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Two de-I-fuzzification procedures for intuitionistic fuzzy information

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Abstract: In this paper, two procedures are proposed that transform intuitionistic fuzzy information into fuzzy information. Using the results obtained with the de-I-fuzzification procedures, formulas for intuitionistic fuzzy entropy are constructed.
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1 Introduction

L. A. Zadeh introduced the concept of fuzzy representation of an information [14]. Thus a parameter $\mu \in [0,1]$ is associated with information. This parameter describes the degree of truth of the information. In the case of fuzzy representation, the degree of falsity ν is calculated using the negation of the degree of truth, that is $\nu = 1 - \mu$.

K. T. Atanassov proposed an extension of the fuzzy representation, namely intuitionistic fuzzy representation [2]. This representation uses two parameters, namely: the degree of truth $\mu \in [0,1]$ and the degree of falsity $\nu \in [0,1]$. There is a constraint with the two parameters:

$$\mu + \nu \leq 1. \tag{1}$$

Taking into account (1), Atanassov defined the degree of hesitation π with the next formula:

$$\pi = 1 - \mu - \nu. \tag{2}$$

At a practical level, often fuzzy information must be transformed into crisp information. For this, defuzzification procedures were developed [13]. Similarly, for intuitionistic fuzzy information, procedures for de-I-fuzzification must be developed, i.e. procedures that transform intuitionistic fuzzy information (μ, ν) into fuzzy information (μ^*, ν^*). We have to find a function $\varphi : [0, 1]^2 \to [0, 1]$ in order to define the fuzzy information, that is:

$$\mu^* = \varphi(\mu, \nu), \tag{3}$$

$$\nu^* = \varphi(\nu, \mu). \tag{4}$$

In addition, the the function φ must verify the following conditions:

$$\varphi(\mu, \nu) + \varphi(\nu, \mu) = 1, \tag{5}$$

$$\varphi(\mu, \nu) \geq \mu, \tag{6}$$

$$\varphi(\nu, \mu) \geq \nu. \tag{7}$$

Also, $\varphi(\mu, \nu)$ increases with μ and decreases with ν. Similarly $\varphi(\nu, \mu)$ decreases with μ and increases with ν.

Some de-I-fuzzification procedures are presented in [1], [3], [4] and [5].

Next, the paper has the following structure: Section 2 presents the two de-I-fuzzification procedures together with some experimental results; Section 3 presents the calculation formulas for intuitionistic fuzzy entropy based on de-I-fuzzification procedures; Section 4 presents the conclusions, while, the last section is that of the references.

2 Two procedures for de-I-fuzzification

2.1 First procedure

Consider points $T = (1, 0)$, $F = (0, 1)$, $P = (\mu, \nu)$, $P^* = (\mu^*, \nu^*)$ and $C = (1, 1)$ where point P^* is the intersection between TF and PC (Figure 1).

Note that point P is associated with intuitionistic fuzzy information while point P^* is associated with fuzzy information. Since points P, P^* and C are collinear the following equality results:

$$\frac{1 - \mu}{1 - \nu} = \frac{1 - \mu^*}{1 - \nu^*}, \tag{8}$$

but

$$\mu^* + \nu^* = 1 \tag{9}$$

and (8) becomes

$$\frac{1 - \mu}{1 - \nu} = \frac{\nu^*}{\mu^*}. \tag{10}$$

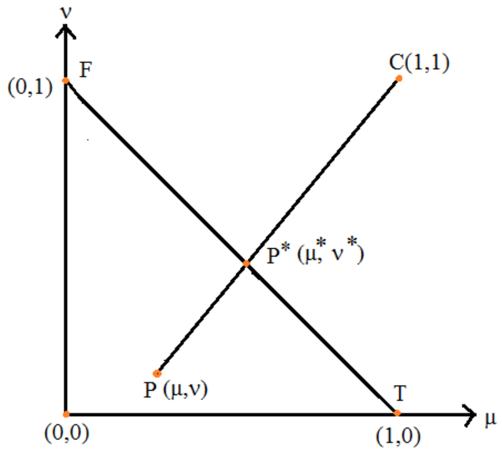

Figure 1. The mapping P^* as constructed by the first procedure.

