**Notes on Intuitionistic Fuzzy Sets**
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# To what extent can intuitionistic fuzzy options be ranked?

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**Abstract:** In this paper, we continue our considerations concerning the ranking of intuitionistic fuzzy alternatives (options, variants, ...). We complete our previous considerations by showing in another way why the method proposed by us gives proper results. We stress when the method should be applied and emphasize its transparency.

**Keywords:** Intuitionistic fuzzy sets, Ranking intuitionistic fuzzy alternatives, Conditions.
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## 1 Introduction

Intuitionistic fuzzy sets (IFSs for short) are a very convenient tool when decision making, with their natural ability to express alternatives/options rendering their pros, cons, and lack of knowledge. Having different options we want to rank them and point out the best of them. We have proposed a method of ranking the intuitionistic fuzzy alternatives (Szmidt and Kacprzyk [27]) and later explained in a geometrical way (Szmidt and Kacprzyk [36]) its advantages in comparison with other methods. Also, transparency and a common sense of the rule applied in the method of ranking the alternatives have been emphasized. In this paper, we stress when the proposed by us (see [27]) method of ranking should be used, and compare the results obtained by it with the results of other well-known and still cited methods of ranking. In our considerations, we make use of the scenarios (Szmidt and Kacprzyk [33]) which are possible to construct when we take into account all three terms describing the intuitionistic fuzzy alternatives, namely, membership values, non-membership values, and the hesitation margins. We recall the examples of not justified results given by other methods and compare them with the results by our method.

# 2 A brief introduction to IFSs

One of the possible generalizations of a fuzzy set in $X$ (Zadeh [43]) given by

$$A^{'} = \{\langle x, \mu_{A'}(x)\rangle | x \in X\} \tag{1}$$

where $\mu_{A'}(x) \in [0,1]$ is the membership function of the fuzzy set $A^{'}$, is an IFS (Atanassov [1,3,4]) $A$ is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \tag{2}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{3}$$

and $\mu_A(x), \nu_A(x) \in [0,1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively (see Szmidt and Baldwin [12] for deriving memberships and non-memberships for A-IFSs from data).

An additional concept for each A-IFS in $X$, that is not only an obvious result of (2) and (3) but which is also relevant for applications, we will call (Atanassov [3])

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{4}$$

a *hesitation margin* of $x \in A$ which expresses a lack of knowledge of whether $x$ belongs to $A$ or not (cf. Atanassov [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The two-term representation (without a hesitation margin) and the three-term representation (with a hesitation margin) of IFSs with some conceptual and analytic aspects were presented in [31–33, 35].

The hesitation margin was found important while considering the distances, entropy and similarity (cf. [13, 16, 17, 23, 25, 26, 29], etc.), i.e., the measures that play a crucial role in virtually all information processing tasks (Szmidt [11]).

Moreover, it has numerous applications in many fields like: image processing [7], classification with imbalanced and overlapping classes [37–39], classification through intuitionistic fuzzy trees (cf. [6]), multiagent decisions, negotiations, voting, group decision making, etc. (cf. [5, 14, 15, 18–22, 24, 28]), selection of the attributes [34], ranking alternatives [36], genetic algorithms [10] or the discovery of drugs for new therapeutic indications [9]. Sometimes the concept of the hesitation margin is just indispensable, for example, for a proper definition of the Hausdorff distance [30], attribute selection [34], or ranking alternatives [36].

# 3 Natural way of representing and ranking alternatives expressed by IFSs

Each element $x = \langle \mu_x, \nu_x, \pi_x \rangle$ of an IFS can in a very natural way express pros, cons, and lack of knowledge concerning an option/alternative. Pros (advantages) are expressed by the membership values $\mu_x$, cons (disadvantages) are expressed by non-membership values $\nu_x$, and lack of knowledge is expressed by hesitation margin $\pi_x$. For example, when considering application of a medicine $x$, its membership value $\mu_x$ expresses positive effects of applying the medicine, $\nu_x$ expresses negative effects (side effects), and $\pi_x$ expresses lack of knowledge of how a concrete patient will react on the medicine.

To make a proper decision (e.g. to point out the best medicine for a patient), we should be able to order the options/alternatives.

The best way to order the intuitionistic fuzzy alternatives is the following definition.

