Notes on Intuitionistic Fuzzy Sets
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My meetings with Prof. Lotfi Zadeh

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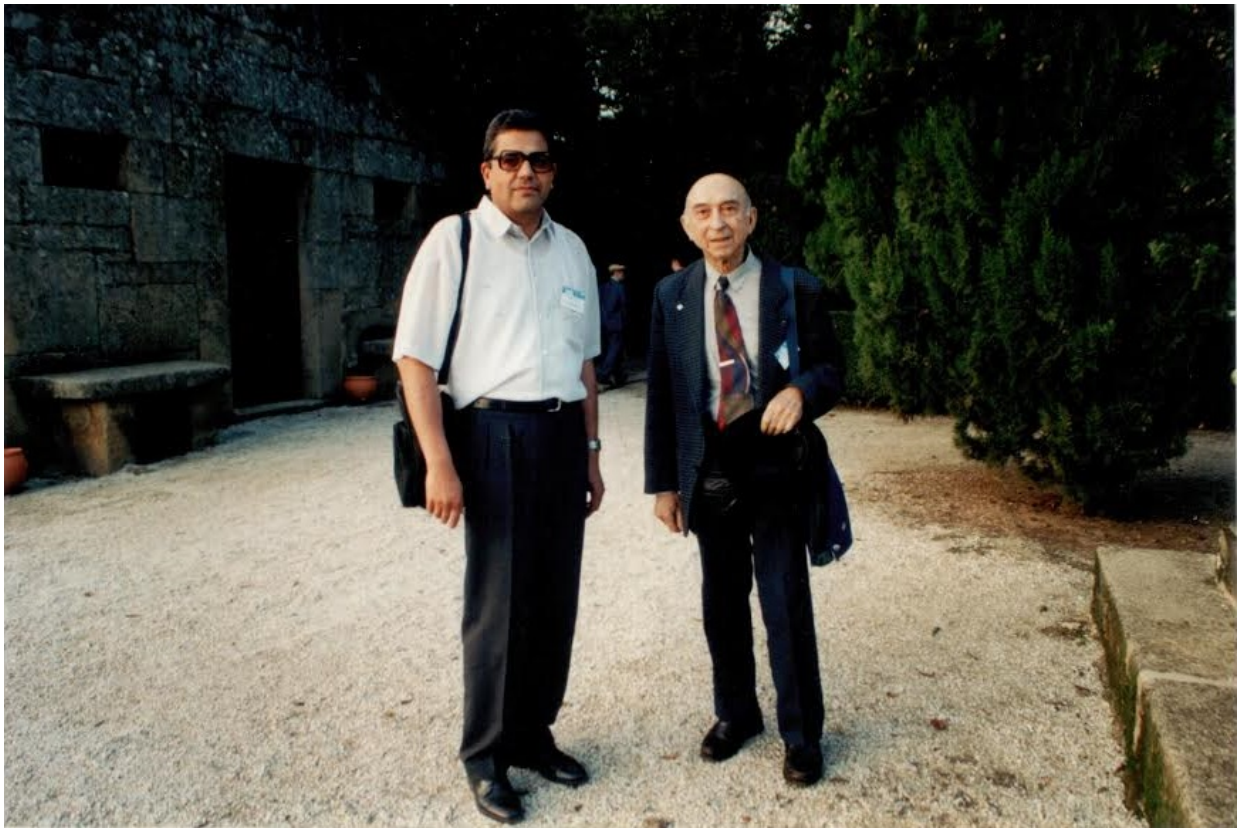
In 2021, we celebrate the 100th anniversary of the birth of Prof. Lotfi Zadeh. On this occasion, in this short note, I will share memories from my brief meetings with one of the few persons whose endless enthusiasm, hard work, purposefulness and capacity to innovate I have always admired.

