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Connection between generalized nets with characteristics of the places and intuitionistic fuzzy generalized nets of type 1 and type 2

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Abstract: In Generalized nets with characteristics of the places (GNCP) the places can receive characteristics and keep information about the flow of the tokens in the net. The class of all GNCP Σ_{CP} is a conservative extension of the class of all Generalized nets Σ. In this paper we study the connection between GNCP and the Intuitionistic fuzzy generalized nets of type 1 (IFGN1) and type 2 (IFGN2).

It is proved that the functioning and the result of the work of every GNCP can be represented by IFGN1 and IFGN2. The opposite statement is also proved to be true.
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1 Introduction

Generalized nets with characteristics of the places (GNCP) are defined in [1]. Again there it is proved that Σ_{CP} – the class of all GNCP – is a conservative extension of the class Σ. In a GNCP, places can receive characteristics related to the number of tokens of different types that have entered them, the time moments when the tokens entered the places and other information about the flow of the tokens into the net. The formal definition of a GNCP is:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle$$

where

(a) A is the set of transitions of the net;

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \to \mathcal{N}$, where $\mathcal{N} = \{0, 1, 2, ...\}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to \mathcal{N}$, where L is the set of all GN-places;

(d) c is a function giving the capacities of the places, i.e., $c : L \to \mathcal{N}$;

(e) f is a function that calculates the truth values of the predicates of the transition's conditions;

(f) function

$$\theta_1 = \bigcup_{Z \in A} \theta_1^Z$$

where θ_1^Z is a function giving the next time moment when the transition Z can be activated, i.e., $\theta_1^Z(t) = t'$, where $t = pr_3 Z, t' \in [T, T + t^*]$ and for $1 \le i \le n$, $pr_i\{x_1, x_2, ..., x_n\} = x_i$. The value of this function is calculated at the moment when the transition terminates its current functioning;

(g) function

$$\theta_2 = \bigcup_{Z \in A} \theta_2^Z$$

and θ_2^Z is a function giving the duration of the active state of a given transition Z, i.e., $\theta_2^Z(t) = t'$, where $t = pr_3 Z \in [T, T + t^*]$ and $t' \ge 0$. The value of this function is calculated at the moment when the transition starts functioning;

(h) K is the set of the GN's tokens;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \to \mathcal{N}$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time moment when the GN starts functioning; this moment is determined with respect to a fixed (global) time scale;

(l) t^0 is an elementary time-step, related to the fixed (global) time scale;

(m) t^* is the duration of the GN functioning;

(n) X is a function which assigns initial characteristics to every token when it enters input place of the net. If $\alpha \in K$, then it enters the GN with initial characteristic x_0^α;

(o) Y is function which assigns initial characteristics to the places;

(p) Φ is a characteristic function that assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition. If $\alpha \in K$, then it, entering an output place of some GN-transition and having as current characteristic x_{cu}^α, obtains the next characteristic x_{cu+1}^α;

(q) Ψ is a characteristic function that assigns new characteristics to the places when a token enters the place (this is the new component which cannot be find in the definition of the ordinary GN);

(r) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \to \mathcal{N}$.

The algorithms for the functioning of transition and GNCP are the same as in the standard GNs (see[2]). The only difference is that now the characteristic function Ψ assigns characteristic to the output places for every token that has been transferred. For the definition of GN transition and the algorithms for functioning of transition and GN the reader can refer to [2, 3]. Previously, the idea of assigning characteristics to the places has been known from the Intuitionistic fuzzy generalized nets of type 2 (IFGN2). An IFGN2 has the form:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi\rangle\rangle,$$

where A is the set of transitions which have the ordinary GN form but the capacities of the arcs given by the index matrix M are real numbers. The function c which in the ordinary GNs gives the capacities of the places in non-negative integers now assigns a real value to each place. This real number corresponds to the volume of the place - that is the quantity of matter it can collect. The function f which calculates the truth values of the predicates has the form:

$$f(r_{i,j}) = \langle \mu(r_{i,j}), \nu(r_{i,j})\rangle,$$

where $\mu(r_{i,j})$ is the degree of truth of the predicate $r_{i,j}$ and $\nu(r_{i,j})$ is the degree of falsity. $\mu(r_{i,j}) \in [0, 1]$ and $\nu(r_{i,j}) \in [0, 1]$ satisfy the condition:

$$\mu(r_{i,j}) + \nu(r_{i,j}) \leq 1.$$

Here the tokens are some kind of quantities that flow into the net and do not have initial or other characteristics. Instead the function Φ assigns to every place characteristics - the quantities of the tokens from each type in the place. For the algorithm for transition's functioning in IFGN2 the reader can refer to [4].

