On some ways of determining membership and non-membership functions characterizing intuitionistic fuzzy sets

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1. Introduction

In the theory of fuzzy sets, various methods are discussed for the generation of values of the membership function (for instance, see [5, 6, 8, 9]).

Here we will discuss a way of generation of the two degrees – of membership and of non-membership that exist in the intuitionistic fuzzy sets (IFSs). For other approaches of assigning membership and non-membership functions of IFSs see [7].

2. Determining membership and non-membership functions of IFSs

The definition and the basic properties of the IFSs are given in [1, 2]. The IFSs have two functions – a membership function \( \mu_A \), giving the degree of membership of each element \( x \in E \), where \( E \) is a fixed universe, to a fixed set \( A \subseteq E \), and a non-membership function \( \nu_A \), giving the degree of non-membership of \( x \) to \( A \). These functions satisfy the conditions

\[
\mu_A(x), \nu_A(x) \in [0, 1],
\]

\[
\mu_A(x) + \nu_A(x) \leq 1,
\]

for every \( x \in E \).

Let us have \( k \) different generators \( G_1, G_2, ..., G_k \) of fuzzy estimations for \( n \) different objects \( O_1, O_2, ..., O_n \). In [5] these generators are called “estimators”.

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Let the estimations are collected in the Index Matrix (IM; see [3, 4])

\[
\begin{array}{cccccc}
G_1 & O_1 & O_2 & \ldots & O_j & \ldots & O_n \\
\alpha_{1,1} & \alpha_{1,2} & \ldots & \alpha_{1,j} & \ldots & \alpha_{1,n} \\
G_2 & \alpha_{2,1} & \alpha_{2,2} & \ldots & \alpha_{2,j} & \ldots & \alpha_{2,n} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
G_i & \alpha_{i,1} & \alpha_{i,2} & \ldots & \alpha_{i,j} & \ldots & \alpha_{i,n} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
G_k & \alpha_{k,1} & \alpha_{k,2} & \ldots & \alpha_{k,j} & \ldots & \alpha_{k,n} \\
\end{array}
\]

On the basis of the values of the IM we can constructs the following two types of fuzzy sets:

\[
O_1^* = \{ \langle G_i, \alpha_{i,1} \rangle | 1 \leq i \leq k \},
\]

\[
O_2^* = \{ \langle G_i, \alpha_{i,2} \rangle | 1 \leq i \leq k \},
\]

\[
\ldots
\]

\[
O_n^* = \{ \langle G_i, \alpha_{i,n} \rangle | 1 \leq i \leq k \},
\]

and

\[
G_1^* = \{ \langle O_j \alpha_{1,j} \rangle | 1 \leq j \leq n \},
\]

\[
G_2^* = \{ \langle O_j \alpha_{2,j} \rangle | 1 \leq j \leq n \},
\]

\[
\ldots
\]

\[
G_k^* = \{ \langle O_j \alpha_{k,j} \rangle | 1 \leq j \leq n \}.
\]

Now, using these sets we will construct different new – already – IFSs. First, we construct the IFSs:

\[
O_1^I = \{ \langle G_i, \alpha_{i,1} \sum_{2 \leq s \leq n} \alpha_{i,s} \rangle | 1 \leq i \leq k \},
\]

\[
O_2^I = \{ \langle G_i, \alpha_{i,2} \sum_{1 \leq s \leq n; s \neq 2} \alpha_{i,s} \rangle | 1 \leq i \leq k \},
\]

\[
\ldots
\]

\[
O_n^I = \{ \langle G_i, \alpha_{i,n} \sum_{1 \leq s \leq n-1} \alpha_{i,s} \rangle | 1 \leq i \leq k \},
\]

or

\[
O_j^I = \{ \langle G_i, \alpha_{i,j} \sum_{1 \leq s \leq n; s \neq j} \alpha_{i,s} \rangle | 1 \leq i \leq k \},
\]

for \( j = 1, 2, \ldots, n; \)

and

\[
G_1^I = \{ \langle O_j, \alpha_{1,j} \sum_{2 \leq s \leq n} \alpha_{s,j} \rangle | 1 \leq j \leq n \},
\]

\[
G_2^I = \{ \langle O_j, \alpha_{2,j} \sum_{1 \leq s \leq n; s \neq 2} \alpha_{s,j} \rangle | 1 \leq j \leq n \},
\]

\[
\ldots
\]

\[
G_k^I = \{ \langle O_j, \alpha_{k,j} \sum_{1 \leq s \leq n-1} \alpha_{s,j} \rangle | 1 \leq j \leq n \},
\]
or
\[ G'_i = \{ (O_j, \alpha_{i,j}, \sum_{1 \leq s \leq n; s \neq i} \alpha_{i,s}) | 1 \leq j \leq n \}, \text{ for } j = 1, 2, \ldots, k; \]

Second, we construct the IFSs:
\[ G_{\text{max, min}}^I = \{ (O_j, \max_{1 \leq i \leq n} \alpha_{i,j}, \min_{1 \leq i \leq n} \alpha_{i,j}) | 1 \leq j \leq n \}, \]
\[ G_{\text{av}}^I = \{ (O_j, \frac{1}{k} \sum_{i=1}^{k} \alpha_{i,j}, \frac{1}{k} \sum_{1 \leq s \leq n; s \neq j} \sum_{i=1}^{k} \alpha_{i,s}) | 1 \leq j \leq n \}, \]
\[ G_{\text{min, max}}^I = \{ (O_j, \min_{1 \leq i \leq n} \alpha_{i,j}, \max_{1 \leq i \leq n} \alpha_{i,j}) | 1 \leq j \leq n \}. \]

Now, we will illustrate the constructions, introduced by us.

