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Intuitionistic fuzzy Prolog

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Abstract: A logic programming system which uses a theory of intuitionistic fuzzy sets to model various forms of uncertainty is presented. To represent uncertainty of facts and rules, a pair of two different real numbers (degree of truth and degree of falsity) are associated. The problem of propagating uncertainty through logical inference and various models of interpretation are considered. The framework discussed allows knowledge representation and inference under uncertainty in the form of rules suitable for expert systems.

Keywords: Expert systems; intuitionistic fuzzy logic; intuitionistic fuzzy sets; logic programming.

1. Introduction

On the basis of the research from [1–7] in this paper we shall describe an extended variant of the well-known logic programming language PROLOG, which will include the elements of intuitionistic fuzziness. Two different real numbers μ and γ for which

$$0 \leq \mu + \gamma \leq 1$$

are associated to every fact. They mark the degree of truth and the degree of falsity of the fact and of the rule. For example, let us consider the following:

[0.5] [0.2] likes(John, Mary)

which can be interpreted as: there is a belief 0.5 and disbelief 0.2 in the hypothesis that "John likes Mary".

The degree of truth is usually different from the degree of falsity, but obviously, if we take $\mu = 1 - \gamma$ this will result in fuzzy logic.

The logic programming paradigm presented in this paper does not rely upon a closed-world form of knowledge representation. It means that the facts which are not in the knowledge base are not assumed to be necessarily false. A fact for which there is no information will have total ignorance ($\mu = \gamma = 0$), i.e. the grade of truth and the grade of falsity are equal to zero.

The logical operations & (conjunction) and $\vee$ (disjunction) and the modal operators 'necessity' and 'possibility' (which we denote by # and @, respectively) can be included in the clauses body.

In this sense Intuitionistic Fuzzy PROLOG (InF-PROLOG) will contain the ideology of the modal PROLOG [16], as well as that of the fuzzy PROLOGS (see e.g. [8, 13–15]).

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2. Knowledge representation

The InF-PROLOG program consists of a finite set of InF-clauses with an InF-structure associated to it. The InF-structures are:

(i) an interval valued object $\langle M, N\rangle$, where $M=[\mu_i, \mu_s]$ and $N=[\gamma_i, \gamma_s]$ are subsets of the interval $[0,1]$ and $\mu_s+\gamma_s=\sup M+\sup N\leqslant 1$;

(ii) a threshold pair $\langle t_\mu, t_\gamma\rangle$, where $0\leqslant t_\mu+t_\gamma\leqslant 1$;

(iii) a pair of values $\langle \mu, \gamma\rangle$, where $0\leqslant \mu+\gamma\leqslant 1$.

The InF-clauses have the standard PROLOG-form, but they are extended with the InF-structures. Generally they look like this:

$$\langle H_\mu, H_\gamma\rangle\ H\ :-\ B\ \langle B_\mu, B_\gamma\rangle.$$

We distinguish different types of clauses by the use of different InF-structures. Calculation of the clauses is based upon the operators from intuitionistic fuzzy set theory.

2.1. Interval rule

$$[\mu_i^h, \mu_s^h][\gamma_i^h, \gamma_s^h]\ H\ :-\ B_1, \ldots, B_n\ [\mu_i^b, \mu_s^b][\gamma_i^b, \gamma_s^b],$$

where H is an atom called clause head and $B_1, \ldots, B_n$ are subgoals. The intervals $[\mu_i^h, \mu_s^h]$, $[\gamma_i^h, \gamma_s^h]$, $[\mu_i^b, \mu_s^b]$ and $[\gamma_i^b, \gamma_s^b]$ are InF-structures from case (i). They can be given the following interpretation. For each assignment of each variable occurring in the clause, if B are all true with degrees within the intervals $[\mu_i^b, \mu_s^b]$ (for the degree of truth) and $[\gamma_i^b, \gamma_s^b]$ (for the degree of falsity), the head H of the clause has truth (μ_H) and falsity (γ_H) degrees within the intervals $[\mu_i^h, \mu_s^h]$ and $[\gamma_i^h, \gamma_s^h]$ respectively. Naturally, the calculated degrees μ_H and γ_H satisfy the constraint $0\leqslant \mu_H+\gamma_H\leqslant 1$.

