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On some ways of determining membership and non-membership functions characterizing intuitionistic fuzzy sets

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1. Introduction

In the theory of fuzzy sets, various methods are discussed for the generation of values of the membership function (for instance, see [5, 6, 8, 9]).

Here we will discuss a way of generation of the two degrees – of membership and of non-membership that exist in the intuitionistic fuzzy sets (IFSs). For other approaches of assigning membership and non-membership functions of IFSs see [7].

2. Determining membership and non-membership functions of IFSs

The definition and the basic properties of the IFSs are given in [1, 2]. The IFSs have two functions – a membership function μ_A, giving the degree of membership of each element $x \in E$, where E is a fixed universe, to a fixed set $A \subseteq E$, and a non-membership function ν_A, giving the degree of non-membership of x to A. These functions satisfy the conditions

$$\mu_A(x), \nu_A(x) \in [0, 1],$$

$$\mu_A(x) + \nu_A(x) \leq 1,$$

for every $x \in E$.

Let us have k different generators $G_1, G_2, ..., G_k$ of fuzzy estimations for n different objects $O_1, O_2, ..., O_n$. In [5] these generators are called "estimators".

Let the estimations are collected in the Index Matrix (IM; see [3, 4]

$$\begin{array}{c|cccccc} & O_1 & O_2 & ... & O_j & ... & O_n \\ \hline G_1 & \alpha_{1,1} & \alpha_{1,2} & ... & \alpha_{1,j} & ... & \alpha_{1,n} \\ G_2 & \alpha_{2,1} & \alpha_{2,2} & ... & \alpha_{2,j} & ... & \alpha_{2,n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ G_i & \alpha_{i,1} & \alpha_{i,2} & ... & \alpha_{i,j} & ... & \alpha_{i,n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ G_k & \alpha_{k,1} & \alpha_{k,2} & ... & \alpha_{k,j} & ... & \alpha_{k,n} \end{array}$$

On the basis of the values of the IM we can constructs the following two types of fuzzy sets:

$$O_1^* = \{\langle G_i, \alpha_{i,1}\rangle | 1 \leq i \leq k\},$$
$$O_2^* = \{\langle G_i, \alpha_{i,2}\rangle | 1 \leq i \leq k\},$$
$$\dots$$
$$O_n^* = \{\langle G_i, \alpha_{i,n}\rangle | 1 \leq i \leq k\},$$

and

$$G_1^* = \{\langle O_j \alpha_{1,j}\rangle | 1 \leq j \leq n\},$$
$$G_2^* = \{\langle O_j \alpha_{2,j}\rangle | 1 \leq j \leq n\},$$
$$\dots$$
$$G_k^* = \{\langle O_j \alpha_{k,j}\rangle | 1 \leq j \leq n\}.$$

Now, using these sets we will construct different new – already – IFSs.

First, we construct the IFSs:

$$O_1^I = \{\langle G_i, \alpha_{i,1}, \sum_{2 \leq s \leq n} \alpha_{i,s}\rangle | 1 \leq i \leq k\},$$

$$O_2^I = \{\langle G_i, \alpha_{i,2}, \sum_{1 \leq s \leq n;\ s \neq 2} \alpha_{i,s}\rangle | 1 \leq i \leq k\},$$
$$\dots$$
$$O_n^I = \{\langle G_i, \alpha_{i,n}, \sum_{1 \leq s \leq n-1} \alpha_{i,s}\rangle | 1 \leq i \leq k\},$$

or

$$O_j^I = \{\langle G_i, \alpha_{i,j}, \sum_{1 \leq s \leq n;\ s \neq j} \alpha_{i,s}\rangle | 1 \leq i \leq k\},\ \text{for } j = 1, 2, ..., n;$$

and

$$G_1^I = \{\langle O_j, \alpha_{1,j}, \sum_{2 \leq s \leq n} \alpha_{s,j}\rangle | 1 \leq j \leq n\},$$

$$G_2^I = \{\langle O_j, \alpha_{2,j}, \sum_{1 \leq s \leq n;\ s \neq 2} \alpha_{s,j}\rangle | 1 \leq j \leq n\},$$
$$\dots$$
$$G_k^I = \{\langle O_j, \alpha_{k,j}, \sum_{1 \leq s \leq n-1} \alpha_{j,s}\rangle | 1 \leq j \leq n\},$$

or

$$G_i^I = \{\langle O_j, \alpha_{i,j}, \sum_{1\le s\le n;\ s\ne i} \alpha_{j,s}\rangle | 1 \le j \le n\},\ \text{for } j = 1, 2, ..., k;$$

Second, we construct the IFSs:

$$G^I_{\text{max,min}} = \{\langle O_j, \max_{1\le i\le n} \alpha_{i,j}, \min_{1\le i\le n} \alpha_{i,j}\rangle | 1 \le j \le n\},$$

$$G^I_{\text{av}} = \{\langle O_j, \frac{1}{k}\sum_{i=1}^{k} \alpha_{i,j}, \frac{1}{k}\sum_{1\le s\le n; s\ne j}\ \sum_{i=1}^{k} \alpha_{i,s}\rangle | 1 \le j \le n\},$$

$$G^I_{\text{min,max}} = \{\langle O_j, \min_{1\le i\le n} \alpha_{i,j}, \max_{1\le i\le n} \alpha_{i,j}\rangle | 1 \le j \le n\}.$$

Now, we will illustrate the constructions, introduced by us.