In Intuitionistic fuzzy generalized nets of type 1 (IFGN1), the function f which evaluates the predicates of the transitions has the same form as in IFGN2. The tokens, however, are regarded in the classical GN sense and they obtain characteristics. Also, the capacities of the arcs, i.e. the elements of the index matrix M are non-negative integers, whereas in $IFGN2$ they are non-negative real numbers. For the algorithm of functioning of the transitions in IFGN1 the reader can refer to [4].

In [2], it is proved that $\Sigma_{IFGN1} \equiv \Sigma$ and $\Sigma_{IFGN2} \equiv \Sigma$. The same result is proved in [1] for the class Σ_{CP}, i.e. $\Sigma_{CP} \equiv \Sigma$. From all said, it is clear that GNCP and IFGN2 are closely related. In particular, the places in both types can receive characteristics. However, there are significant differences in the definition of the two types and in the way they function. In GNCP the tokens obtain characteristics while in IFGN2 they do not. Also, the capacities of the arcs in IFGN2 are real numbers, while in GNCP they are non-negative integers. The function f that determines the truth values of the predicates in GNCP has values in the set $\{$"$false$", "$true$"$\}$ while in IFGN2 its values are ordered couples of real numbers $\langle \mu(r_{i,j}), \nu(r_{i,j})\rangle$, $\mu(r_{i,j}), \nu(r_{i,j}) \in [0, 1]$ which, as mentioned above, satisfy the condition $\mu(r_{i,j}) + \nu(r_{i,j}) \leq 1$. The function c which in the ordinary GNs gives the capacities of the places in non-negative integers now assigns a non-negative real value to each place. It is interesting to study in details the connection between the class Σ_{CP} and the classes Σ_{IFGN2} and Σ_{IFGN1} and to see how we can represent the work of a GNCP in terms of IFGN1 and IFGN2.

2 Connection between Σ_{CP} and Σ_{IFGN1}

To see how the class Σ_{CP} is related to Σ_{IFGN1} we will first show how it is possible to represent an arbitrary IFGN1 in terms of GNCP.

Theorem 1 *The functioning and the result of work of every IFGN1 can be represented by a GNCP.*

Proof: Let a IFGN1 G be given.

$$G = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi, b\rangle\rangle \tag{1}$$

We will construct a GNCP E which represents the functioning and the result of the work of G. The new net has the following components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f^*, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle \tag{2}$$

All components of E with the exception of f^* and the function Ψ are the same as in G. Before we define f^* we shall note that in the general algorithm for transition functioning in IFGN1 (see [4]) the transfer of tokens from input to output places is determined by one of the following conditions:

C1 $\mu(r_{i,j}) = 1,\ \nu(r_{i,j}) = 0$ (the case of ordinary GN)

C2 $\mu(r_{i,j}) > \frac{1}{2}\ (> \nu(r_{i,j}))$

C3 $\mu(r_{i,j}) \geq \frac{1}{2}\ (\geq \nu(r_{i,j}))$

C4 $\mu(r_{i,j}) > \nu(r_{i,j})$

C5 $\mu(r_{i,j}) \geq \nu(r_{i,j})$

C6 $\mu(r_{i,j}) > 0$

C7 $\nu(r_{i,j}) < 1$, i.e. at least $\pi(r_{i,j}) > 0$, where $\pi(r_{i,j}) = 1 - \mu(r_{i,j}) - \nu(r_{i,j})$ is the degree of uncertainty(indeterminancy).

The condition for transfer of the tokens which will be used is determined for every transition before the firing of the net. In order to preserve this condition in E, where the function f^* assigns to the predicates values from the set $\{0, 1\}$, for the condition $C1$ we define f^* in the following way:

C1* $f^*(r_{i,j}) = \lfloor pr_1 f(r_{i,j})\rfloor$.
where $\lfloor x\rfloor$ is the floor function which maps a real number x to the largest integer smaller or equal to x.