Let μ_B and γ_B be the already calculated truth and falsity degrees of the body of an InF-clause. The calculation of the degrees of the head μ_H and γ_H in terms of the interval rule goes as follows:

$$\mu_H=\mu_i^h+\alpha_\mu(\mu_s^h-\mu_i^h), \qquad \gamma_H=\gamma_i^h+\alpha_\gamma(\gamma_s^h-\gamma_i^h),$$

where

$$\alpha_\mu=\begin{cases}\dfrac{\mu_B-\mu_i^b}{\mu_s^b-\mu_i^b} & \text{if } \mu_s^b>\mu_i^b,\\ \frac{1}{2} & \text{otherwise,}\end{cases} \qquad \alpha_\gamma=\begin{cases}\dfrac{\gamma_B-\gamma_i^b}{\gamma_s^b-\gamma_i^b} & \text{if } \gamma_s^b<\gamma_i^b,\\ \frac{1}{2} & \text{otherwise.}\end{cases}$$

The interval rule is the rule of the form:

$$[\mu_i^h, \mu_s^h][\gamma_i^h, \gamma_s^h]\ H\ :-\ B_1, \ldots, B_n\ [\mu_B][\gamma_B].$$

where μ_B and γ_B are InF-structures from case (ii). The interpretation is as above, except that the truth and falsity degrees of the atoms $B_1, \ldots, B_n$ must be greater than or equal to μ_B and less than or equal to γ_B respectively. The calculation is as follows:

$$\alpha_\mu=\begin{cases}\dfrac{\mu_B-b_\mu}{1-b_\mu} & \text{if } b_\mu<1,\\ \frac{1}{2} & \text{otherwise,}\end{cases} \qquad \alpha_\gamma=\begin{cases}\dfrac{\gamma_B}{b_\gamma} & \text{if } b_\gamma>0,\\ \frac{1}{2} & \text{otherwise.}\end{cases}$$

μ_H and γ_H are calculated in the same way.

The interval rules described above are provided in this framework to give greater expressing power for representing uncertainty in the knowledge. Indeed, instead of the use of fuzzy numbers to express the truth and falsity degrees for a rule or fact, the only thing we specify in the case of interval rules is the interval in which the fuzzy degrees must appear. Moreover some special constraint techniques are available for keeping the process of goals derivation in the desired direction. The following two examples are supposed to clarify the use of intervals for fuzzy degrees and constraints on them.

Example 1. Suppose we have the following InF-PROLOG program:

```
/* Example 1 rules */

[0.6, 0.8] [0.1,  0.2] d(X) :- p(X), l(X) [0.4,  0.7] [0, 0.2].
[0.4, 0.7] [0.1,  0.2] d(X) :- c(X)       [0.5,  0.8] [0, 0.1].
[0.5, 0.8] [0.15, 0.2] p(X) :- e(X), r(X) [0.3, 0.75] [0, 0.2].

/* Example 1 facts */

[0.7] [0.2] r(a).
[0.8] [0.2] l(a).
[0.6] [0.1] c(a).
[0.6] [0.2] e(a).
[0.8] [0.1] r(b).
[0.9] [0  ] l(b).
[0.9] [0.1] e(b).
[0.3] [0.6] c(b).
```

All InF-clauses are based on interval rules with constraints of the interval form too. The question about goal $d(X)$ we ask looks like:

```
?- d(X).
   SUCCESS 0.850:0.150
   X = a
```

