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INTUITIONISTIC FUZZY DATABASE

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Abstract. We introduce a concept of intuitionistic fuzzy database(IFDB). We study some intuitionistic fuzzy queries on an intuitionistic fuzzy database.

Keywords: Intuitionistic fuzzy set(IFS), intuitionistic fuzzy relation(IFR), α-similar, intuitionistic fuzzy database(IFDB).

Introduction

Out of several higher order fuzzy sets[2, 21, 22, 23, 24, 25], the notion of intuitionistic fuzzy sets (IFS) defined by Atanassov [2] is interesting and useful. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true [2]. Besides, it has been cultured in [15] that vague sets[26] are nothing but intuitionistic fuzzy sets. IFS theory has been applied in different areas viz., Logic Programming [8], Decision Making Problems [18, 28], Optimization Problem[1], Medical Diagnosis [19] etc.. In the present paper, we study intuitionistic fuzzy relations and introduce the concept of intuitionistic fuzzy database(IFDB).

1 Preliminaries

We present here relevent preliminaries required for the progress of this paper.

Definition 2.1

Let a set E be fixed. An intuitionistic fuzzy set or IFS A in E is an object having the form

A= $\{ < x, \mu_A (x), \nu_A (x) > | x \in E \}$

where the function $\mu_A : E \rightarrow [0, 1]$ and $\nu_A : E \rightarrow [0, 1]$ define the degree of membership

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and degree of non-membership respectively of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

The amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part, which may cater either membership value or non-membership value or to both.

Definition 2.2

If A and B are two IFSs of the set E, then

$A \subset B$ iff $\forall x \in E$, $[\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)]$

$A \subset B$ iff $B \supset A$

$A = B$ iff $\forall x \in E$, $[\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)]$

$\bar{A} = \{<x, \nu_A(x), \mu_A(x)> \mid x \in E\}$

$A \cap B = \{<x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))> \mid x \in E\}$

$A \cup B = \{<x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))> \mid x \in E\}$

Obviously every fuzzy set has the form

$\{<x, \mu_A(x), \mu_{A^c}(x)> \mid x \in E\}$

Definition 2.3

Let X and Y be two sets. An intuitionistic fuzzy relation (IFR) R from X to Y is an IFS of $X \times Y$ characterized by the membership function μ_R and non-membership function ν_R. An IFR R from X to Y will be denoted by $R(X \to Y)$ and defined by

$$R = \{ <(x, y), \mu_R(x, y), \nu_R(x, y)> \mid x \in X, y \in Y \}$$

where $\mu_R : X \times Y \to [0, 1]$ and $\nu_R : X \times Y \to [0, 1]$ satisfy the condition

$0 \leq \mu_R(x, y) + \nu_R(x, y) \leq 1$ for every $(x, y) \in X \times Y$.

The complementary relation of R is

$$R_c = \{ <(x, y), \nu_R(x, y), \mu_R(x, y)> \mid x \in X, y \in Y \}$$

Definition 2.4

Let $Q(X \to Y)$ and $R(Y \to Z)$ be two IFRs. The max-min-max composition RoQ is the intuitionistic fuzzy relation from X to Z, defined by the membership function

$$\mu_{Q \circ R}(x, z) = \bigvee_y [\mu_Q(x, y) \wedge \mu_R(y, z)]$$

and the non-membership function

$$\nu_{Q \circ R}(x, z) = \bigwedge_y [\nu_Q(x, y) \vee \nu_R(y, z)]$$

$\forall (x, z) \in X \times Z$ and $\forall y \in Y$.

Definition 2.5

An IFR $R(X \times X)$ is said to be

(i) reflexive : iff $\forall x \in X$, $\mu_R(x, x) = 1$,

(ii) Symmetric : iff $\forall x_1, x_2 \in X$, $\mu_R(x_1, x_2) = \mu_R(x_2, x_1)$ and $\nu_R(x_1, x_2) = \nu_R(x_2, x_1)$,

(iii) transitive : if $R^2 \subseteq R$ where $R^2 = R \circ R$.