Similarly, in the other cases we define $f^*(r_{i,j})$ as follows:

$$\mathbf{C2}^* \quad f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) > \frac{1}{2} \\ 0\,, & \text{otherwise} \end{cases}$$

$$\mathbf{C3}^* \quad f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) \geq \frac{1}{2} \\ 0\,, & \text{otherwise} \end{cases}$$

$$\mathbf{C4}^* \quad f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) > \nu(r_{i,j}) \\ 0\,, & \text{otherwise} \end{cases}$$

$$\mathbf{C5}^* \quad f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) \geq \nu(r_{i,j}) \\ 0\,, & \text{otherwise} \end{cases}$$

$$\mathbf{C6}^* \quad f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \mu(r_{i,j}) > 0 \\ 0\,, & \text{otherwise} \end{cases}$$

$$\mathbf{C7}^* \quad f^*(r_{i,j}) = \begin{cases} 1\,, & \text{if } \nu(r_{i,j}) < 1 \\ 0\,, & \text{otherwise} \end{cases}$$

Since in IFGN1 the places do not obtain characteristics we can consider that the characteristic functions Y and Ψ of E does not assign any charcteristics to the places, i.e. $\Psi(l) = \emptyset$, $Y(l) = \emptyset$ for all places l. Although the GNCP E has the same graphic structure as G, for every transition $Z_i \in pr_1pr_1G$ we will denote with Z_i^* its corresponding transition in E (that is a transition with the same components as Z_i). Let now $Z \in pr_1pr_1G$ and $Z^* \in pr_1pr_1E$ be two corresponding transitions. To prove that the GNCP E reperesents the work of G we will use the theorem for the completeness of the GN transitions which states that every GN can be constructed only from the set of its transitions and operations union, concatenation and iteration defined over transitions (see [3]). Both transitions have the same number of input and output places, the same time components, the same index matrices with predicates and capacities of the arcs. Let $\alpha \in pr_1pr_2G$ and $\alpha^* \in pr_1pr_2E$ be two tokens with equal characteristics that are in two corresponding input places $l_i \in pr_1Z$ and $l_i^* \in pr_1Z^*$. Depending on the execution of the operator for permission or prohibition of tokens' splitting the token α will be transferred either to all permitted output places or to the place with the highest priority among all output places. The transfer of α is determined by one of the conditions $C1,\ C2,\ ...,\ C7$. Let the conditions allow the transfer to output place l_j (the case where splitting of tokens is allowed is analogous). At the same time α^* in place l_i^* will be transferred to the output place l_j^* because the function $f^*(r_{i,j}) = 1$ if the corresponding condition for the transfer from l_i to l_j is satisfied. Upon entering l_j^* the token α^* obtains the same characteristic as the token α because the characteristic function Φ is the same for the two nets. If α can not be transferred to any output places of Z, then the token α^* also can not be transferred because of the definition of f^*. Therefore the two transitions function equally. Since the corresponding transitions Z and Z^* are arbitrarily chosen, from the Theorem for the completeness of the GN transitions it follows that the GNCP E represents the functioning and result of the work of G. □

Theorem 2 *The functioning and the results of the work of every GNCP can be represented by an IFGN1.*

Proof: Let E be a GNCP with components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle$$

We will construct an IFGN1 G which preserves the result of the work of E. Let $Z \in pr_1 pr_1 E$ be arbitrary transition with components $Z = \langle L', L'', t_1, t_2, r, M, \square\rangle$(see Fig.1)

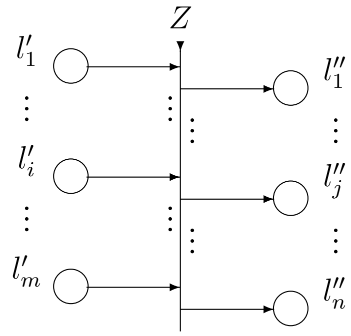

Fig. 1.