Formula (10) is equivalent to the following two formulas:

$$\frac{1-\mu}{2-\mu-\nu} = \frac{\nu^*}{\mu^*+\nu^*}, \tag{11}$$

$$\frac{1-\nu}{2-\mu-\nu} = \frac{\mu^*}{\mu^*+\nu^*}. \tag{12}$$

From (9), (11), (12) and (2) it results:

$$\mu^* = \frac{\mu+\pi}{1+\pi}, \tag{13}$$

$$\nu^* = \frac{\nu+\pi}{1+\pi}. \tag{14}$$

In this case the function φ is defined by the formula:

$$\varphi(\mu,\nu) = \frac{1-\nu}{2-\mu-\nu}. \tag{15}$$

It is obvious that the function defined by (15) verifies conditions (5), (6) and (7). Also, it increases with μ and decreases with ν.

Numerical examples.

a) $(\mu,\nu) = (0.4, 0.2)$, $(\mu^*,\nu^*) = (0,5714, 0,4286)$
b) $(\mu,\nu) = (0.5, 0.3)$, $(\mu^*,\nu^*) = (0,5833, 0,4167)$
c) $(\mu,\nu) = (0.4, 0.5)$, $(\mu^*,\nu^*) = (0,4545, 0,5455)$
d) $(\mu,\nu) = (0.7, 0.1)$, $(\mu^*,\nu^*) = (0,7500, 0,2500)$

2.2 Second procedure

We consider the points $T = (1,0)$, $F = (0,1)$, $P = (\mu,\nu)$ and $P^* = (\mu^*,\nu^*)$. The point P^* is found on the segment TF at the intersection with the bisector of the angle $\angle TPF$. In other words, P^* is chosen such that $\angle TPP^* = \angle FPP^*$ (Figure 2). Again, we mention that point P is associated with intuitionistic fuzzy information while point P^* is associated with fuzzy information.

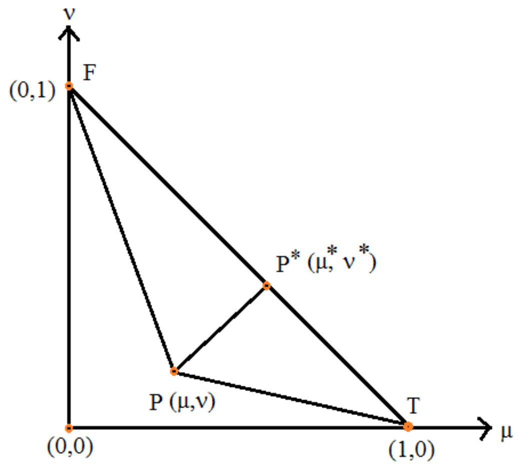

Figure 2. The mapping P^* as constructed by the second procedure.

We consider the Euclidean distance, namely, for all $Q_1 = (x_1, y_1) \in (-\infty, \infty)^2$ and for all $Q_2 = (x_2, y_2) \in (-\infty, \infty)^2$,

$$d(Q_1, Q_2) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \tag{16}$$

We determine the point P^* with the following condition given by the angle bisector theorem:

$$\frac{d(P, F)}{d(P, T)} = \frac{d(P^*, F)}{d(P^*, T)} \tag{17}$$

From (9) the following formulas result:

$$\mu^* = \frac{d(P^*, F)}{d(P^*, F) + d(P^*, T)}, \tag{18}$$

$$\nu^* = \frac{d(P^*, T)}{d(P^*, F) + d(P^*, T)}. \tag{19}$$

From (17), (18) and (19) the following formulas result:

$$\mu^* = \frac{d(P, F)}{d(P, F) + d(P, T)}, \tag{20}$$

$$\nu^* = \frac{d(P, T)}{d(P, F) + d(P, T)}. \tag{21}$$

Namely:

$$\mu^* = \frac{\sqrt{(1-\nu)^2 + \mu^2}}{\sqrt{(1-\mu)^2 + \nu^2} + \sqrt{(1-\nu)^2 + \mu^2}}, \tag{22}$$

$$\nu^* = \frac{\sqrt{(1-\mu)^2 + \nu^2}}{\sqrt{(1-\mu)^2 + \nu^2} + \sqrt{(1-\nu)^2 + \mu^2}}. \tag{23}$$