The transitive closure of an IFR R on $X \times X$ is $\hat{R}$ defined by

$$\hat{R} = R \cup R^2 \cup R^3 \cup \cdots.$$

An intuitionistic fuzzy relation R on the cartesian set $(X \times X)$, is called

1. an intuitionistic tolerance relation on $X \times X$ if R is reflexive and symmetric.

2. an intuitionistic similarity (intuitionistic equivalence) relation on X $\times$ X if R is reflexive, symmetric and transitive.

Definition 2.6 [17]

Let A be an IFS of the set E. For $\alpha \in [0, 1]$, the α-cut of A is the crisp set A_α defined by

$A_\alpha = \{ x : x \in E$, either $\mu_A(x) \geq \alpha$ or $\nu_A(x) \leq 1 - \alpha \}$.

It may be noted that the condition $\mu_A(x) \geq \alpha$ ensures $\nu_A(x) \leq 1 - \alpha$ but not conversely.

So, we can define α-cut of A as $A_\alpha = \{ x : x \in E, \nu_A(x) \leq 1 - \alpha \}$.

2 On Intuitionistic Fuzzy Relations

In this section, we study some properties of IFR. We start with some definitions.

Definition 3.1

If T be an intuitionistic fuzzy tolerance relation on X, then given an $\alpha \in [0, 1]$, two elements x, y $\in$ X are α-similar (denoted by
$x\ T_\alpha\ y$) if and only if $\nu_T(x, y) \leq 1 - \alpha$.

Definition 3.2

If T be an intuitionistic fuzzy tolerance relation on X, then given an $\alpha \in [0, 1]$, two elements x, z $\in$ X are α-tolerate (denoted by $x\ T_\alpha^+\ z$) if and only if either $x\ T_\alpha\ z$ or there exists a sequence $y_1, y_2, \cdots y_r \in X$ such that $x\ T_\alpha\ y_1\ T_\alpha\ y_2\ T_\alpha\ y_3 \cdots T_\alpha\ y_r\ T_\alpha\ z$.

Lemma 3.1

If T be an intuitionistic fuzzy tolerance relation on X, then T_α^+ is an equivalence relation. For any $\alpha \in [0, 1]$, T_α^+ partitions X into disjoint equivalence classes. If T is an intuitionistic fuzzy similarity relation on X then T_α is an equivalence relation for any $\alpha \in [0, 1]$.

Lemma 3.2

Let T is an intuitionistic fuzzy similarity relation on X and $\alpha \in [0, 1]$ be fixed. Y $\subseteq$ X is an equivalence class in the partition determined by T_α with respect to T if and only if Y is a maximal subset obtained by merging elements from X that satisfy

$\max_{x,y \in Y} [\ \nu_T(x, y)\ ] \leq 1 - \alpha$.

Lemma 3.3

If T is an intuitionistic fuzzy similarity relation on X, then for any $\alpha \in [0, 1]$, T_α and T_α^+ generate identical equivalence classes.

Lemma 3.4

The transitive closure $\hat{T}$ of an intuitionistic fuzzy tolerance relation is the minimal intuitionistic fuzzy similarity relation containing T.

Proof : $\hat{T}$ is an proximity relation. Also, $\hat{T}$ is transitive. Minimality is obvious. Hence proved.

Example 3.1

Consider the intuitionistic fuzzy tolerance relation T on X= { x_1, x_2, x_3, x_4 } given by

T=

	x_1	x_2	x_3	x_4
x_1	(1, 0)	(.8, .1)	(.6, .2)	(.3, .4)
x_2	(.8, .1)	(1, 0)	(.4, .5)	(.5, .3)
x_3	(.6, .2)	(.4, .5)	(1, 0)	(.6, .3)
x_4	(.3, .4)	(.5, .3)	(.6, .3)	(1, 0)

It can be computed that

for α= 1, the partition of X determined by T_α given by { { x_1 }, { x_2 }, { x_3 }, { x_4 } },

for α= .9, the partition of X determined by T_α given by { { x_1, x_2 }, { x_3 }, { x_4 } },

for α= .8, the partition of X determined by T_α given by { { x_1, x_2, x_3 }, { x_4 } }, and

for α= .7, the partition of X determined by T_α given by { { x_1, x_2, x_3, x_4 } }.