For every such transition Z we construct a corresponding transition Z^* with components $Z^* = \langle L'^*, L''^*, t_1, t_2, r^*, M^*, \square^*\rangle$(see Fig.2)

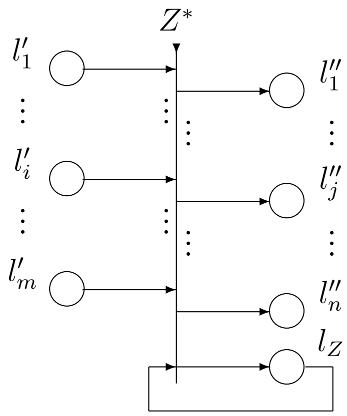

Fig. 2.

where Z^* is obtained from Z with the addition of a new place l_Z which is input and output for the transition.

$$L'^* = L' \cup \{l_Z\}$$

$$L''^* = L^{''} \cup \{l_Z\}$$

We use the same notation for the places in Z and Z^* to avoid complicating the notation. Both transitions become active at the same time and have equal durations of their active states. In place l_Z a token α_Z will loop and keep the characteristics of the output places of Z. The rest of the components of Z^* are defined as follows. If

$$r = pr_5 Z = [L', L'', \{r_{l'_i, l''_j}\}]$$

is the IM of the transition's condition, then

$$r^* = pr_5 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r^*_{l'_i, l''_j}\}]$$

where

$$(\forall l'_i \in L')(\forall l''_j \in L'')(r^*_{l'_i,l''_j} = r_{l'_i,l''_j})$$

$$(\forall l'_i \in L')(\forall l''_j \in L'')(r^*_{l'_i,l_Z} = r^*_{l_Z,l''_j} = "false"),$$
$$r^*_{l_Z,l_Z} = "true";$$

If

$$M = pr_6 Z = [L', L'', \{m_{l'_i,l''_j}\}]$$

has the form of an IM, then

$$M^* = pr_6 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m^*_{l'_i,l''_j}\}],$$

where

$$(\forall l'_i \in L')(\forall l''_j \in L'')(m^*_{l'_i,l''_j} = m_{l'_i,l''_j}),$$
$$(\forall l'_i \in L')(\forall l''_j \in L'')(m^*_{l'_i,l_Z} = m^*_{l_Z,l''_j} = 0),$$
$$m^*_{l_Z,l_Z} = 1$$

Let A^* be the set of all transitions obtained after reapeating the above procedure for all transitions of E. Let the components of G be

$$G = \langle\langle A^*, \pi^*_A, \pi^*_L, c^*, f, \theta_1, \theta_2\rangle, \langle K^*, \pi^*_K, \theta^*_K\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b^*\rangle\rangle$$

where

$$(\forall Z^*_i \in A^*)(\pi^*_A(Z^*_i) = \pi_A(Z_i))$$
$$\pi^*_L = \pi_L \cup \pi_{\{l_Z|Z\in A\}},$$

where function $\pi_{\{l_Z|Z\in A\}}$ determines the priorities of the new places that are elements of set $\{l_Z|Z \in A\}$ and the priorities of the places l_Z for every transition $Z \in A$ are the minimal among the priorities of all other places of the transition Z.

$$c^* = c \cup c_{\{l_Z|Z\in A\}},$$

where function $c_{\{l_Z|Z\in A\}}$ satisfies the equality

$$c_{\{l_Z|Z\in A\}}(l_Z) = 1,$$

We shall note that if Q^I is the set of the input places of the GN, then, as it is noted in [3] the set K can be represented by

$$K = \bigcup_{l\in Q^I} K_l,$$

where K_l is the set of the GN-tokens that enter the GN through place l. Now,

$$K^* = (\bigcup_{l\in Q^I} K_l)\bigcup\{\alpha_Z|Z \in A\},$$

i.e. the set of the tokens of G consists of all tokens of E and all additional tokens in the l_Z-places.

$$\pi_K^* = \pi_K \cup \pi_{\{l_Z|Z \in A\}}$$

where the function $\pi_{\{l_Z|Z \in A\}}$ determines the priorities of the α_Z tokens. The α_Z tokens stay in the l_Z places during the entire period of functioning of the net and no other tokens can enter the l_Z places. The α_Z tokens should have the lowest priority among all tokens of the net.

$$\theta_K^* = \theta_K \cup \theta_{\{l_Z|Z \in A\}},$$

where $\theta_{\{l_Z|Z \in A\}}$ determines that each α_Z-token stays in the initial time-moment T in the l_Z place.