**Definition 1.** ([2, 3]) *For two intuitionistic fuzzy alternatives $x_1 = \langle \mu_1, \nu_1 \rangle$ and $x_2 = \langle \mu_2, \nu_2 \rangle$ we say that $x_1 \leq x_2$ if*

$$\mu_1 \leq \mu_2 \quad \text{and} \quad \nu_1 \geq \nu_2. \tag{5}$$

If the conditions of Definition 1 are fulfilled, we obtain an order witch is well justified and accepted without any doubts.

The drawback of Definition 1 is that lots of elements belonging to an IFS do not fulfill the condition (5). For example, having the alternatives $x_1 = \langle 0.5, 0.3, 0.2 \rangle$ and $x_2 = \langle 0.6, 0.4, 0.0 \rangle$, we are not able to use Definition 1.

**Remark.** Later we will use the notation $x(\mu, \nu, \pi)$ instead of $x = \langle \mu, \nu, \pi \rangle$.

As Definition 1 can not be always used we have introduced another measure $R$ (6). The motivation of introducing $R$ (6) is given in [27]. In [36] we presented geometrical representations of $R$, and other well-known methods explaining advantages of $R$. Here we even deeper explain the reasons of proper answers given by $R$ (when Definition 1 can not be applied).

The measure $R$ of ordering the alternatives $x(\mu_x, \nu_x, \pi_x)$ is given as [27]:

$$R(x) = 0.5(1 + \pi_x) l_{IFS}(M, x), \tag{6}$$

where $l_{IFS}(M, x)$ is the Hamming distance $x$ from ideal positive alternative $M(1, 0, 0)$.

The Hamming distance between two intuitionistic fuzzy sets $A$ and $B$ in $X = \{x_1, x_2, \ldots, x_n\}$ is equal to [16]:

$$l_{IFS}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|),$$

thus

$$l_{IFS}(M, x) = \frac{1}{2}(|1 - \mu_x| + |0 - \nu_x| + |0 - \pi_x|) = 1 - \mu_x \tag{7}$$

In result, (6) is given as:

$$R(x) = 0.5(1 + \pi_x)(1 - \mu_x) \tag{8}$$

Table 1. Chosen scenarios (9)–(10) for $x_1(0.3, 0.5, 0.2)$, and $x_2(0.3, 0.6, 0.1)$

| $\alpha$ | $x_1(0.3,0.5,0.2)$ | $x_2(0.3,0.6,0.1)$ | Result |
|---|---|---|---|
| $\alpha = 1$ | (0.5, 0.5) | (0.4, 0.6) | $x_1 > x_2$ |
| $\alpha = 0$ | (0.3, 0.7) | (0.3, 0.7) | $x_1 = x_2$ |
| $\alpha = 0.5$ | (0.4, 0.6) | (0.35, 0.65) | $x_1 > x_2$ |

Equation (8) expresses the "quality" of an alternative. The lower the value of $R(x)$, (8), the better the alternative. From (8) we conclude that for the best alternatives the amount of positive information included $\mu_x$ is as big as possible, and the reliability of information $\pi_x$ is as small as possible.

Returning to the alternatives mentioned above: $x_1(0.5, 0.3, 0.2)$, $x_2(0.6, 0.4, 0)$, from (8) we obtain $R(x_1) = 0.3$, $R(x_2) = 0.2$ which means that $x_1$ is a worse alternative than $x_2$.

It is necessary to stress that when the conditions of Definition 1 are fulfilled we can not use the measure $R$ (8). We should pay attention to the equal values of the membership values and non-membership values in the conditions of Definition 1. The equal values mean that Definition 1 should be used.

For example, for $x_1(0.3, 0.5, 0.2)$, $x_2(0.3, 0.6, 0.1)$, we have equal values of membership values, so from Definition 1, as the non-membership value of $x_2$, i.e., 0.6, is bigger than the non-membership value of $x_1$, i.e., 0.5, we conclude that $x_1$ is better than $x_2$.

On the other hand, if we use (8) and calculate $R$, we obtain $R(x_1) = 0.42$, $R(x_2) = 0.385$ which would suggest the opposite answer and is not justified. Using Definition 1 is fully justified by its conditions. However, we can show also in another way that the answer obtained by Definition 1 is correct. To do so, we recall that the three term representation of IFSs can be expressed by two intervals, namely (Szmidt and Kacprzyk [33]:

$$[\mu(x), \mu(x) + \alpha\pi(x)], \tag{9}$$

$$[\nu(x), \nu(x) + (1-\alpha)\pi(x)], \tag{10}$$

where $\alpha \in [0, 1]$.