In this case, the function φ is defined by the formula:

$$\varphi(\mu,\nu) = \frac{\sqrt{(1-\nu)^2+\mu^2}}{\sqrt{(1-\mu)^2+\nu^2}+\sqrt{(1-\nu)^2+\mu^2}}. \tag{24}$$

The function defined by (24) verifies the condition (5).
For (6) we have

$$\sqrt{(1-\mu)^2+\nu^2} \ \le \ (1-\mu)\sqrt{2}, \tag{25}$$

$$\sqrt{(1-\nu)^2+\mu^2} \ \ge \ \mu\sqrt{2}. \tag{26}$$

Since $\varphi(\mu,\nu)$ increases with $\sqrt{(1-\nu)^2+\mu^2}$ and decreases with $\sqrt{(1-\mu)^2+\nu^2}$, it follows that:

$$\varphi(\mu,\nu) \ge \frac{\mu\sqrt{2}}{(1-\mu)\sqrt{2}+\mu\sqrt{2}}, \tag{27}$$

namely:

$$\varphi(\mu,\nu) \ge \mu. \tag{28}$$

Since $\frac{\sqrt{(1-\nu)^2+\mu^2}}{\sqrt{(1-\mu)^2+\nu^2}}$ increases with μ and decreases with ν it also follows that $\varphi(\mu,\nu)$ increases with μ and decreases with ν.

Next, we will define the parameter ρ with the following formula:

$$\rho = \frac{1+\mu^2+\nu^2}{2} \tag{29}$$

Taking into account (29), formulas (22) and (23) become:

$$\mu^* = \frac{\sqrt{\rho-\nu}}{\sqrt{\rho-\mu}+\sqrt{\rho-\nu}}, \tag{30}$$

$$\nu^* = \frac{\sqrt{\rho-\mu}}{\sqrt{\rho-\mu}+\sqrt{\rho-\nu}}. \tag{31}$$

Numerical examples.
a) $(\mu,\nu) = (0.4, 0.2)$, $(\mu^*,\nu^*) = (0,5858, 0,4142)$.
b) $(\mu,\nu) = (0.5, 0.3)$, $(\mu^*,\nu^*) = (0,5960, 0,4040)$.
c) $(\mu,\nu) = (0.4, 0.5)$, $(\mu^*,\nu^*) = (0,4505, 0,5495)$.
d) $(\mu,\nu) = (0.7, 0.1)$, $(\mu^*,\nu^*) = (0,7829, 0,2171)$.

3 Entropy formulas for intuitionistic fuzzy information

In this section, using the formulas obtained through the two procedures presented in Section 2, we will construct calculation formulas for the intuitionistic fuzzy entropy.

In this sense we will use the Kauffman entropy [7], the Kosko entropy [8] and the Shannon entropy [6, 11]. Thus, for the intuitionistic fuzzy information (μ,ν) we will use the fuzzy information (μ^*, ν^*) obtained by the de-I-fuzzification procedures. The calculation formula is the following:

$$E(\mu, \nu) = e(\mu^*, \nu^*). \quad (32)$$

The Kaufmann fuzzy entropy is given by the following formula:

$$e_{kf} = 1 - |\mu^* - \nu^*|. \quad (33)$$

From (33), (13) and (14) the following formula presented in [9] results for intuitionistic fuzzy entropy:

$$E_{kf} = 1 - \frac{|\mu - \nu|}{1 + \pi}. \quad (34)$$

From (33), (30) and (31) it results a new formula for intuitionistic fuzzy entropy:

$$E_{kf} = 1 - \frac{|\sqrt{\rho - \nu} - \sqrt{\rho - \mu}|}{\sqrt{\rho - \mu} + \sqrt{\rho - \nu}}. \quad (35)$$