$$X^* = X \cup \{x_0^{\alpha_Z}|Z \in A\},$$

where $x_0^{\alpha_Z}$ is the initial characteristic of token α_Z and it is a list of all output places for the transition Z with their initial characteristics:

$$\text{“}\langle l_1'', Y(l_1'')\rangle, \langle l_2'', Y(l_2'')\rangle, \cdots \langle l_n'', Y(l_n'')\rangle\text{”}$$

$$\Phi^* = \Phi' \cup \Psi^*_{\{l_Z|Z \in A\}},$$

where the function $\Psi^*_{\{l_Z|Z \in A\}}$ determines the characteristics of the α_Z-tokens in the form

$$\Psi^*_{\{l_Z|Z \in A\}}(\alpha_Z) = \text{“}\{\langle l_j'', \Psi(l_j'')\rangle | l_j'' \in L''\}\text{”}.$$

The characteristic function Φ^* assigns to every token α_Z the characteristics of the places of the transition Z of E. The function Φ' which assigns the same characteristics to the tokens that have been transferred as the function Φ in E. If we strictly follow the definition of IFGN1, we should define $\Phi'(\alpha) = \langle \Phi(\alpha), \langle \mu(r_{i,j}), \nu(r_{i,j})\rangle\rangle$. However, in this case the couple $\langle \mu(r_{i,j}), \nu(r_{i,j}\rangle = \langle 1, 0\rangle$ as this is the only way for the token to be transferred.

$$b^* = b \cup b_{\{\alpha_Z|Z \in A\}}$$

where the function $b_{\{\alpha_Z|Z \in A\}}(\alpha) = \infty$ determines the number of characteristics the α_Z tokens can keep.

To prove that the so constructed IFGN1 G represents the funtioning and the results of work of E let us take an arbitrary pair of corresponding transitions $Z_i \in pr_1 pr_1 E$ and $Z_i^* \in pr_1 pr_1 G$. Let $\alpha \in K$ and $\beta \in K^*$ be two tokens of the same type with equal characteristics which are at two corresponding places of the transitions at some moment of time. Apparently neither of them is α_Z token. From the definition of the transition Z_i^* it is clear that the token β can be transferred to some output place $l_j''^*$ if and only if the token α can be transferred to the output place l_j'' (here $l_j''^*$ and l_j'' are two corresponding output places of the two transitions). The two tokens will receive the same characteristics because the characteristic functions Φ and Φ' coincide in all places except the l_Z places. The characteristic funtion Ψ assigns characteristic to the place l_j''. The same characteristic is assigned to the α_Z token. Since the transitions and the tokens were arbitrarily chosen, we can conclude that G represents the funtioning and the result of work of E. The case where splitting of tokens is allowed is analogous. □

From the two theorems above it follows

Theorem 3 $\Sigma_{CP} \equiv \Sigma_{IFGN1}$.

3 Connection between Σ_{CP} and Σ_{IFGN2}

We already pointed out the basic differences between GNCP and IFGN2. Before we show how we can represent the work of IFGN2 in terms of GNCP and vice versa we need to mention the general algorithm for transition functioning in IFGN2 proposed in [4]. As in the case of IFGN1, the transfer of the tokens from input place to output place of a given transition is determined by one of the following conditions:

C1 $\mu(r_{i,j}) = 1, \nu(r_{i,j}) = 0$ (the case of ordinary GN)

C2 $\mu(r_{i,j}) > \frac{1}{2}$ $(> \nu(r_{i,j}))$

C3 $\mu(r_{i,j}) \geq \frac{1}{2}$ $(\geq \nu(r_{i,j}))$

C4 $\mu(r_{i,j}) > \nu(r_{i,j})$

C5 $\mu(r_{i,j}) \geq \nu(r_{i,j})$

C6 $\mu(r_{i,j}) > 0$

C7 $\nu(r_{i,j}) < 1$, i.e. at least $\pi(r_{i,j}) > 0$,
where $\pi(r_{i,j}) = 1 - \mu(r_{i,j}) - \nu(r_{i,j})$ is the degree of uncertainty (indeterminancy).

The condition for transfer of the tokens which will be used is determined for every transition before the firing of the net. When the condition for transfer is satisfied the token with the highest priority in the i-th input place will be distributed to j-th output place according to the value of $\mu(r_{i,j})$. The quantity that remains in the i-th input place corresponds to the degree of falsity $\nu(r_{i,j})$. Along the arc connecting the i-th input place and the j-th output place remains the rest of the matter which is given by $\pi(r_{i,j}) = 1 - \mu(r_{i,j}) - \nu(r_{i,j})$. This interpretation of the degrees of truth and falsity of the predicates requires the following restriction to be imposed:

$$\sum_j \mu(r_{i,j}) \leq 1.$$

First we will prove the following theorem.

