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Intuitionistic Fuzzy Extension Principle

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Abstract
The present paper proposes an idea of how to define a generalised extension principle for the case of intuitionistic fuzzy sets. In relation to this, two new ways of defining a Cartesian product over IFS are proposed.

I. Introduction

Fuzzy number theory and its mathematical representation, fuzzy arithmetic expression, and calculus of fuzzy quantities were introduced in Zadeh's prominent paper [1]. All these achievements are based on the so-called *extension principle.*

The extension principle described by Zadeh [1] provides a natural way for extending the domain of a mapping or a relation defined on a set U to fuzzy subsets of U. It is particularly useful in connection with the computation of linguistic variables, the calculus of linguistic probabilities, arithmetic of fuzzy numbers, and, more generally, in applications which call an extension of the domain of a relation.

The extension principle over intuitionistic fuzzy (IF) sets is a natural generalisation of the extension principle over ordinary fuzzy sets. According to the definition of IF sets (IFS), given by K. Atanassov (see [2]), every element x of a given set A is assigned the numbers μ, ν, and π, called "degree of membership", "degree of non-membership", and "degree of uncertainty" of the element $x \in A$ to some IFS B. In this way, taking A as a universe, a new IFS B is defined over it.

II. Extension principle over IFS

Let us be given universes X and Y, and a function $f : X \to Y$.
Let A be an intuitionistic fuzzy (IF) set over X, and B be an IF set over Y.
Definition.

$$\begin{aligned}
\mu_{f(A)}(y) &= \begin{cases} \sup_{f(x)=y} \mu_A(x), & y \in f(X) \\ 0, & y \notin f(X) \end{cases} \\
\nu_{f(A)}(y) &= \begin{cases} \inf_{f(x)=y} \nu_A(x), & y \in f(X) \\ 1, & y \notin f(X) \end{cases} \\
\pi_{f(A)}(y) &= 1 - \mu_{f(A)}(y) - \nu_{f(A)}(y).
\end{aligned}$$

Correctness of the definition. To prove its correctness, let us assume that for an arbitrary $y \in f(X)$ there exist $x_1, x_2 \in \{x \in X | f(x) = y\}$ such that

$$\sup_{f(x)=y} \mu_A(x) = \mu_A(x_1),$$

$$\inf_{f(x)=y} \nu_A(x) = \nu_A(x_2)$$

and

$$\mu_A(x_1) + \nu_A(x_2) > 1.$$

However, $\mu_A(x_1) + \nu_A(x_1) \leq 1$ and therefore
$\mu_A(x_1) + \nu_A(x_2) > \mu_A(x_1) + \nu_A(x_1)$.
Hence we have that $\nu_A(x_2) > \nu_A(x_1)$, which contradicts our assumption.
Definition.

$$\begin{aligned} \mu_{f^{-1}(B)}(x) &= \mu_B(f(x)) \\ \nu_{f^{-1}(B)}(x) &= \nu_B(f(x)) \\ \pi_{f^{-1}(B)}(x) &= \pi_B(f(x)) \end{aligned}$$

Definition.

$$\begin{aligned} \mu_{(A\times B)}(x,y) &= \min\{\mu_A(x), \mu_B(y)\} \\ \pi'_{(A\times B)}(x,y) &= \min\{\pi_A(x), \pi_B(y)\} \\ \nu'_{(A\times B)}(x,y) &= 1 - \mu_{(A\times B)}(x,y) - \pi'_{(A\times B)}(x,y) \end{aligned}$$

Definition.

$$\begin{aligned} \mu_{(A\times B)}(x,y) &= \min\{\mu_A(x), \mu_B(y)\} \\ \pi''_{(A\times B)}(x,y) &= \max\{\pi_A(x), \pi_B(y)\} \\ \nu''_{(A\times B)}(x,y) &= 1 - \mu_{(A\times B)}(x,y) - \pi''_{(A\times B)}(x,y) \end{aligned}$$

Bibliography

[1] L. A. Zadeh, *Fuzzy sets*, Information and Control, Vol. 8, 1965, 338 – 353.

[2] K. Atanassov, *Intuitionistic fuzzy sets,*Fuzzy sets and Systems, Vol. 20, 1986, No. 1, 87-96.