Moreover, we see that

when $\alpha \in (.9, 1]$, the partition of X determined by T_α is { { x_1 }, { x_2 }, { x_3 }, { x_4 } },

when $\alpha \in (.8, .9]$, the partition of X determined by T_α is { { x_1, x_2 }, { x_3 }, { x_4 } },

when $\alpha \in (.7, .8]$, the partition of X determined by T_α is { { x_1, x_2, x_3 }, { x_4 } } and

when $\alpha \in [0, .7]$, the partition of X determined by T_α is { { x_1, x_2, x_3, x_4 } }.

In the next section we introduce the concept of intuitionistic fuzzy database.

3 Intuitionistic Fuzzy Database

A fuzzy relational database introduced by Buckles and Petry [14] is a generalization of the classical database. A fuzzy relational database is defined as a set of relations where each relation is a set of tuples. If t_i represents the i-th tuple, it has the form (d_{i1}, d_{i2}, $\cdots$, d_{im}). In classical relational database, each component d_{ij}, of the tuple is an element of the corresponding scalar (or discrete finite) domain D_j i.e., $d_{ij} \in D_j$. But in the case of fuzzy relational database, the elements of tuples consist of either singleton or crisp subsets of the scalar domain i.e., $d_{ij} \subseteq D_j$ ($d_{ij} \neq \emptyset$).

The fuzzy relational model of Buckles and Petry [14] is based on similarity relation [30] for each domain of the fuzzy database. Shenoi and Melton [27] generalize the model by allowing fuzzy proximity relation in each domain in place of fuzzy equivalence relation. We here generalize fuzzy database by incorporating intuitionistic fuzzy tolarance relation in place of fuzzy proximity relation. The reason behind such attempt of generalization lies in the fact that there is always a fair chance of the existence of some indeterministic part while evaluating the relation between two elements of a domain set in a database.

Definition 4.1

An intuitionistic fuzzy database relation R is a subset of the cross product $2^{D_1} \times 2^{D_2} \times \cdots 2^{D_m}$, where $2^{D_j} = 2^{D_1} - \emptyset$.

Definition 4.2

Let R $\subseteq 2^{D_1} \times 2^{D_2} \times \cdots 2^{D_m}$ be an intuitionistic fuzzy database relation. An intuitionistic fuzzy tuple (with respect to R) is an element of R.

Let t_i= (d_{i1}, d_{i2}, $\cdots$, d_{im}) be an intuitionistic fuzzy tuple. An interpretation of t, is a tuple θ= (a_1, a_2, $\cdots$, a_m) where $a_j \in d_{ij}$ for each domain D_j.

For each domain D_j, if T_j be the intuitionistic fuzzy tolerance relation then the membership function is given by $\mu_{T_j} : D_j \times D_j \rightarrow [0, 1]$ and the non-membership function is given by $\nu_{T_j} : D_j \times D_j \rightarrow [0, 1]$.

Let us make a hypothetical case study below :

We consider a criminal data file. Suppose that one murder has taken place at some area in a deem light. The police suspects that the murderer is also from the same area; and so police refer to a data file of all the suspected criminals of the that area. Listening to the eye-witness, the police has discovered that the criminal for that murder case has more or less full big hair coverage, more or less curly hair texture and he has moderately large build. From the criminal data file, the information table with attributes 'HAIR COVERAGE', 'HAIR TEXTURE' and 'BUILD' is given by

NAME	HAIR COVERAGE	HAIR TEXTURE	BUILD
Arup	Full Small(FS)	Stc.	Large
Boby	Rec.	Wavy	Very Small(VS)
Chandra	Full Small(FS)	Straight(Str.)	Small(S)
Dutta	Bald	Curly	Average(A)
Esita	Bald	Wavy	Average(A)
Falguni	Full Big(FB)	Stc.	Very Large(VL)
Gautom	Full Small	Straight	Small(S)
Halder	Rec.	Curly	Average(A)

