, respectively. Then, by routine computations, we have

$$\mu_{A_2}(S_1) = \mu_{A_1}(v_1) = 0.2,\ \mu_{A_2}(S_2) = \mu_{A_1}(v_2) = 0.3,\ \mu_{A_2}(S_1) = \mu_{A_1}(v_3) = 0.4,$$

$$\nu_{A_2}(S_1) = \nu_{A_1}(v_1) = 0.3,\ \nu_{A_2}(S_2) = \nu_{A_1}(v_2) = 0.4,\ \nu_{A_2}(S_1) = \nu_{A_1}(v_3) = 0.5,$$

$$\mu_{B_2}(S_1S_2) = \mu_{B_1}(v_1v_2) = 0.1,\ \mu_{B_2}(S_2S_3) = \mu_{B_1}(v_2v_3) = 0.1,\ \mu_{B_2}(S_3S_1) = \mu_{B_1}(v_3v_1) = 0.1,$$

$$\nu_{B_2}(S_1S_2) = \nu_{B_1}(v_1v_2) = 0.6,\ \nu_{B_2}(S_2S_3) = \nu_{B_1}(v_2v_3) = 0.7,\ \nu_{B_2}(S_3S_1) = \nu_{B_1}(v_3v_1) = 0.6.$$

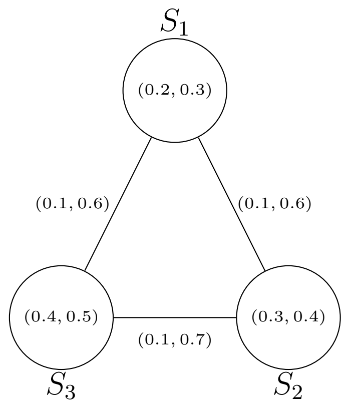

Figure 2: Intuitionistic fuzzy intersection graph

By routine computations, it is easy to see that $P(G)$ is an intuitionistic fuzzy intersection graph.

Proposition 3.4. *Let $G = (A_1, B_1)$ be a intuitionistic fuzzy graph of G^* and let $P(G) = (A_2, B_2)$ be an intuitionistic fuzzy intersection graph of $P(S)$. Then*

(a) *an intuitionistic fuzzy intersection graph is an intuitionistic fuzzy graph.*

(b) *an intuitionistic fuzzy graph is isomorphic to an intuitionistic fuzzy intersection graph.*

Proof. (a) From Definition 3.2, it follows that

$$\mu_{B_2}(S_iS_j) = \mu_{B_1}(v_iv_j) \leq \min(\mu_{A_1}(v_i), \mu_{A_1}(v_j)),$$

$$\nu_{B_2}(S_iS_j) = \nu_{B_1}(v_iv_j) \geq \max(\nu_{A_1}(v_i), \nu_{A_1}(v_j)).$$

This shows that an intuitionistic fuzzy intersection graph is an intuitionistic fuzzy graph.

(b) Define $\varphi : V \to S$ by $\varphi(v_i) = s_i$, for $i = 1, 2 \ldots, n$. Clearly, φ is a one-to-one function of V onto S. Now $v_iv_j \in E$ if and only if $s_is_j \in T$ and $T = \{\varphi(v_i)\varphi(v_j) \mid v_iv_j \in E\}$. Also

$$\mu_{A_2}(\varphi(v_i)) = \mu_{A_2}(S_i) = \mu_{A_1}(v_i),$$

$$\nu_{A_2}(\varphi(v_i)) = \nu_{A_2}(S_i) = \nu_{A_1}(v_i),$$

$$\mu_{B_2}(\varphi(v_i)\varphi(v_j)) = \mu_{B_2}(S_iS_j) = \mu_{B_2}(v_iv_j),$$

$$\nu_{B_2}(\varphi(v_i)\varphi(v_j)) = \nu_{B_2}(S_iS_j) = \nu_{B_2}(v_iv_j).$$

Thus φ is an isomorphism of G onto $P(G)$.

