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Properties of the intuitionistic fuzzy modal operator $\otimes_{\alpha,\beta,\gamma,\delta}$

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Abstract. Some new properties of the intuitionistic fuzzy modal operator $\otimes_{\alpha,\beta,\gamma,\delta}$ are formulated and proved. Some open problems are formulated.
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1 Introduction

In a previous paper of the authors [4], a new type of intuitionistic fuzzy modal operator, denoted by $\otimes_{\alpha,\beta,\gamma,\delta}$, was introduced and some of its properties were studied. Now, new properties of this operator are investigated.

Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

Different relations, operations and operators are introduced over the IFSs. One of them is the classical negation, defined by

$$\neg A = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}.$$

For the needs of the present research, we introduce the most general form of the extended modal operators (see, e.g. [1, 2]):

$$X_{a,b,c,d,e,f}(A) = \{\langle x, a.\mu_A(x) + b.(1 - \mu_A(x) - c.\nu_A(x)),$$

$$d.\nu_A(x) + e.(1 - f.\mu_A(x) - \nu_A(x))\rangle | x \in E\}$$

where $a, b, c, d, e, f \in [0,1]$ and there, the following two conditions are given:

$$a + e - e.f \le 1,$$

$$b + d - b.c \le 1.$$

In addition, in [3] it is demonstrated that it is also necesary to add the following third condition:

$$b + e \le 1.$$

In [2] another type of modal operators are described. The most general of them is the intuitionistic fuzzy modal operator

$$\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta} A = \{\langle x, \alpha.\mu_A(x) - \varepsilon.\nu_A(x) + \gamma,$$

$$\beta.\nu_A(x) - \zeta.\mu_A(x) + \delta\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0,1]$ and

$$\max(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \le 1, \min(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \ge 0.$$

In [4], we introduced the following new operator from modal type:

$$\otimes_{\alpha,\beta,\gamma,\delta} A = \{\langle x, \alpha.\mu_A(x) + \gamma.\nu_A(x), \beta.\mu_A(x) + \delta.\nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta, \gamma, \delta \in [0,1]$ and $\alpha + \beta \le 1, \gamma + \delta \le 1$.

According to this definition, on one hand, the operator reduces by α the degree of membership $\mu_A(x)$ original IFS A's and sums it up with a part of the degree of non-membership ($\gamma.\nu_A(x)$), and in the same time it reduces the original A's degree of non-membership ($\nu_A(x)$) by δ and sums it up with a part of the degree of membership ($\beta.\mu_A(x)$).

As it is mentioned in [4], it is easy to see that

$$\otimes_{1,0,0,1} A = A,$$

$$\otimes_{0,1,1,0}A = \neg A,$$

and the set $\otimes_{\alpha,\beta,\gamma,\delta}A$ is an IFS. There, they are proved also that for every IFS A and for every four real numbers $\alpha, \beta, \gamma, \delta \in [0, 1]$ such that $\alpha + \beta \leq 1, \gamma + \delta \leq 1$

$$\neg \otimes_{\alpha,\beta,\gamma,\delta} \neg A = \otimes_{\delta,\gamma,\beta,\alpha}A$$

and for every two IFSs A and B and for every four real numbers $\alpha, \beta, \gamma, \delta \in [0, 1]$ such that $\alpha + \beta \leq 1, \gamma + \delta \leq 1$, it holds that

(a) $\otimes_{\alpha,\beta,\gamma,\delta} (A \cup B) = \otimes_{\alpha,\beta,\gamma,\delta}A \cup \otimes_{\alpha,\beta,\gamma,\delta}B$,
(b) $\otimes_{\alpha,\beta,\gamma,\delta} (A \cap B) = \otimes_{\alpha,\beta,\gamma,\delta}A \cap \otimes_{\alpha,\beta,\gamma,\delta}B$,
(c) $\otimes_{\alpha,\beta,\gamma,\delta} (A + B) = \otimes_{\alpha,\beta,\gamma,\delta}A + \otimes_{\alpha,\beta,\gamma,\delta}B$,
(d) $\otimes_{\alpha,\beta,\gamma,\delta} (A.B) = \otimes_{\alpha,\beta,\gamma,\delta}A. \otimes_{\alpha,\beta,\gamma,\delta} B$.

2 Main results

Here, we formulate and prove some new assertions, related to the intuitionistic fuzzy modal operator $\otimes_{\alpha,\beta,\gamma,\delta}$.