Theorem 4 *The functioning and the result of the work of every IFGN2 can be represented by a GNCP.*

Proof: Let an IFGN2 G be given.

$$G = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi\rangle\rangle$$

We will construct a GNCP E based on G which represents the functioning and the results of the work of G. Let $Z = \langle L', L'', t_1, t_2, r, M, \Box\rangle$ be arbitrary transition of G. We will construct a corresponding transition $Z^* = \langle L', L'', t_1, t_2, r, M^*, \Box\rangle$ which has the same graphic structure as Z, the same number of input and output places, the same time components, the same predicates and the same transition type $\square$. The only difference is that all elements of the index matrix M^* of the capacities of the arcs in Z^* are ∞. Let A^* be the set of all transitions obtained from the transitions of G by the above procedure. Let the GNCP E have the following components:

$$E = \langle\langle A^*, \pi_{A^*}, \pi_L, c^*, f^*, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi^*, \Psi, b\rangle\rangle$$

The function π_{A^*} which determines the priorities of the transitions assigns to every transition of E the priority of its corresponding transition in G: $\pi_{A^*}(Z_i^*) = \pi_A(Z_i)$ for all $Z_i^* \in A^*$. The priorities of the places in E and the functions θ_1 and θ_2 are also the same. We use the same notation for them in E. Since the capacities of the places in E must be positive integers, we define $c^*(l_i) = \lceil c(l_i)\rceil$ for all places $l_i \in L$, where $\lceil x\rceil$ is the ceiling function which maps a real number x to the smallest integer greater or equal to it. The codomain of the function f^* which determines the degrees of truth and falsity of the predicates in the case of GNCP is the set $\{0, 1\}$. As it was in the case of IFGN1 the values of $f^*(r_{i,j})$ depend on the conditions for the transfer $C1, C2, ..., C7$.

$\mathbf{C1}^*$ $f^*(r_{i,j}) = \lfloor pr_1 f(r_{i,j})\rfloor$.

where $\lfloor x\rfloor$ is the floor function which maps a real number x to the largest integer smaller or equal to x.

$\mathbf{C2}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

$\mathbf{C3}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

$\mathbf{C4}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

$\mathbf{C5}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) \geq \nu(r_{i,j}) \\ 0, & \text{otherwise} \end{cases}$

$\mathbf{C6}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \mu(r_{i,j}) > 0 \\ 0, & \text{otherwise} \end{cases}$

$\mathbf{C7}^*$ $f^*(r_{i,j}) = \begin{cases} 1, & \text{if } \nu(r_{i,j}) < 1 \\ 0, & \text{otherwise} \end{cases}$

Here, $\mu(r_{i,j})$ and $\nu(r_{i,j})$ are the degrees of validity and non-validity of the predicate $r_{i,j}$ which are determined by $f(r_{i,j}) = \langle\mu(r_{i,j}), \nu(r_{i,j})\rangle$. All other components of E are the same as in G except Y, Φ^* and Ψ. The tokens in $IFGN2$ do not receive characteristics and therefore we define $\Phi^*(\alpha) = \emptyset$ for all $\alpha \in pr_1 pr_2 E$. The places in E does not have initial characteristics: $Y(l) = \emptyset$ for all places l. The new characteristic function Ψ in E assigns the same characteristics to the places in E as the function Φ assigns to the places in G: $\Psi(l) = \Phi(l)$ for all places l.

To prove that the so constructed GNCP E represents the functioning and the result of the work of G we will use the theorem for the completeness of the GN transitions. Let $Z \in pr_1 pr_1 G$ and $Z^* \in pr_1 pr_1 E$ be two corresponding transitions, i.e. Z^* is obtained from Z by the procedure described above.