Let five experts \( E_1, E_2, E_3, E_4 \) and \( E_5 \) offer their evaluations of the percentage of votes, obtained by the political parties \( P_1, P_2 \) and \( P_3 \):

<table>
<thead>
<tr>
<th></th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 )</td>
<td>32%</td>
<td>9%</td>
<td>37%</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>27%</td>
<td>7%</td>
<td>39%</td>
</tr>
<tr>
<td>( E_3 )</td>
<td>26%</td>
<td>11%</td>
<td>35%</td>
</tr>
<tr>
<td>( E_4 )</td>
<td>31%</td>
<td>8%</td>
<td>39%</td>
</tr>
<tr>
<td>( E_5 )</td>
<td>29%</td>
<td>9%</td>
<td>41%</td>
</tr>
</tbody>
</table>

Now, we are able to generate the fuzzy sets
\[ P_1^* = \{ (E1, 0.32), (E2, 0.27), (E3, 0.26), (E4, 0.31), (E5, 0.29) \}, \]
\[ P_2^* = \{ (E1, 0.09), (E2, 0.07), (E3, 0.11), (E4, 0.08), (E5, 0.09) \}, \]
\[ P_3^* = \{ (E1, 0.37), (E2, 0.39), (E3, 0.35), (E4, 0.39), (E5, 0.41) \}, \]
\[ E_1^* = \{ (P1, 0.32), (P2, 0.09), (P3, 0.37) \}, \]
\[ E_2^* = \{ (P1, 0.27), (P2, 0.07), (P3, 0.39) \}, \]
\[ E_3^* = \{ (P1, 0.26), (P2, 0.11), (P3, 0.35) \}, \]
\[ E_4^* = \{ (P1, 0.31), (P2, 0.08), (P3, 0.39) \}, \]
\[ E_5^* = \{ (P1, 0.29), (P2, 0.09), (P3, 0.41) \}. \]

We can aggregate the last five sets, e.g., by operation \( @ \) and will obtain the fuzzy set
\[ E_{FS} = \{ (P1, 0.29), (P2, 0.088), (P3, 0.382) \}. \]

Now, we show why we can use the above information for constructing IFSs.

It is easily to figure out that if expert \( E_1 \) believes that party \( P_1 \) would obtain 32% of the election votes, then he thinks that 68% of the voters are against this party. If we take for granted that all the five experts are equally competent, i.e. their opinions are of equal worth, then we may conclude that party \( P_1 \) will receive between 26% and 32% of the votes,
therefore, the opposers of this party will count between 68% and 74% of the voters. Now, an IFS can be constructed for the universe \( \{P_1, P_2, P_3\} \) that would have the form:

\[
E_{IFS,1} = \{\langle P_1, 0.26, 0.68 \rangle, \langle P_2, 0.07, 0.89 \rangle, \langle P_3, 0.35, 0.59 \rangle\}.
\]

This shows that at least 26% of the voters would support party \( P_1 \) and at least 68% would oppose it.

Another possible IFS that we can construct on the basis of the above data, is

\[
E_{IFS,2} = \{\langle P_1, 0.29, 0.47 \rangle, \langle P_2, 0.088, 0.672 \rangle, \langle P_3, 0.382, 0.378 \rangle\}.
\]

The \( \mu \)-components of this IFS are obtained directly from \( E_{FS} \), while the \( \nu \)-components are sums of the \( \mu \)-components of the other two parties.

Following the above formulae, we can construct the next IFSs:

\[
P_1^* = \{\langle E_1, 0.32, 0.46 \rangle, \langle E_2, 0.27, 0.46 \rangle, \langle E_3, 0.26, 0.46 \rangle, \langle E_4, 0.31, 0.47 \rangle, \langle E_5, 0.29, 0.50 \rangle\},
\]

\[
P_2^* = \{\langle E_1, 0.09, 0.59 \rangle, \langle E_2, 0.07, 0.66 \rangle, \langle E_3, 0.11, 0.61 \rangle, \langle E_4, 0.08, 0.70 \rangle, \langle E_5, 0.09, 0.70 \rangle\},
\]

\[
P_3^* = \{\langle E_1, 0.37, 0.41 \rangle, \langle E_2, 0.39, 0.34 \rangle, \langle E_3, 0.35, 0.37 \rangle, \langle E_4, 0.39, 0.39 \rangle, \langle E_5, 0.41, 0.38 \rangle\},
\]

\[
E_1^* = \{\langle P_1, 0.32, 0.46 \rangle, \langle P_2, 0.09, 0.59 \rangle, \langle P_3, 0.37, 0.41 \rangle\},
\]

\[
E_2^* = \{\langle P_1, 0.27, 0.46 \rangle, \langle P_2, 0.07, 0.66 \rangle, \langle P_3, 0.39, 0.34 \rangle\},
\]

\[
E_3^* = \{\langle P_1, 0.26, 0.46 \rangle, \langle P_2, 0.11, 0.61 \rangle, \langle P_3, 0.35, 0.37 \rangle\},
\]

\[
E_4^* = \{\langle P_1, 0.31, 0.47 \rangle, \langle P_2, 0.08, 0.70 \rangle, \langle P_3, 0.39, 0.39 \rangle\},
\]

\[
E_5^* = \{\langle P_1, 0.29, 0.50 \rangle, \langle P_2, 0.09, 0.70 \rangle, \langle P_3, 0.41, 0.38 \rangle\}.
\]

Obviously, the estimations of the fuzzy sets, like those of the IFS, are constructive objects, as this is discuss in [2].

3. Conclusions

We have presented some ways of determining membership and non-membership functions characterizing IFSs.

References