My first meeting with his fuzzy sets was in February 1983. During one hospital stay, I had taken as a piece of leisure reading the first book on fuzzy sets which had appeared in Bulgaria – the Russian translation [4] of the book of Prof. Arnold Kaufmann [3]. Several years before that, as a university student, I had heard about that notion, which several logicians from the Mathematical Logic Department at my Faculty of Mathematics had referred to as "nonsense". This said, years later, I came up to the conclusion that back then all of them had failed to fully grasp the theory of fuzzy sets; so novel and different—and game changing—it must have been for them and it was indeed.

As I have already had several times the opportunity to write on various occasions, e.g., [2], for me it was a kind of a "mathematical game" to play with the definition of Zadeh's fuzzy set, adding up a second degree—that of non-membership—to the degree of membership, characteristic of the fuzzy sets. In my "game", the sum of both degrees was giving as a result a non-negative real number less than or equal to 1. Studying its most immediate properties, I realized that the newly defined set is significantly different from the fuzzy set, and after showing my notes to my former Assistant Professor at the Faculty, George Gargov, he proposed to name it *intuitionistic fuzzy set*. Once my mathematical "offspring" had a name, I naturally decided to learn more about the standard fuzzy sets in order to be able to develop the intuitionistic fuzzy sets appropriately. Over the next several years, I penned several letters to Prof. Zadeh asking him for copies of his papers, and he was so kind and generous to send me the papers that I had asked for. In this way I became aware of Prof. Zadeh's works [5–7] and in a way, was able to drink from the source.

My first face-to-face meeting with Prof. Zadeh was in 2001 in Vila Real, Portugal. I was invited by Prof. Pedro Melo-Pinto to participate in a conference he organized at the University



de Trás-os-Montes e Alto Douro. In the morning, after the conference opening, Prof. Zadeh spoke for 3 hours, walking between the display where the slides were projected, the table where his paper notes were placed, and the slide projector. After lunch, he chaired the discussion over applications of fuzzy sets for another 3 hours. At that time he was 80 years old! After the end of the discussion, I introduced myself to him and we had a long and interesting conversation. When he learned that I am a Bulgarian with a not-so-good command of English, he switched to Russian. During the years when I worked at the Institute of Physics at the Bulgarian Academy of Sciences, I had met many Russian researchers visiting our institute, but I had never heard from any of them such melodical Russian as the one Zadeh produced.

In the evening, when I was leaving Vila Real for Sofia I saw through the train window two cars that were moving in two parallel roads which eventually merged in one road. I could not determine which of the cars passed first and which – second. The sight however reminded me of the question which was raised by Prof. Zadeh during his talk: "When and how does uncertainty appear?". As a result of that scene, after arriving in Sofia, I wrote a short text about the question of parallelism and sent it to Prof. Zadeh. We arranged to discuss the details at our next meeting which was to take place in Annecy, France, in 2002. Unfortunately, I arrived there at the day of my presentation while he had already delivered his talk and and had left.

Our second meeting was in Zittau, Germany, in 2003. It was a brief one as I arrived again right before his departure. I had invited him the year before to take part in the 2nd International IEEE Conference of "Intelligent Systems" which was scheduled to take place in Varna, Bulgaria, in 2004. When I asked him in Zittau to confirm that he will come, he said "Wait a moment, please," and took out of his bag a big notebook, glanced at it and said "Oh, this notebook is for

2005". Then he took out another notebook—this time for 2004. At that time he was already 82 years old and yet he was planning his conference participations for 2-3 years ahead! I have told this story many times to illustrate his lively spirit.

The third time I met Zadeh was in Varna in 2004. He arrived on the day before the opening of the "Intelligent Systems" conference. I met him at the airport and took him for a tour around the city. Then we arrived at the hotel where he opened the 8$^{\text{th}}$ International Conference on Intuitionistic Fuzzy Sets, which took place in parallel to the IEEE Conference.

In our conversation back then we touched the parallelism theme. He said that he had thought over this idea and he planned to write about it. I had to write about some of the things we dicussed but at that time I was working on other themes and in the end the idea of parallelism remained unfinished and not completely talked through. Now, for the first time I am showing the text written in 2001 that was the object of our discussions.