Making use of the above representation (9)–(10) we can consider different scenarios for $x_1(0.3, 0.5, 0.2)$, and $x_2(0.3, 0.6, 0.1)$ (cf. Table 1). We can notice from Table 1 that both for the best possibility ($\alpha = 1$) and for the worst possibility ($\alpha = 0$), the alternative $x_1$ is better than $x_2$. When the hesitation margin is divided equally between membership values and non-membership values of $x_1$ and $x_2$, i.e., for $\alpha = 0.5$, both alternatives become equal. In result we obtain result the same as given by Definition 1. When the conditions of Definition 1 are fulfilled, we can not use measure $R$ (8).

The same situation repeats when we consider options with equal non-membership values. An example for $x_1(0.7, 0.1, 0.2)$, and $x_2(0.8, 0.1, 0.1)$ is presented in Table 2. The result is again the same as obtained from Definition 1.

Table 2. Chosen scenarios (9)–(10) for $x_1(0.7, 0.1, 0.2)$, and $x_2(0.8, 0.1, 0.1)$

| $\alpha$ | $x_1(0.7, 0.1, 0.2)$ | $x_2(0.8, 0.1, 0.1)$ | Result |
|---|---|---|---|
| $\alpha = 1$ | (0.9, 0.1) | (0.9, 0.1) | $x_1 = x_2$ |
| $\alpha = 0$ | (0.7, 0.3) | (0.8, 0.2) | $x_1 < x_2$ |
| $\alpha = 0.5$ | (0.8, 0.2) | (0.85, 0.15) | $x_1 < x_2$ |

Table 3. Chosen scenarios (9)–(10) for $x_1(0.7, 0.3, 0)$, and $x_2(0.7, 0.15, 0.15)$

| $\alpha$ | $x_1(0.7, 0.3)$ | $x_2(0.7, 0.15, 0.15)$ | Result |
|---|---|---|---|
| $\alpha = 1$ | (0.7, 0.3) | (0.85, 0.15) | $x_1 < x_2$ |
| $\alpha = 0$ | (0.7, 0.3) | (0.7, 0.3) | $x_1 = x_2$ |
| $\alpha = 0.5$ | (0.7, 0.3) | (0.775, 0.225) | $x_1 < x_2$ |

The above results indicate the source of some doubts concerning measure $R$ (8) pointed out in some papers. The authors do not apply Definition 1 when its conditions are fulfilled, and instead, use $R$ (8) obtaining doubtful results. But it is not a drawback of $R$ (8) but using it when it should not be used. The measure $R$ (8) works in a proper way only when Definition 1 can not be applied.

In Table 3 we have an example of two alternatives $x_1$ and $x_2$ with equal membership values (equal to 0.7), and with the non-membership value less for $x_2$ (equal to 0.15) than that for $x_1$ (equal to 0.3). As the conditions of Definition 1 are fulfilled, we conclude that $x_2 > x_1$. Consideration of the scenarios (Table 3) confirms the same result. If we use measure $R$, the result would be counterintuitive ($R(x_1) = 0.15$; $R(x_2) = 0.1725$, i.e., $R(x_1) < R(x_2)$ which means that $x_1 > x_2$, the obviously wrong result of using $R$ whereas Definition 1 should be used).

The above examples were to stress firmly, that first we verify conditions of Definition 1 and use the Definition if possible. Only if it is not possible, we use measure $R$ (8). Measure $R$ (8) always points out the options with bigger membership values, and lower hesitation margins.

In our previous paper [36] we pointed out the drawbacks of some well-known methods ranking intuitionistic fuzzy alternatives. We have presented a general geometrical representation of these methods and gave some numerical examples showing that these methods do not work properly. Now we will show that our method using measure $R$ (8) gives proper results in the same situations.

## 3.1 Xu's method [41]

In (Szmidt and Kacprzyk [36]), two intuitionistic fuzzy alternatives are considered, namely, $a_1(0.5, 0.4, 0.1)$ and $a_2(0.3, 0.1, 0.6)$. From Xu's method [41] it is concluded [36] that $a_1$ is smaller than $a_2$. However, the membership value of $a_1$ is bigger than that of $a_2$. Moreover, the hesitation margin of $a_2$ equal to 0.6 is greater than the hesitation margin of $a_1$ equal to 0.1. In other words, it is difficult to agree that $a_1$ is smaller than $a_2$.