Let five experts E_1, E_2, E_3, E_4 and E_5 offer their evaluations of the percentage of votes, obtained by the political parties P_1, P_2 and P_3:

	P_1	P_2	P_3
E_1	32%	9%	37%
E_2	27%	7%	39%
E_3	26%	11%	35%
E_4	31%	8%	39%
E_5	29%	9%	41%

Now, we are able to generate the fuzzy sets

$$P_1^* = \{\langle E1, 0.32\rangle, \langle E2, 0.27\rangle, \langle E3, 0.26\rangle, \langle E4, 0.31\rangle, \langle E5, 0.29\rangle\},$$

$$P_2^* = \{\langle E1, 0.09\rangle, \langle E2, 0.07\rangle, \langle E3, 0.11\rangle, \langle E4, 0.08\rangle, \langle E5, 0.09\rangle\},$$

$$P_3^* = \{\langle E1, 0.37\rangle, \langle E2, 0.39\rangle, \langle E3, 0.35\rangle, \langle E4, 0.39\rangle, \langle E5, 0.41\rangle\},$$

$$E_1^* = \{\langle P1, 0.32\rangle, \langle P2, 0.09\rangle, \langle P3, 0.37\rangle\},$$

$$E_2^* = \{\langle P1, 0.27\rangle, \langle P2, 0.07\rangle, \langle P3, 0.39\rangle\},$$

$$E_3^* = \{\langle P1, 0.26\rangle, \langle P2, 0.11\rangle, \langle P3, 0.35\rangle\},$$

$$E_4^* = \{\langle P1, 0.31\rangle, \langle P2, 0.08\rangle, \langle P3, 0.39\rangle\},$$

$$E_5^* = \{\langle P1, 0.29\rangle, \langle P2, 0.09\rangle, \langle P3, 0.41\rangle\}.$$

We can aggregate the last five sets, e.g., by operation @ and will obtain the fuzzy set

$$E_{FS} = \{\langle P1, 0.29\rangle, \langle P2, 0.088\rangle, \langle P3, 0.382\rangle\}.$$

Now, we show why we can use the above information for constructing IFSs.

It is easily to figure out that if expert E_1 believes that party P_1 would obtain 32% of the election votes, then he thinks that 68% of the voters are against this party. If we take for granted that all the five experts are equally competent, i.e. their opinions are of equal worth, then we may conclude that party P_1 will receive between 26% and 32% of the votes, therefore, the opposers of this party will count between 68% and 74% of the voters. Now, an IFS can be constructed for the universe $\{P_1, P_2, P_3\}$ that would have the form:

$$E_{IFS,1} = \{\langle P_1, 0.26, 0.68\rangle, \langle P_2, 0.07, 0.89\rangle, \langle P_3, 0.35, 0.59\rangle\}.$$

This shows that at least 26% of the voters would support party P_1 and at least 68% would oppose it.

Another possible IFS that we can construct on the basis of the above data, is

$$E_{IFS,2} = \{\langle P1, 0.29, 0.47\rangle, \langle P2, 0.088, 0.672\rangle, \langle P3, 0.382, 0.378\rangle\}.$$

The μ-components of this IFS are obtained directly from E_{FS}, while the ν-components are sums of the μ-components of the other two parties.

Following the above formulae, we can construct the next IFSs:

$$P_1^* = \{\langle E1, 0.32, 0.46\rangle, \langle E2, 0.27, 0.46\rangle, \langle E3, 0.26, 0.46\rangle, \langle E4, 0.31, 0.47\rangle, \langle E5, 0.29, 0.50\rangle\},$$

$$P_2^* = \{\langle E1, 0.09, 0.59\rangle, \langle E2, 0.07, 0.66\rangle, \langle E3, 0.11, 0.61\rangle, \langle E4, 0.08, 0.70\rangle, \langle E5, 0.09, 0.70\rangle\},$$

$$P_3^* = \{\langle E1, 0.37, 0.41\rangle, \langle E2, 0.39, 0.34\rangle, \langle E3, 0.35, 0.37\rangle, \langle E4, 0.39, 0.39\rangle, \langle E5, 0.41, 0.38\rangle\},$$

$$E_1^* = \{\langle P1, 0.32, 0.46\rangle, \langle P2, 0.09, 0.59\rangle, \langle P3, 0.37, 0.41\rangle\},$$

$$E_2^* = \{\langle P1, 0.27, 0.46\rangle, \langle P2, 0.07, 0.66\rangle, \langle P3, 0.39, 0.34\rangle\},$$

$$E_3^* = \{\langle P1, 0.26, 0.46\rangle, \langle P2, 0.11, 0.61\rangle, \langle P3, 0.35, 0.37\rangle\},$$

$$E_4^* = \{\langle P1, 0.31, 0.47\rangle, \langle P2, 0.08, 0.70\rangle, \langle P3, 0.39, 0.39\rangle\},$$

$$E_5^* = \{\langle P1, 0.29, 0.50\rangle, \langle P2, 0.09, 0.70\rangle, \langle P3, 0.41, 0.38\rangle\}.$$

Obviously, the estimations of the fuzzy sets, like those of the IFS, are constructive objects, as this is discuss in [2].

3. Conclusions

We have presented some ways of determining membership and non-membership functions characterizing IFSs.

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