In 2001, at the end of his speech in Vila Real, Prof. Zadeh showed slides with quotes from papers of Prof. Kurt Gödel and Prof. Stephen Cole Kleene in which concerns and criticisms were raised regarding the notion of fuzzy sets. On the one hand, I was amazed that such renowned scientists had not judged on its merits this new mathematical object which had proven to have a large number of applications. On the other hand, I understood why some Bulgarian specialists in logic opposed to the fuzzy sets. In practice, only George Gargov among them was not an opposer, and as long as he was alive, he was supportive of intuitionistic fuzzy sets and my efforts to work in the field. Over the next decade, I spent a lot of time thinking about this weird (at least for me) situation, and I finally arrived to the conclusion that Gödel and Kleene were concerned that Zadeh proposed that the evaluations of the evaluated objects were expressed as numbers in the $[0, 1]$ interval, which to a large extent are non-constructable objects. Indeed, there is a difference whether the evaluations are 0 or 1 as proposed by Aristotle; 0, $1/2$ or 1; or even $0, \frac{1}{n-1}, \frac{2}{n-1}, \ldots, \frac{n-2}{n-1}, 1$, as proposed by Jan Łukasiewicz; or $\sqrt{2}$–1, $\frac{1}{e}$, or some even more "terrifying" transcendental number. But, in practice, no expert would ever evaluate that the weather tomorrow will be sunny in $\frac{1}{\pi}$ parts of the day! He would say "The weather will be sunny in 10% of the time" or "in 20% of the time", or even if he is completely certain that the weather will be bright he would say "in 101% of the time", or he would use another integer or in the worst case rational, i.e., a constructable number.

When I realized this, I wrote a paper in which I pointed out that if the degrees of membership and non-membership of an intuitionistic fuzzy set are rational numbers, then if all operators (modal, topological, level operators) have as parameters rational numbers in the interval $[0, 1]$, when applied to this set, they will preserve the rational character of its degrees. Also, if we have two intuitionistic fuzzy sets, whose degrees of membership and non-membership are rational numbers, then all operations over them will yeild as a result rational numbers again. Therefore, the fuzzy sets do not send us to non-constructiveness. I sent the paper to Prof. Zadeh and he approved it, encouraging me to publish it. I presented it as a talk at the International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets in Warsaw [1], and after that I included it in my monograph [2].

I deeply regret that I did not have the opportunity to meet Prof. Zadeh ever after, but it was until the last year of his life that we maintained correspondence by email. Many times, I would

ask for his opinion on particular ideas, and every time he would answer in his accurate, enriching and insightful way.

Ever since my first encounter with his concept of fuzzy sets almost 40 years ago, I have seen in Lotfi a Teacher of mine. A feeling which, I suppose, many other colleagues who had the chance the meet him, have also felt by themselves. His idea, his bright mind redefined to a large extent my scientific career and my life.

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Remark on the concepts of parallelism and fuzziness

The Mathematics from ancient times up to 1962 is mathematics of algorithms, mathematics of the sequential processes. Solving a Diophantine equation or solving of a system of (arbitrary many and complex) differential equations, constructing a geometrical figure, or the functioning of the Turing machine – the solutions of all these problems are realized step by step.

Petri nets, introduced in 1962 by *Carl-Adam Petri* were the first tools for describing of *parallel* processes. Of course, the ordinary Petri nets were very simple instruments for modelling, but their next extensions and modifications such as E-Nets, Time Petri nets, Colour Petri nets, Predicative-Transition nets, Self-Modifying Petri nets, Generalized nets, Fuzzy Petri nets. They are suitable to describe the real processes in (enough) details.

The twentieth century set theory enriched mathematics with the ideas of D. Hilbert, E. Zermelo, A. Fraenkel, J. von Neumann, J. Bernaus, K. Gödel, J. Łukasiewicz, P. Cohen.

In 1965 *Lotfi Zadeh* gave his contribution to the future of mathematics.

Today, mathematicians, biologists, sociologists, physicist, psyhologists – all of these specialists in different areas know L. Zadeh's apparatus of fuzzy sets.