We will trace the behavior of two corresponding tokens $\alpha \in pr_1pr_2G$ and $\alpha^* \in pr_1pr_2E$ which are in two corresponding input places l'_i and l'^*_i at the same moment of time. Here we denote the corresponding to l'_i input place l'^*_i to avoid any ambiguity. Without loss of generality we can consider that the two input places have the same characteristics. We can do this because at least for the transitions which have input places for the two nets it is true – the characteristic functions which assign initial characteristics to the tokens coincide. By induction from the proof that Z and Z^* function similarly (which we will prove) it will follow that the corresponding places have the same characteristics. If the quantity of matter α is distributed to output places $l''_j, l''_{j+1}, \cdots, l''_{j+k}$ in Z, then the corresponding token α^* splits into $k+1$ tokens which enter the corresponding places $l''^*_j, l''^*_{j+1}, \cdots, l''^*_{j+k}$. This is so because $f^*(r_{i,j}) = 1$ if the conditions for transfer in G - $C1, C2, \cdots, C7$ allow the transfer of the token. The characteristic functions for the positions Φ and Ψ assign equal characteristics to the corresponding places. At the end of the current time step the two tokens will be transferred to corresponding places and these places will have the same characteristics. Therefore the two transitions function in the same way. From the theorem for the completeness of the GN transitions it follows that E represents the functioning and the results of the work of G. $\square$

Next we will prove that for every GNCP E there exists an IFGN2 which describes the functioning and the result of work of E.

Theorem 5 *The functioning and the result of work of every GNCP can be represented by an IFGN2.*

Proof: Let E be a GNCP with components:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle$$

For every transition $Z = \langle L', L'', t_1, t_2, r, M, \square \rangle$ of E we construct corresponding transition Z^* in the same way as in the proof of Theorem 2. It has one more place l_Z which is input and output for the transition (see Fig. 2). All components are of Z^* are defined in the same way as in the proof of Theorem 2. In place l_Z a token α_Z loops and the place does not obtain new characteristics. Its initial characteristic is a list with the initial characteristics of all places of the transition determined by the function Y. Let A^* be the set of all transitions obtained by the above procedure. Let G be IFGN2 with components:

$$G = \langle\langle A^*, \pi^*_A, \pi^*_L, c^*, f, \theta_1, \theta_2\rangle, \langle K^*, \pi^*_K, \theta^*_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi^*\rangle$$

The components of the net are defined in the same way as in the proof of Theorem 2. The function Φ^* assigns to the places of G both the characteristics of the tokens that are transferred to the output places in E which are determined by function Φ and the characteristics of the output places determined by Ψ.

$$\Phi^*(l_j) = \langle\langle \alpha_1, \Phi_{l_j}(\alpha_1)\rangle, \langle \alpha_2, \Phi_{l_j}(\alpha_2)\rangle, \cdots, \langle \alpha_k, \Phi_{l_j}(\alpha_k)\rangle, \Psi(l_j)\rangle$$

where $\alpha_1, ..., \alpha_k$ are the tokens that has entered the output place l_j and $\Psi(l_j)$ is the characteristic of the corresponding place of E. Let Z_i and Z_i^* be two corresponding transitions of E and G. To compare their work let a token α be in the input place l'_i of Z_i and a corresponding token α^* be in the corresponding to l'_i input place l'^*_i of G. Apparently neither of them is α_Z token. Without loss of generality we can consider that the characteristics of α and l'_i coincide with characteristics of the place l'^*_i. If the splitting of tokens is prohibited, the truth value of the predicate $r_{i,j}$ is "$true$" and if the other conditions for the transfer allow it, the token α will be transferred to place $l''_j \in pr_2 Z_i$. The token α^* will be transferred to the corresponding output place $l''^*_j \in pr_2 Z_i^*$. The token α and the place l''_j in Z_i will receive the characteristics $\Phi_{l_j}(\alpha)$ and $\Psi(l_j)$. From the definition of the characteristic function Φ^* it is clear that this characteristics will be included in the characteristics of l''^*_j. The case when splitting of tokens is allowed is analogous. Therefore the two transitions function similarly and all information relevant to Z_i is also present in Z_i^*. From the theorem for completeness of the GN transitions it follows that G represents the functioning and the results of the work of E. $\square$

From the two theorems above it follows

Theorem 6 $\Sigma_{IFGN2} \equiv \Sigma_{CP}$.

4 Conclusion

The theorems in this paper state that given a GNCP we can construct IFGN1 and IFGN2(or vice versa), which preserve the functioning and the results of the work of the given net. Since the proofs are constructive, they are important for the applications of GNs in the modelling of real processes. In future, we intend to define intuitionistic fuzzy GNCP and study their relations to IFGN1 and IFGN2.

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