□

Definition 3.5. [4] Let $L(G^*) = (Z, W)$ be a line graph of a crisp graph $G^* = (V, E)$. Let $A_1 = (\mu_{A_1}, \nu_{A_1})$ and $B_1 = (\mu_{B_1}, \nu_{B_1})$ be intuitionistic fuzzy sets on V and E, $A_2 = (\mu_{A_2}, \nu_{A_2})$ and $B_2 = (\mu_{B_2}, \nu_{B_2})$ on Z and W, respectively. Then an *intuitionistic fuzzy line graph* of the intuitionistic fuzzy graph $G = (A_1, B_1)$ is an intuitionistic fuzzy graph $L(G) = (A_2, B_2)$ such that

(i) $\mu_{A_2}(S_x) = \mu_{B_1}(x) = \mu_{B_1}(u_xv_x)$,

(ii) $\nu_{A_2}(S_x) = \nu_{B_1}(x) = \nu_{B_1}(u_xv_x)$,

(iii) $\mu_{B_2}(S_xS_y) = \min(\mu_{B_1}(x), \mu_{B_1}(y))$,

(iv) $\nu_{B_2}(S_xS_y) = \max(\nu_{B_1}(x), \nu_{B_1}(y))$

for all $S_x, S_y \in Z$, $S_xS_y \in W$.

Proposition 3.6. $L(G) = (A_2, B_2)$ *is an intuitionistic fuzzy line graph of some intuitionistic fuzzy graph* $G = (A_1, B_1)$ *if and only if*

$$\mu_{B_2}(S_xS_y) = \min(\mu_{A_2}(S_x), \mu_{A_2}(S_y)),$$

$$\nu_{B_2}(S_xS_y) = \max(\nu_{A_2}(S_x), \nu_{A_2}(S_y))$$

for all $S_x, S_y \in W$.

Definition 3.7. Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two intuitionistic fuzzy graphs. A *homomorphism* $\varphi : G_1 \to G_2$ is a mapping $\varphi : V_1 \to V_2$ such that

(a) $\mu_{A_1}(x_1) \leq \mu_{A_2}(\varphi(x_1)), \quad \nu_{A_1}(x_1) \geq \nu_{A_2}(\varphi(x_1))$,

(b) $\mu_{B_1}(x_1y_1) \leq \mu_{B_2}(\varphi(x_1)\varphi(y_1)), \quad \nu_{B_1}(x_1y_1) \geq \nu_{B_2}(\varphi(x_1)\varphi(y_1))$

for all $x_1 \in V_1$, $x_1y_1 \in E_1$.

A bijective homomorphism $\varphi : G_1 \to G_2$ of intuitionistic fuzzy graphs is called a *weak vertex-isomorphism*, if

(c) $\mu_{A_1}(x_1) = \mu_{A_2}(\varphi(x_1)), \quad \nu_{A_1}(x_1) = \nu_{A_2}(\varphi(x_1)),$

for all $x_1 \in V_1$, and a *weak line-isomorphism* if

(d) $\mu_{B_1}(x_1y_1) = \mu_{B_2}(\varphi(x_1)\varphi(y_1)), \quad \nu_{B_1}(x_1y_1) = \nu_{B_2}(\varphi(x_1)\varphi(y_1))$

for all $x_1y_1 \in V_1$. A bijective homomorphism $\varphi : G_1 \to G_2$ satisfying (c) and (d) is called a *weak isomorphism* of intuitionistic fuzzy graphs G_1 and G_2. A weak isomorphism preserves the weights of the vertices but not necessarily the weights of the edges.

The following fact is obvious.