In our approach, conditions of Definition 1 are not fulfilled, so we use measure $R$ (8). It is enough to notice that the membership of $a_1$ is bigger than that of $a_2$, and the hesitation margin of

$a_1$ is smaller than that of $a_2$. Thus we conclude that $a_1 > a_2$.

Another feature of the method [41] is not a proper trend in ranking. To be more precise, when the membership values increase and the hesitation margins decrease, one could expect that the alternatives are better. Unfortunately, the results generated are counter-intuitive. For the alternatives:

$$\begin{aligned} a_1 &= (0.2, 0.11, 0.69), \\ a_2 &= (0.3, 0.22, 0.48), \\ a_3 &= (0.4, 0.33, 0.27), \end{aligned}$$

from [41] we obtain [36]:

$$a_1 > a_2 > a_3.$$

On the other hand, from $R$ (8), we obtain

$$a_3 > a_2 > a_1,$$

which is justified with the rule that an alternative with bigger membership values and lower hesitation margins are better.

By changing a little the non-membership values in the above example, i.e. by considering:

$$\begin{aligned} a_1 &= (0.2, 0.1, 0.7), \\ a_2 &= (0.3, 0.2, 0.5), \\ a_3 &= (0.4, 0.3, 0.3), \end{aligned}$$

we obtain from method given in [41] the reverse order (cf. [36] for detailed calculations), namely:

$$a_3 > a_2 > a_1.$$

On the other hand, from $R$ (8) we obtain, due to the rule that the best options are those with the biggest membership and the lowest hesitation margin, the order is still the same as it was before small changes of the non-membership values, i.e.:

$$a_3 > a_2 > a_1.$$

The conclusion is that $R$ (8) avoided the drawbacks of Xu's method [41].

## 3.2 Zhang and Xu [42] method

In [36] an example is given showing not justified results for Zhang and Xu [42] method. Two alternatives were examined: $a_1(0.1, 0.1, 0.8)$ and $a_2(0.4, 0.6, 0)$.

By Zhang and Xu [42] method $a_1 > a_2$ (see [36] for details) which is counterintuitive as the membership of $a_1$, (equal to 0.1) is less than the membership of $a_2$ (equal to 0.4), and the hesitation margin of $a_1$ is bigger than the hesitation margin for $a_2$ (equal to 0). On the contrary, result given by $R$ (8) is opposite, i.e., $a_1 < a_2$ ($R(a_1) = 0.81 > R(a_2) = 0.3$).

### 3.3 Guo's [8] method

In [36] the following example is considered in the context of Guo's [8] method: $a_1(0.42, 0, 0.58)$ and for $a_2(0.5, 0.5, 0)$. Guo's [8] method points out that both alternatives are the same. It is difficult to agree with the result. Considering $R$ (8) we obtain $R(a_1) = 0.458 > R(a_2) = 0.25$ which means that $a_1 < a_2$ (alternative $a_2$ has a bigger membership value and lower hesitation margin that the respective values for $a_1$).

### 3.4 Xing, Xiong, Liu [40] method

In the case of Xing, Xiong, Liu [40] method we have a similar situation discussed in [36]. For two options $a_1(0.2, 0, 0.8)$ and for $a_2(0.53, 0.47, 0)$, making use of Xing, Xiong, Liu [40] method we obtain answer that the two options are the same (cf. [36]) although the options are quite different. The difference is properly seen by $R$ (8) from which we obtain $R(a_1) = 0.72 > R(a_2) = 0.235$ which means that $a_1 < a_2$.

## 4 Conclusions

We explained the problem of ranking intuitionistic fuzzy alternatives. The first step is to use Definition 1. It means that we prefer options with the biggest advantages and the smallest disadvantages. If the condition of Definition 1 are not fulfilled, and only then, we use measure $R$ (8). It means that we rank higher the options with higher membership values (bigger advantages) and lower hesitation margins (less lack of knowledge). It is a transparent rule and seems reasonable. Moreover, in the situation when for two options we receive the same values of $R$ (8), as we noticed in (Szmidt et al. [36]), the properties of $R$ (8) make it possible to order such options.

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