Like the Petri nets, fuzzy sets gave essentially new and more suitable possibilities to model reality.

The author's opinion is that between the *parallelism* and *fuzziness* there are essentially more important connections.

Now, we shall discuss one possibility for the appearance of fuzziness (uncertainty, vagueness) generated by the parallelism of two (or more) processes.

Let an observer observe two processes flowing parallelly in time (see Fig. 1). If the events of both processes happen at different time-moments, then the observer *will be able to register* them. But when two events take place at one and the same time-moment, then it would be difficult for the observer to record them (see Fig. 2). For example, when there is a full solar eclipse, we do not see the Sun.

In the case of a particular eclipse we see simultaneously both – the Sun and the Moon, but someone may assert that one sees only the Sun that for several minutes changes its form. Therefore, when we have two events happening at one time-moment, then we can observe both or only one of them.

The same is the situation with Bohr's Principle of Additionality, with Heisenberg's Principle of Uncertainty, or with Pauli's Effect. Why are we unable to determine simultaneously the coordinates and the velocity of the electron? We are, because determining both of them needs simultaneous realization of two (different) processes (algorithms).

Parallelism and fuzziness have similar effects over electrocardiograms and electroencephalograms, in telecommunication connections, etc.

The presence of noise deforming the measures is the result of two parallel processes: the process of measurement and the process of accounting the first process error level.

Therefore, we can formulate the following

Principle: *Parallelism generates fuzziness.*

Now, we can see that in case of longer duration of the joint functioning of two events we obtain an example of L. Zadeh's granules. On Fig. 2 any of these granules are represented by the symbol "•". The situation is essentially more interesting when the two granules exist during different time-intervals (see Fig. 3).

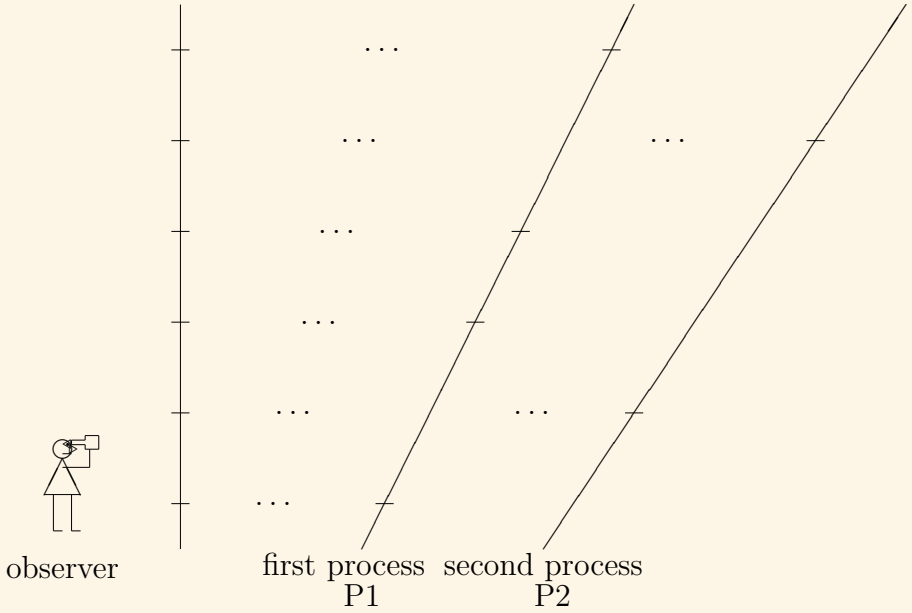

Fig. 1.