Now, consider the intuitionistic fuzzy tolerance relation T_{D_1} where D_1 = 'HAIR COVERAGE', which is given by

	FB	FS	Rec.	Bald
FB	(1, 0)	(.8, .1)	(.4, .4)	(0, 1)
FS	(.8, .1)	(1, 0)	(.5, .4)	(0, .9)
Rec.	(.4, .4)	(.5, .4)	(1, 0)	(.4, .4)
Bald	(0, 1)	(0, .9)	(.4, .4)	(1, 0)

where, HAIR COVERAGE= { FB, FS, Rec., Bald }.

Intuitionistic fuzzy tolerance relation T_{D_2} where D_2= 'HAIR TEXTURE', is given by

	Str.	Stc.	Wavy	Curly
Str.	(1, 0)	(.7, .3)	(.2, .7)	(.1, .7)
Stc.	(.7, .3)	(1, 0)	(.3, .4)	(.5, .2)
Wavy	(.2, .7)	(.3, .4)	(1, 0)	(.4, .4)
Bald	(.1, .7)	(.5, .2)	(.4, .4)	(1, 0)

where, HAIR TEXTURE= { Str., Stc., Wavy, Curly }.

Also, intuitionistic fuzzy Tolerance relation T_{D_3} where D_3= 'BUILD', is given by

	VL	L	A	S	VS
VL	(1, 0)	(.8, .2)	(.5, .4)	(.3, .6)	(0, 1)
L	(.8, .2)	(1, 0)	(.6, .4)	(.4, .5)	(0, .9)
A	(.5, .4)	(.6, .4)	(1, 0)	(.6, .3)	(.3, .6)
S	(.3, .6)	(.4, ..5)	(.6, .3)	(1, 0)	(.8, .2)
VS	(0, 1)	(0, .9)	(.3, .6)	(.8, .2)	(1, 0)

where, BUILD= { VL, L, A, S, VS }.

Now, the job is to find out a list of those criminals who resemble with more or less big hair coverage with more or less curly hair texture and moderately large build. This list will be useful to the police for further investigation. It can be translated into relational algebra in the following form :

(Project (Select (CRIMINALS DATA FILE)
where HAIR COVERAGE= "FULL BIG",
HAIR TEXTURE= "CURLY"
BUILD= "LARGE"
with LEVEL(HAIR COVERAGE)= 0.8
LEVEL(HAIR TEXTURE)= 0.8
LEVEL(BUILD)= 0.7)
with LEVEL(NAME)=0.0,
LEVEL(HAIR COVERAGE)= 0.8,
LEVEL(HAIR TEXTURE)= 0.8,
LEVEL(BUILD)= 0.7
giving LIKELY MURDERER).

Result : It can be computed that the above intuitionistic fuzzy query gives rise to the following relation :

LIKELY MURDERER

NAME	HAIR COVERAGE	HAIR TEXTURE	BUILD
{ Arup, Falguni }	{ Full Big, Full Small }	{ Curly, Stc. }	{Large, Very Large }

Therefore, according to the information obtained from the eye-witness, police concludes that Arup or Falguni are the likely murderers. And, further investigation now is to be done on them only, instead of dealing with a huge list of criminals.

4 CONCLUSION

There is always a fair chance of the existence of some indeterministic part while evaluating the relation between two elements of a domain value set in a database. As a consequence, the non-membership functions have signifacant importance compared to the complement of fuzzy sets in finding out the partitions of a domain value set. Intuitionistic fuzzy set theory takes care of such indeterministic part in connection with each references point of its universe. In the present paper we have introduced a concept of intuitionistic fuzzy data base (IFDB) and have shown by an exmple the usefulness of intuitionistic fuzzy queries on a intuitionistic fuzzy database.

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