Theorem 1. Let A be an IFS and let $a, b, c, d, e, f, g, h \in [0, 1]$, so that $a+b, c+d, e+f, g+h \in [0, 1]$. Then

$$\otimes_{e,f,g,h}(\otimes_{a,b,c,d}(A) = \otimes_{ae+bg,af+bh,ce+dg,cf+dh}(A). \tag{1}$$

Proof. Let set A and real numbers a, b, c, d, e, f, g, h satisfy the conditions from Theorem 1. Then

$$\otimes_{e,f,g,h}(\otimes_{a,b,c,d}(A))$$

$$= \otimes_{e,f,g,h}(\{\langle x, a.\mu_A(x) + c.\nu_A(x), b.\mu_A(x) + d.\nu_A(x)\rangle | x \in E\})$$

$$= \{\langle x, a.e.\mu_A(x) + c.e.\nu_A(x) + b.g.\mu_A(x) + d.g.\nu_A(x),$$

$$a.f.\mu_A(x) + c.f.\nu_A(x) + b.h.\mu_A(x) + d.h.\nu_A(x)\rangle | x \in E\}$$

$$= \{\langle x, (a.e + b.g).\mu_A(x) + (c.e + d.g).\nu_A(x), (a.f + b.h).\mu_A(x) + (c.f + d.h).\nu_A(x)\rangle | x \in E\}$$

$$= \otimes_{ae+bg,af+bh,ce+dg,cf+dh}(A).$$

Therefore, (1) is valid. □

Theorem 2. Let A be an IFS and let $a, d, e, h \in (0, 1], b, c, f, g \in (0, 1]$, so that $a + b, c + d, e + f, g + h \in [0, 1]$ and

$$bg = cf, \tag{2}$$

$$ag + ch = ce + dg. \tag{3}$$

Then

$$\otimes_{e,f,g,h}(\otimes_{a,b,c,d}(A)) = \otimes_{a,b,c,d}(\otimes_{e,f,g,h}(A)). \tag{4}$$

Proof. Let set A and real numbers a, b, c, d, e, f, g, h satisfy the conditions from Theorem 2. First, we see, that from (2) and (3) it follows:

$$af + bh - be - df = af + \frac{cf}{g}h - \frac{cf}{g}e - df = \frac{f}{g}(ag + ch - ce - dg) = 0.$$

But, by the above conditions, $f, g > 0$. Therefore,

$$af + bh - be - df = 0,$$

i.e.,

$$af + bh = be + df. \tag{5}$$

Now,

$$\otimes_{e,f,g,h}(\otimes_{a,b,c,d}(A))$$

$$= \otimes_{e,f,g,h}(\{\langle x, a.\mu_A(x) + c.\nu_A(x), b.\mu_A(x) + d.\nu_A(x)\rangle | x \in E\})$$

$$= \{\langle x, a.e.\mu_A(x) + c.e.\nu_A(x) + b.g.\mu_A(x) + d.g.\nu_A(x),$$

$$a.f.\mu_A(x) + c.f.\nu_A(x) + b.h.\mu_A(x) + d.h.\nu_A(x)\rangle | x \in E\}$$

(from (3) and (5))

$$= \{\langle x, a.e.\mu_A(x) + a.g.\nu_A(x) + c.f.\mu_A(x) + c.h.\nu_A(x),$$

$$b.e.\mu_A(x) + b.g.\nu_A(x) + d.f.\mu_A(x) + d.h.\nu_A(x)\rangle | x \in E\}$$

$$= \otimes_{a,b,c,d}(\{\langle x, e.\mu_A(x) + g.\nu_A(x), f.\mu_A(x) + h.\nu_A(x)\rangle | x \in E\})$$

$$= \otimes_{a,b,c,d}(\otimes_{e,f,g,h}(A)).$$

Therefore, (4) is valid. $\square$

Theorem 3. Let A be an IFS, $\alpha, \beta, \gamma, \delta \in [0, 1]$, so that $\alpha + \beta, \gamma + \delta \in [0, 1]$, $a, b, c, d, e, f \in [0, 1]$ so that $a + e - e.f \leq 1$, $b + d - b.c \leq 1$ and $b + e \leq 1$. Then

(a) $$\boxed{\circ}_{a,b,c,d,e,f}(\otimes_{\alpha,\beta,\gamma,\delta}(A)) = \boxed{\circ}_{a\alpha - e\beta, b\beta - f\alpha, c, d, e\delta - a\gamma, f\gamma - b\delta}(A),$$

(b) $$\otimes_{\alpha,\beta,\gamma,\delta}(\boxed{\circ}_{a,b,c,d,e,f}(A)) = \boxed{\circ}_{a\alpha - f\gamma, a\beta - f\delta, c\alpha + d\gamma, c\beta + d\delta, e\alpha - b\gamma, e\beta - b\delta}(A).$$

3 Conclusion

In the present paper, new properties of the intuitionistic fuzzy modal operator $\otimes_{\alpha,\beta,\gamma,\delta}$ are given. It is different from the rest modal operators, defined over IFSs. Following [4] we mention that it arises some open problems, as the following ones.

Open Problem 1: Can operator $\otimes_{\alpha,\beta,\gamma,\delta}$ be represented by the extended modal operators?

Open Problem 2: Can operator $\otimes_{\alpha,\beta,\gamma,\delta}$ be represented by the modal operator $\boxed{\circ}_{a,b,c,d,e,f}$?

Open Problem 3: Can operator $\otimes_{\alpha,\beta,\gamma,\delta}$ be used for representation of some type of modal operators?

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