The answer is, X is instantiated with the atom a and this is an intuitionistic truth with resulting degrees $\langle \mu, \gamma \rangle = \langle 0.85, 0.15 \rangle$. Let us consider in more details how this result is obtained. According to the way the InF-PROLOG interpreter works, the matching clause for the above question is the first clause on $d(X)$. Suppose for the moment that subgoal $p(X)$ is proved and the next subgoal is $l(X)$. We can see that the goal $l(X)$ would not be proved because of the constraints which restrict the possible values of truth and falsity degrees. In this case the truth degrees $\mu_a = 0.8$ and $\mu_g = 0.9$ for facts $l(a)$ and $l(b)$ respectively do not belong to the interval $[0.4, 0.7]$ which is specified in the constraint on truth degrees. If subgoal $l(X)$ fails, this causes back-tracking to occur for the previous subgoal $p(X)$ and since there are no other alternatives for it, the main goal $d(X)$ fails. This example shows the way in which interval constraints control the goal derivation. The next step is to consider the second clause provided for goal $d(X)$. The only subgoal in this clause is $c(X)$ and fact $c(a)$ has fuzzy degrees $\langle 0.6, 0.1 \rangle$ which belong to the intervals $[0.5, 0.8]$ and $[0, 0.1]$ respectively. This means that derivation of goal $d(X)$ may continue with calculating its fuzzy degrees by means of the interval rule. There are no other answers of the question because the second fact $c(b)$ is not to be considered in the terms of the constraints.

Example 2. Suppose we have the following InF-PROLOG program;

```
/* Example 2 rules */

[0.6, 0.8] [0.1,  0.2] d(X) :- p(X), l(X) [0.4] [0.2].
[0.4, 0.7] [0.1,  0.2] d(X) :- c(X)       [0.5] [0.1].
[0.5, 0.8] [0.15, 0.2] p(X) :- e(X), r(X) [0.3] [0.2]

/* Example 2 facts */

[0.7] [0.2] r(a).
[0.8] [0.2] l(a).
[0.6] [0.1] c(a).
[0.6] [0.2] e(a).
```

[0.8] [0.1] $r(b)$.
[0.9] [0] $l(b)$.
[0.9] [0.1] $e(b)$.
[0.3] [0.6] $c(b)$.

All InF-clauses are based on interval rules with constraints of the form of threshold. The question about goal $d(X)$ we ask looks like:

?- $d(X)$.
SUCCESS 0.676:0.200
$X = a$
;
SUCCESS 0.705:0.187
$X = b$

Let us consider in more detail how this result is obtained. According to the way the InF-Prolog interpreter works, the matching clause for the above question is the first clause on $d(X)$. The first subgoal in the clause body is $p(X)$. Derivation continues with the goal $e(X)$ and it is proved by using the fact $e(a)$ with degrees $\langle 0.6, 0.2\rangle$. Note that these degrees satisfy the threshold constraints because $0.6 > 0.3$ and $0.2 \leq 0.2$. The second subgoal is $r(X)$ (with $X = a$), which is proved too and has degrees $\langle 0.7, 0.2\rangle$. The resulting degree for the clause body is calculated as $\langle 0.6, 0.3\rangle$ by the use of conjunction based on MinMax operations. Applying formulae for the interval rule for the threshold pair we obtain for the degrees of the goal $p(X)$ (with $X = a$)

$$\langle \mu p(a), \gamma p(a)\rangle.$$

The process of goals derivation continues with the same actions applied for the body of the first clause on $d(X)$.

2.2. F-rule

$$[\alpha, B]\ H{:-}\ B_1, \ldots, B_n\ \langle B_\mu, B_\gamma\rangle,$$

where α and β are real numbers for which $0 \leq \alpha + \beta \leq 1$ and B_μ, B_γ are InF-structures from cases (i) and (ii). The meaning of the constraints B_μ and B_γ (which may be absent) is obvious and was mentioned above. The calculation of the degree of the clause head is based on operator $F_{\alpha,\beta}$, for $0 \leq \alpha + \beta \leq 1$, and is as follows:

$$\langle \mu_H, \gamma_H\rangle = F_{\alpha,\beta}(\mu_B, \gamma_B) = \langle \mu_B + \alpha\pi_B, \gamma_B + \beta\pi_B\rangle,$$

where $\pi_B = 1 - \mu_B - \gamma_B$ is called certainty factor.

