Fourteeth Int. Conf. on IFSs, Sofia, 15-16 May 2010
NIFS Vol. 16 (2010), 2, 35-38

On the new intuitionistic fuzzy operator $x_{a,b,c,d,e,f}$

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Over Intuitionistic Fuzzy Sets (IFSs, see [2]) different operators from modal type are defined. Here, we shall introduce an extension of a group of these operators and will discuss some of its properties.

Let $\alpha, \beta \in [0, 1]$. We will define (see [3]) six operators over a given IFS

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

by:

$$\begin{array}{ll} f_{\alpha,\beta}(A) & = \{\langle x, \nu_A(x) + \alpha.\pi_A(x), \mu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1, \\ g_{\alpha,\beta}(A) & = \{\langle x, \alpha.\nu_A(x), \beta.\mu_A(x)\rangle | x \in E\}, \\ h_{\alpha,\beta}(A) & = \{\langle x, \alpha.\nu_A(x), \mu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \\ h^*_{\alpha,\beta}(A) & = \{\langle x, \alpha.\nu_A(x), \mu_A(x) + \beta.(1 - \alpha.\nu_A(x) - \mu_A(x))\rangle | x \in E\}, \\ j_{\alpha,\beta}(A) & = \{\langle x, \nu_A(x) + \alpha.\pi_A(x), \beta.\mu_A(x)\rangle | x \in E\}, \\ j^*_{\alpha,\beta}(A) & = \{\langle x, \nu_A(x) + \alpha.(1 - \nu_A(x) - \beta.\mu_A(x)), \beta.\mu_A(x)\rangle | x \in E\}, \end{array}$$

For every two IFSs A and B a lot of relations and operations are defined (see, e.g. [2]), the most important of which are:

$$\begin{array}{lll} A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\ A \supset B & \text{iff} & B \subset A; \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\ A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\ A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\ A@B & = & \{\langle x, (\frac{\mu_A(x)+\mu_B(x))}{2}, \frac{(\nu_A(x)+\nu_B(x))}{2}\rangle | x \in E\}; \\ A & = & \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}. \end{array}$$

Following [2] we shall mention that in 1991 the author published in [1] the following operator which is universal for the extended modal operators $F_{\alpha,\beta}, G_{\alpha,\beta}, H_{\alpha,\beta}, H^*_{\alpha,\beta}, J_{\alpha,\beta}, J^*_{\alpha,\beta}$, defined in [2]. Let:

$$\begin{array}{ll} X_{a,b,c,d,e,f}(A) = & \{\langle x, a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)), \\ & d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x))\rangle | x \in E\} \end{array}$$

where $a, b, c, d, e, f \in [0, 1]$ and:

$$a + e - e.f \leq 1, \tag{1}$$

$$b + d - b.c \leq 1. \tag{2}$$

All the above mentioned operators can be represented by operator X, at suitably chosen values of its parameters. These representations are the following:

$$\begin{array}{ll} F_{a,b}(A) & = X_{1,a,1,1,b,1}(A), \\ G_{a,b}(A) & = X_{a,0,r,b,0,s}(A), \\ H_{a,b}(A) & = X_{a,0,r,1,b,1}(A), \\ H^*_{a,b}(A) & = X_{a,0,r,1,b,a}(A), \\ J_{a,b}(A) & = X_{1,a,1,b,0,r}(A), \\ J^*_{a,b}(A) & = X_{1,a,b,b,0,r}(A), \end{array}$$

where r and s are arbitrary real numbers in interval $[0, 1]$.

By analogy with this operator, here we will introduce an operator, that is universal for the extended modal operators $f_{\alpha,\beta}, g_{\alpha,\beta}, h_{\alpha,\beta}, h^*_{\alpha,\beta}, j_{\alpha,\beta}, j^*_{\alpha,\beta}$ by

$$\begin{array}{ll} x_{a,b,c,d,e,f}(A) = & \{\langle x, a.\nu_A(x) + b.(1 - \nu_A(x) - c.\mu_A(x)), \\ & d.\mu_A(x) + e.(1 - f.\nu_A(x) - \mu_A(x))\rangle | x \in E\} \end{array}$$

where $a, b, c, d, e, f \in [0, 1]$ satisfy (1) and (2).

The notation $X_{a,b,c,d,e,f}$ was proposed about 20 years ago and its current modification is denoted here by $x_{a,b,c,d,e,f}$. The author believes that this notation, concurrently existing with the notation of variable x as an element of the universe, will not hinder the readers' understanding of the results.

Theorem 1: For every IFS A and for every $a, b, c, d, e, f \in [0, 1]$ satisfying (1) and (2), $x_{a,b,c,d,e,f}(A)$ is an IFS.

Proof. Let the IFS A and six real numbers $a, b, c, d, e, f \in [0, 1]$ satisfying (1) and (2), be given. Let

$$Y \equiv a.\nu_A(x) + b.(1 - \nu_A(x) - c.\mu_A(x)) + d.\mu_A(x) + e.(1 - f.\nu_A(x) - \mu_A(x))$$

$$= (d - b.c - e).\mu_A(x) + (a - b - e.f).\nu_A(x) + b + e.$$

If $\mu_A(x) = 0$ and $\nu_A(x) = 1$, then from (1) we obtain

$$Y = (a - b - e.f).\nu_A(x) + b + e = a + e - e.f \leq 1$$

and, obviously, $Y \geq 0$.