Proposition 3.8. *A weak isomorphism of intuitionistic fuzzy graphs G_1 and G_2 is an isomorphism of their crisp graphs G_1^* and G_2^*.*

Theorem 3.9. *Let $L(G) = (A_2, B_2)$ be the intuitionistic fuzzy line graph corresponding to the intuitionistic fuzzy graph $G = (A_1, B_1)$. Suppose that $G^* = (V, E)$ is connected. Then*

(i) *there exists a week isomorphism of G onto $L(G)$ if and only if G^* is a cyclic and for all $v \in V$, $x \in E$, $\mu_{A_1}(v) = \mu_{B_1}(x)$, $\nu_{A_1}(v) = \nu_{B_1}(x)$, i.e., $A_1 = (\mu_{A_1}, \nu_{A_1})$ and $B_1 = (\mu_{B_1}, \nu_{B_1})$ are constant functions on V and E, respectively, taking on the same value.*

(ii) *If φ is a weak isomorphism of G onto $L(G)$, then φ is an isomorphism.*

Proof. Assume that φ is a weak isomorphism of G onto $L(G)$. From Proposition 3.6, it follows that $G^* = (V, E)$ is a cycle [12, Theorem 8.2, p.72]. Let $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{x_1 = v_1v_2,\ x_2 = v_2v_3, \ldots, x_n = v_nv_1\}$, where $v_1v_2v_3 \ldots v_n$ is a cycle. Define intuitionistic fuzzy sets

$$\mu_{A_1}(v_i) = s_i, \quad \nu_{A_1}(v_i) = s_i'$$

and

$$\mu_{B_1}(x_i) = \mu_{B_1}(v_iv_{i+1}) = r_i, \quad \nu_{B_1}(x_i) = \nu_{B_1}(v_iv_{i+1}) = r_i', \quad i = 1, 2, \ldots, n, \quad v_{n+1} = v_1.$$

Then for $s_{n+1} = s_1$, $s_{n+1}' = s_1'$, we have

$$r_i \le \min(s_i, s_{i+1}), \quad r_i' \ge \max(s_i', s_{i+1}'), \quad i = 1, 2, \ldots, n. \tag{1}$$

Now

$$Z = \{S_{x_1}, S_{x_1}, S_{x_2}, \ldots, S_{x_n}\}, \quad W = \{S_{x_1}S_{x_2},\ S_{x_2}S_{x_3}, \ldots, S_{x_n}S_{x_1}\}.$$

Thus for $r_{n+1} = r_1$, we obtain

$$\mu_{A_2}(S_{x_i}) = \mu_{B_1}(x_i) = r_i, \quad \nu_{A_2}(S_{x_i}) = \nu_{B_1}(x_i) = r_i',$$

$$\mu_{B_2}(S_{x_i}S_{x_{i+1}}) = \min(\mu_{B_1}(x_i), \mu_{B_1}(x_{i+1})) = \min(r_i, r_{i+1}),$$

$$\nu_{B_2}(S_{x_i}S_{x_{i+1}}) = \max(\nu_{B_1}(x_i), \nu_{B_1}(x_{i+1})) = \max(r_i', r_{i+1}')$$

for $i = 1, 2, \ldots, n$, $v_{n+1} = v_1$. Since φ is an isomorphism of G^* onto $L(G^*)$, φ is a bijective map of V onto Z. Also φ preserves adjacency. Hence φ induces a permutation π of $\{1, 2, \ldots, n\}$ such that

$$\varphi(v_i) = S_{v_{\pi(i)} v_{\pi(i)+1}}$$

and

$$v_i v_{i+1} \to \varphi(v_i)\varphi(v_{i+1}) = S_{v_{\pi(i)} v_{\pi(i)+1}} S_{v_{\pi(i+1)} v_{\pi(i+1)+1}}, \quad i = 1, 2, \ldots, n-1.$$