2.3. G-rule

$$[\alpha][\beta]\ H\ :-\ B_1, \ldots, B_n\ \langle B_\mu, B_\gamma\rangle,$$

where $\alpha, b \in [0, 1]$ are real numbers. B_μ and B_γ are InF-structures from cases (i) and (ii). The meaning of the constraints B_μ and B_γ (which may be absent) is obvious and was mentioned above. The calculation of the degrees of the clause head is based on operator $G_{\alpha,\beta}$ and is as follows:

$$\langle \mu_H, \gamma_H\rangle = G_{\alpha,\beta}(\mu_B, \gamma_B) = \langle \alpha\mu_B, \beta\gamma_B\rangle.$$

2.4. D-rule

$$[\alpha]\ H\ :-\ B_1, \ldots, B_n\ \langle B_\mu, B_\gamma\rangle,$$

where $\alpha \in [0, 1]$ is a real number and B_μ and B_γ are InF-structures from cases (i) and (ii). The meaning of the constraints B_μ and B_γ (which may be absent) is obvious and was mentioned above. The

calculation of the degrees of the clause head is based on operator D_α and is as follows:

$$\langle \mu_H, \gamma_H \rangle = D_\alpha(\mu_B, \gamma_B) = \langle \mu_B + \alpha\pi_B, \gamma_B + (1-\alpha)\pi_B \rangle.$$

Note that unlike the first three rules the D-rule produces probability measures μ_H and γ_H, for which $\mu_H = 1 - \gamma_H$ unlike of intuitionistic fuzzy measures. This operator is a particular case of F-operator.

Example 3. The following example demonstrates the use of fuzzy constraint techniques with the other types of rules, F-rule, G-rule and D-rule. Note that in this case fuzzy constraint values do not take part in the calculation of the derived goal. They are used only to control the way of derivation. Suppose that the truth-values of goals $p(X)$, $q(X)$ and $r(X)$ are already calculated as follows:

$p(a)$ with degrees $\langle 0.4, 0.3 \rangle$,

$q(a)$ with degrees $\langle 0.7, 0.2 \rangle$,

$r(a)$ with degrees $\langle 0.5, 0.0 \rangle$.

The truth-value of the expression

$$a(X) = (\text{ness}(p(X)); \text{poss}(q(X)), (\text{poss}(p(X)); \text{not}(r(X)))$$

for $X = a$ is $\langle 0.7, 0.2 \rangle$.

Case (*a*). If we have an InF-clause of the form:

$$[0.7]\ h(X)\ :-\ a(X)\ [0.6, 0.8]\ [0.1, 0.2],$$

then the rule will be activated, because the degrees $\langle 0.7, 0.2 \rangle$ satisfy the interval constraints $0.7 \in [0.6, 0.8]$ and $0.2 \in [0.1, 0.2]$. The question about $h(X)$ will have the following answer:

?- $h(X)$.
SUCCESS 0.450:0.400
$X = a$

which is obtained by the use of the D-rule as follows:

$$D_{0.7}\langle 0.7, 0.2 \rangle = \langle 0.77, 0.23 \rangle.$$

Case (*b*). If

$$[0.8, 0.1]\ h(X)\ :-\ a(X)\ [0.6]\ [0.2],$$

then the rule will be activated, because the degrees $\langle 0.7, 0.2 \rangle$ satisfy the threshold constraints $0.7 \geqslant 0.6$ and $0.2 \leqslant 0.2$. The question on $h(X)$ will have the following answer:

?- $h(X)$.
SUCCESS 0.450:0.400
$X = a$

which is obtained by the use of the D-rule as follows:

$$F_{0.8,0.1}\langle 0.7, 0.2 \rangle = \langle 0.78, 0.21 \rangle.$$

Case (*c*). If

$$[0.8]\,[0.1]\ h(X)\ :-\ a(X)\ [0.6, 0.8]\,[0.1, 0.2],$$

then the rule will be activated, because degrees $\langle 0.7, 0.2 \rangle$ satisfy the interval constraints $0.7 \in [0.6, 0.8]$ and $0.2 \in [0.1, 0.2]$. The question on $h(X)$ will have the following answer:

?- $h(X)$.
SUCCESS 0.450:0.400
$X = a$

which is obtained by the use of G-rule as follows:

$$G_{0.8,0.1}\langle 0.7, 0.2\rangle = \langle 0.56, 0.02\rangle.$$

Example 4. Let us consider the following statements:

"It's known that John likes Mary" with $\langle \mu, \gamma\rangle = \langle 0.7, 0.2\rangle$
"It's known that Mary likes John" with $\langle \mu, \gamma\rangle = \langle 0.9, 0.0\rangle$

and we have a rule of the form:

"Two persons say person X and person Y are friends if they like each other"
with degrees of truth and falsity $\langle \mu, \gamma\rangle = \langle 0.8, 0.0\rangle$.