If $\nu_A(x) = 0$ and $\mu_A(x) = 1$, then from (2) we obtain

$$Y = (d - b.c - e).\mu_A(x) + b + e = d + b - b.c \leq 1$$

and, obviously, $Y \geq 0$.

If $0 \leq \mu_A(x), \nu_A(x) < 1$, then from $\mu_A(x) \leq 1 - \nu_A(x)$ we obtain

$$Y = (d - b.c - e).\mu_A(x) + (a - b - e.f).\nu_A(x) + b + e$$

$$\leq (d - b.c - e).\mu_A(x) + (a - b - e.f).(1 - \mu_A(x)) + b + e$$

$$= (d - b.c - e - a + b + e.f).\mu_A(x) + a - b - e.f + b + e$$
$$= (d - b.c - e - a + b + e.f).\mu_A(x) + a - e.f + e$$
$$\leq d - b.c - e - a + b + e.f + a - e.f + e$$
$$= d - b.c + b \leq 1.$$

On the other hand,

$$Y \geq b + e \geq 0.$$

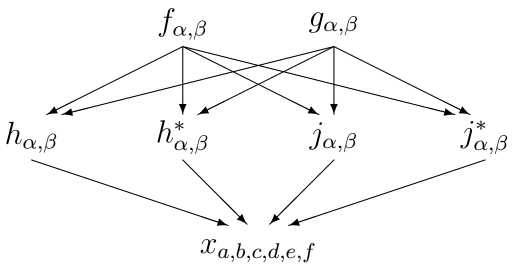
Therefore, $x_{a,b,c,d,e,f}(A)$ is an IFS.

All the above operators can be represented by the operator x at suitably chosen values of its parameters. These representations are the following:

$$\begin{array}{ll}
f_{a,b}(A) & = X_{1,a,1,1,b,1}(A), \\
g_{a,b}(A) & = X_{a,0,r,b,0,s}(A), \\
h_{a,b}(A) & = X_{a,0,r,1,b,1}(A), \\
h^*_{a,b}(A) & = X_{a,0,r,1,b,a}(A), \\
j_{a,b}(A) & = X_{1,a,1,b,0,r}(A), \\
j^*_{a,b}(A) & = X_{1,a,b,b,0,r}(A),
\end{array}$$

where r and s are arbitrary real numbers in interval $[0, 1]$.

All the above new operators can be systematized, constructing the following scheme:



Theorem 2: Let for $a, b, c, d, e, f, g, h, i, j, k, l \in [0, 1]$ it holds that

$$\begin{array}{llllllr}
u & = & a.j & +b & -b.j & -b.e.h + b.c.h.i & \geq 0, \\
v & = & a.k & +b & -b.c.h & -b.k & > 0, \\
w & = & b.k.l & +b.c.h & -a.k.l & -b.c.g & \geq 0, \\
x & = & d.g & +e & -e.f.k & +e.f.k.l - e.g & \geq 0, \\
y & = & d.h & +e & -e.f.k & -e.h & > 0, \\
z & = & e.f.k & +e.f.k & -d.h.i & -e.f.j & \geq 0, \\
v & \geq w, & & & & & \\
y & \geq z. & & & & &
\end{array}$$

Then:

$$X_{a,b,c,d,e,f}(X_{g,h,i,j,k,l}(A)) = X_{u,v,w/v,x,y,z/y}.$$

Theorem 3: For every two IFSs A and B and for every $a, b, c, d, e, f \in [0,1]$ satisfying (1) and (2):

$$(a)\overline{x_{a,b,c,d,e,f}(\overline{A})} = x_{d,e,f,a,b,c}(A),$$

$$(b)x_{a,b,c,d,e,f}(\overline{A}) = X_{a,b,c,d,e,f}(A),$$

$$(c)X_{a,b,c,d,e,f}(A@B) = X_{a,b,c,d,e,f}(A)@X_{a,b,c,d,e,f}(B).$$

Theorem 4: For $a, b, c, d, e, f, g, h, i, j, k, l \in [0,1]$ such that (1), (2) and

$$g + k - k.l \leq 1, \tag{3}$$

$$h + j - h.i \leq 1 \tag{4}$$

are valid, and for a given IFS A:

$$(a)x_{a,b,c,d,e,f}(A) \cap x_{g,h,i,j,k,l}(A) \supset X_{\min(a,g),\min(b,h),\max(c,i),\max(d,j),\max(e,k),\min(f,l)}(A),$$

$$(b)x_{a,b,c,d,e,f}(A) \cup x_{g,h,i,j,k,l}(A) \subset x_{\max(a,g),\max(b,h),\min(c,i),\min(d,j),\min(e,k),\max(f,l)}(A),$$

$$(c)x_{a,b,c,d,e,f}(A)@x_{g,h,i,j,k,l}(A) = x_{\frac{a+g}{2}+\frac{b+h}{2}+\frac{bc+hi}{b+h}+\frac{d+j}{2}+\frac{e+k}{2}+\frac{ef+kl}{e+k}}(A).$$

The validity of the other expressions is checked analogously.

Acknowledgement

The author is grateful for the support provided by the project of the Bulgarian Ministry of Education, Youth and Science (Grant BIn-2/09).

References

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