Thus

$$s_i = \mu_{A_1}(v_i) \leq \mu_{A_2}(\varphi(v_i)) = \mu_{A_2}(S_{v_{\pi(i)} v_{\pi(i)+1}}) = \mu_{B_1}(v_{\pi(i)} v_{\pi(i)+1}) = r_{\pi(i)},$$

$$s'_i = \nu_{A_1}(v_i) \geq \nu_{A_2}(\varphi(v_i)) = \nu_{A_2}(S_{v_{\pi(i)} v_{\pi(i)+1}}) = \nu_{B_1}(v_{\pi(i)} v_{\pi(i)+1}) = r'_{\pi(i)}$$

and

$$\begin{aligned} r_i = \mu_{B_1}(v_i v_{i+1}) &\leq \mu_{B_2}(\varphi(v_i)\varphi(v_{i+1})) \\ &= \mu_{B_2}(S_{v_{\pi(i)} v_{\pi(i)+1}} S_{v_{\pi(i+1)} v_{\pi(i+1)+1}}) \\ &= \min(\mu_{B_1}(v_{\pi(i)} v_{\pi(i)+1}), \mu_{B_1}(v_{\pi(i+1)} v_{\pi(i+1)+1})) \\ &= \min(r_{\pi(i)}, r_{\pi(i+1)}). \end{aligned}$$

Similarly,

$$\begin{aligned} r'_i = \nu_{B_1}(v_i v_{i+1}) &\geq \nu_{B_2}(\varphi(v_i)\varphi(v_{i+1})) \\ &= \nu_{B_2}(S_{v_{\pi(i)} v_{\pi(i)+1}} S_{v_{\pi(i+1)} v_{\pi(i+1)+1}}) \\ &= \max(\nu_{B_1}(v_{\pi(i)} v_{\pi(i)+1}), \nu_{B_1}(v_{\pi(i+1)} v_{\pi(i+1)+1})) \\ &= \max(r'_{\pi(i)}, r'_{\pi(i+1)}) \end{aligned}$$

for $i = 1, 2, \ldots, n$. That is,

$$s_i \leq r_{\pi(i)}, \quad s'_i \geq r'_{\pi}(i) \tag{2}$$

and

$$r_i \leq \min(r_{\pi(i)}, r_{\pi(i+1)}), \quad r'_i \geq \max(r'_{\pi(i)}, r'_{\pi(i+1)}). \tag{3}$$

Thus, $r_i \leq r_{\pi(i)}$, $r'_i \geq r'_{\pi(i)}$, and so $r_{\pi(i)} \leq r_{\pi(\pi(i))}$, $r'_{\pi(i)} \geq r'_{\pi(\pi(i))}$ for all $i = 1, 2, \ldots, n$. Continuing, we obtain

$$r_i \leq r_{\pi(i)} \leq \ldots \leq r_{\pi^j(i)} \leq r_i,$$

$$r'_i \geq r'_{\pi(i)} \geq \ldots \geq r'_{\pi^j(i)} \geq r'_i,$$

where π^{j+1} is the identity map. So, $r_i = r_{\pi(i)}$, $r'_i = r'_{\pi(i)}$ for all $i = 1, 2, \ldots, n$. But, by (3), we also have $r_i \leq r_{\pi(i+1)} = r_{i+1}$ and $r'_i \geq r'_{\pi(i+1)} = r'_{i+1}$, which together with $r_{n+1} = r_1$, $r'_{n+1} = r'_1$, implies $r_i = r_1$ and $r'_i = r'_1$ for all $i = 1, 2, \ldots, n$. Hence by (1) and (2), we get

$$r_1 = \ldots = r_n = s_1 = \ldots = s_n,$$

$$r'_1 = \ldots = r'_n = s'_1 = \ldots = s'_n.$$

Thus we have not only proved the conclusion about A_1 and B_1 being constant function, but we have also shown that (ii) holds. The converse part of (i) is obvious. $\square$

4 Conclusions

Graph theory is becoming increasingly significant as it is applied to other areas of mathematics, science and technology. In computer science, graphs are used to represent networks of communication, data organization, computational devices, the flow of computation. One practical example is the link structure of a website could be represented by a directed graph. The vertices are the web pages available at the website and a directed edge from page A to page B exists if and only if A contains a link to B. It is known that intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compare to the classic and fuzzy models. Thus, we plan to extend our research work to application of intuitionistic fuzzy graphs in (1) neural networks, and (2) database theory.

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