The question is at what degree of truth and falsity the statement "John and Mary are friends" is valid. The appropriate InF-PROLOG program which encodes the above facts and rules looks like:

[0.8] [0.0] friends(X, Y) :– likes(X, Y), likes(Y, X).

[0.7] [0.2] likes(John, Mary).

[0.9] [0.0] likes(Mary, John).

Note that subgoals in the clause body are in conjunction and there is no need for aggregating the fuzzy measures because the facts express different information. The conjunction operation based on MinMax operators is used. To express the question we can use the following:

?- friends(John, X).
SUCCESS 0.560:0.000
X = Mary

It forces the InF-PROLOG interpreter to find an instantiation of the variable X which is in the relation 'friends' with John. After calculating the truth degrees for the clause body which are $\langle 0.7, 0.2\rangle$ by the use of the G-rule, we can conclude that X is instantiated with the atom 'Mary' and the resulting truth and falsity degrees are $\langle 0.56, 0.0\rangle = \langle 0.8 \times 0.7, 0.0 \times 0.2\rangle$. The answer is considered as Intuitionistic fuzzy truth becasue $\mu > \gamma$.

3. Truth and falsity degrees of the body

Let R be an InF-clause denoted by:

$$R :- B_0, B_1, \ldots, B_m.$$

If μ_i^B and γ_i^B are the truth and falsity degrees of the atom B_i, then for the values $\langle \mu_i^B, \gamma_i^B\rangle$ of the body we get

$$\langle \mu_i^B, \gamma_i^B\rangle = \left\langle \min_{1\le i\le m} \mu_i^B, \max_{1\le i\le m} \gamma_i^B \right\rangle,$$

which corresponds to the operation & (conjunction denoted by ',') from intuitionistic fuzzy logic, while in the case of the operation disjunction denoted by ';' for the values $\langle \mu_i^B, \gamma_i^B\rangle$ we get:

$$\langle \mu_i^B, \gamma_i^B\rangle = \left\langle \max_{1\le i\le m} \mu_i^B, \min_{1\le i\le m} \gamma_i^B \right\rangle.$$

Besides the operations conjunction and disjunction in the body of an InF-PROLOG clause, an unordered collection from operations and operators defined over every IFS may appear. Most of them are implemented as built-in predicates which handle the uncertainty of InF-PROLOG goals in different manners. These predicates are called certainty predicates because they remove or modify the

uncertainty degrees of the goals derived. Certainty predicates have a strong theoretical background and correspond to the operators from IFS theory. Here are some of the definitions:

(a) not(G): the goal G is proved with some truth and falsity degrees $\langle \mu^G, \gamma^G \rangle$ and for the resulting degrees μ and γ we get $\langle \mu, \gamma \rangle = \langle \gamma^G, \mu^G \rangle$.

(b) ness(G): the goal G is proved with some truth and falsity degrees $\langle \mu^G, \gamma^G \rangle$ and for the resulting degrees μ and γ we get $\langle \mu, \gamma \rangle = \langle \mu^G, 1 - \mu^G \rangle$. This predicate corresponds to the modal operator 'necessity' from intuitionistic fuzzy logic and removes the uncertainty (which is $=1 - \mu^G - \gamma^G$) adding it to the falsity degree.

(c) poss(G): the same as ness(G) but this predicate corresponds to the modal operator 'possibility' from intuitionistic fuzzy logic and adds the CF to the truth degree. Hence $\langle \mu, \gamma \rangle = \langle 1 - \gamma^G, \gamma^G \rangle$.

Examples of InF-clauses: (1) Consider

[1/2] [1/4] is_comfortable(large_car).

This states that the degree of truth for "large car is comfortable" is 1/2, while the degree of falsity for the same proposition is 1/4. It is easy to conclude that the degree of truth and degree of falsity for the proposition "large car is not comfortable" are equal to 1/4 and 1/2 respectively. Note that in both cases the uncertainty factor is 1/2.