The Kosko fuzzy entropy is given by the following formula:

$$e_{ks} = \frac{\min(\mu^*, \nu^*)}{\max(\mu^*, \nu^*)}. \quad (36)$$

From (36), (13) and (14) the following formula results for intuitionistic fuzzy entropy:

$$E_{ks} = \frac{1 - \max(\mu, \nu)}{1 - \min(\mu, \nu)}. \quad (37)$$

Formula (37) is equivalent to the following version presented in [12]:

$$E_{ks} = \frac{1 - |\mu - \nu| + \pi}{1 + |\mu - \nu| + \pi}. \quad (38)$$

From (36), (30) and (31) the following new formula results for intuitionistic fuzzy entropy:

$$E_{ks} = \frac{\sqrt{\rho - \max(\mu, \nu)}}{\sqrt{\rho - \min(\mu, \nu)}}. \quad (39)$$

Formula (39) is equivalent to:

$$E_{ks} = \frac{\sqrt{\rho - \mu} + \sqrt{\rho - \nu} - |\sqrt{\rho - \nu} - \sqrt{\rho - \mu}|}{\sqrt{\rho - \mu} + \sqrt{\rho - \nu} + \sqrt{\rho - \nu} - \sqrt{\rho - \mu}}. \quad (40)$$

The Shannon fuzzy entropy is given by the following formula [11]:

$$e_{sh} = -\mu^* \ln(\mu^*) - \nu^* \ln(\nu^*). \quad (41)$$

From (41), (13) and (14) the following formula presented in [10] results for intuitionistic fuzzy entropy:

$$E_{sh} = -\frac{\mu + \pi}{1 + \pi} \ln\left(\frac{\mu + \pi}{1 + \pi}\right) - \frac{\nu + \pi}{1 + \pi} \ln\left(\frac{\nu + \pi}{1 + \pi}\right). \quad (42)$$

From (41), (30) and (31) the following new formula results for intuitionistic fuzzy entropy:

$$E_{sh} = -\frac{\sqrt{\rho-\nu}\ln\left(\frac{\sqrt{\rho-\nu}}{\sqrt{\rho-\mu}+\sqrt{\rho-\nu}}\right)}{\sqrt{\rho-\mu}+\sqrt{\rho-\nu}} - \frac{\sqrt{\rho-\mu}\ln\left(\frac{\sqrt{\rho-\mu}}{\sqrt{\rho-\mu}+\sqrt{\rho-\nu}}\right)}{\sqrt{\rho-\mu}+\sqrt{\rho-\nu}}. \tag{43}$$

The six variants (34, 35, 37, 39, 42, 43) presented in this paper for calculating intuitionistic fuzzy entropy verify the following four conditions [10]:

1) *Minimality*: $E(\mu, \nu)$ is minimal if and only if $(\mu, \nu) \in \{(1, 0), (0, 1)\}$.
2) *Maximality*: $E(\mu, \nu)$ is maximal if and only if $\mu = \nu$.
3) *Symmetry*: $E(\mu, \nu) = E(\nu, \mu)$.
4) *Monotonicity*: $E(\mu, \nu)$ decreases both with $|\mu - \nu|$ as well as with $\mu + \nu$.

4 Conclusion

In this paper we have presented two procedures of de-I-fuzzification for intuitionistic fuzzy information. This allows us on a practical level to extend the use of defined computational methods for fuzzy information also for intuitionistic fuzzy information. One such example is presented for calculating the entropy of intuitionistic fuzzy information. The fact that calculation formulas obtained with other methods are obtained in this paper for intuitionistic fuzzy information shows the robustness of the de-I-fuzzification formulas presented in this paper.

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