(2) Consider

[Tt] temperature(X, normal) :− T is $1 - [(150 - X)/150]^2$.

This shows how the membership function (say x) can be given explicitly in the clause body. The example above defines that if X takes the value 150°F then it is certain ($T = 1$) that this temperature is normal, for $X = 0$°F the value of T is 0 and if X is 200°F then $T = 7/8$ (cf. [12]).

4. Combination of degrees from different proof paths

In rule-based expert systems several conclusions concerning the same proposition P may be obtained using different proof paths. When these conclusions are proved with some degree of uncertainty (say μ_i^P, γ_i^P), a combination operation is usually performed in order to realize a consensus between the different sources (rules, facts). Consider that one proof path gives $\langle \mu_1^P, \gamma_1^P \rangle$ and another proof path gives $\langle \mu_2^P, \gamma_2^P \rangle$ for proving the proposition P. The resulting degrees $\langle \mu^P, \gamma^P \rangle$ can be obtained in a different manner. There exist several combination formulas which realise a consensus between different sources of information in various mathematical frameworks [9, 10]. Following the classification given by Dubois [10] we present an intuitionistic fuzzy variant of combination formulas of evidence.

Conjunctive consensus is realized by the use of intuitionistic fuzzy set intersection operation which gives

$$\langle \mu^P, \gamma^P \rangle = \left\langle \min_{1 \le i \le n} \mu_i^P, \max_{1 \le i \le n} \gamma_i^P \right\rangle.$$

The corresponding predicate is minc(G).

Disjunctive consensus is realized by the use of intuitionistic fuzzy set union operation which gives

$$\langle \mu^P, \gamma^P \rangle = \left\langle \max_{1 \le i \le n} \mu_i^P, \min_{1 \le i \le n} \gamma_i^P \right\rangle.$$

The corresponding predicate is maxc (G).

The pooling operation is a convex combination of the uncertainty measures in case of different reliability of the sources (proof paths). There are nonnegative numbers $\alpha_1, \ldots, \alpha_n$, and $\beta_1, \ldots, \beta_n$ for which

$$0 \le \sum_{i=1}^{n} \alpha_i + \sum_{i=1}^{n} \beta_i \le 1,$$

such that

$$\langle \mu_A(x), \gamma_A(x) \rangle = \left\langle \sum_{i=1}^{n} \alpha_i \mu_i(x) + \sum_{i=1}^{n} \beta_i \gamma_i(x) \right\rangle.$$

The α_i and β_i reflect the relative reliability and the relative unreliability of the sources.

Hybrid consensus rules have a hybrid behaviour depending on μ_i^P and γ_i^P. The most popular rule of combination is Dempster's rule of combination.

Here we present a form of Dempster's rule which can be used in intuitionistic fuzzy set theory. Note that applying it to an intuitionistic fuzzy value yield an intuitionistic fuzzy value. This kind of assumption is a natural one if we want to stay in a given mathematical framework.

Let

$$\mu_P = (\mu_1^P \pi_2^P + \mu_2^P \pi_1^P + \mu_1^P \mu_2^P)/d, \qquad \gamma_P = (\gamma_1^P \pi_2^P + \gamma_2^P \pi_1^P + \gamma_1^P \gamma_2^P)/d,$$

where $\pi_i^P = 1 - \mu_i^P - \gamma_i^P$ and $d = 1 - \mu_1^P \gamma_2^P - \mu_2^P \gamma_1^P$. From

$$d - (\mu_1^P \pi_2^P + \mu_2^P \pi_1^P + \mu_1^P \mu_2^P) - (\gamma_1^P \pi_2^P + \gamma_2^P \pi_1^P + \gamma_1^P \gamma_2^P) = (1 - \mu_1^P - \gamma_1^P)(1 - \mu_2^P - \gamma_2^P) \geqslant 0$$

follows the validity of the definitions of μ_P and γ_P (i.e. $\mu_P + \gamma_P \leqslant 1$).

It is not always meaningful to consider all possible proof paths as in [8], while computing the answer in case of more than one proof path, different combination schemes can be applied.

In the next version of InF-PROLOG we shall include other operators over IFSs, too. For example, we shall realize Zimmermann–Zysno's operator from [17, 18].

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