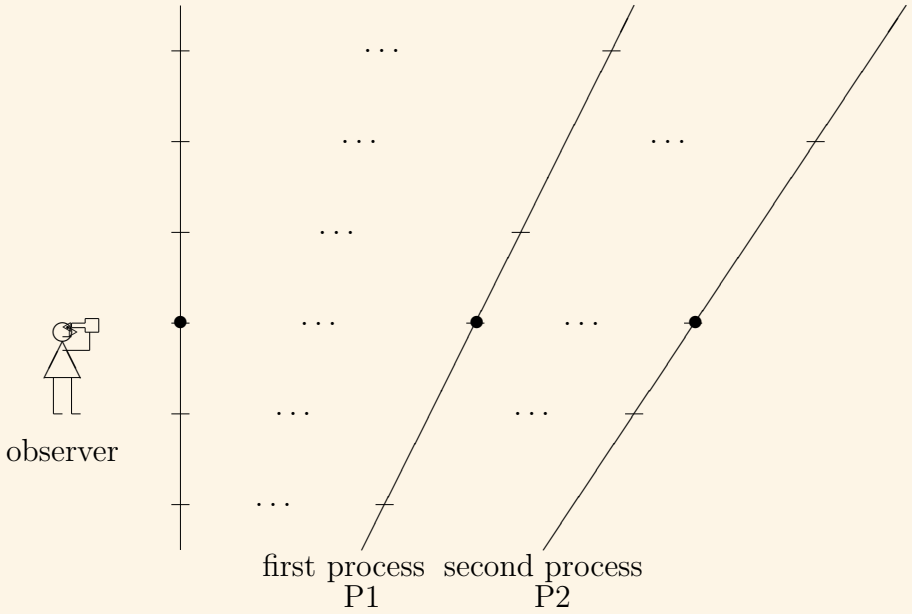

Fig. 2.

In this case the uncertainty is more complex than in the case of Fig. 2. There are three time-intervals – A, B and C for the observer from Fig. 3. He observes the first process in time-interval A, the second process in time-interval C and *both* processes, or *at least one of them* in time-interval B. Astronomy, history, social sciences give rise to many exemplary situations when (it is) not obvious which of the events is being observed. For instance, in the battle of Waterloo there were two clearly determined armies – of Napoleon and of Wellington, but there were two other armies located around – the French and the German and for some time it was not clear which of them would arrive first and would seal the fate of the battle. According to the time-scale (i.e., if the observer watched the battle instantly), if the left of both granules determined the French army (in general) and the right granule – the ally (English and German) armies, then time period A corresponds to the initial French superiority, time period B corresponds to the second phase of the battle, when the German army arrived first, and time period C corresponds to the total French defeat. All of these processes can be described by the means of the fuzzy sets, but they *cannot* be represented by the ordinary tools of the mathematical logic.

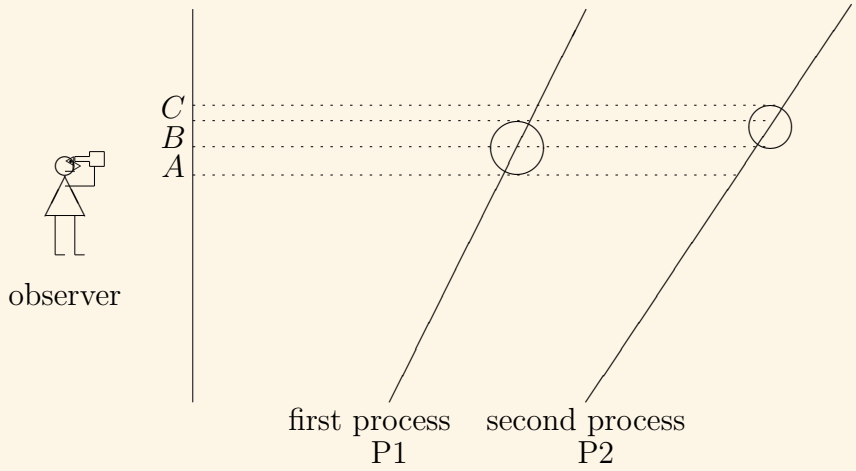

Fig. 3.

Therefore, we can have *degree of observing of the first process*, and *degree of observing of the second process*, and *degree of observing of both processes*, i.e., *degree of uncertainty*. By this reason, we obtain a situation that is specific for Brouwer's intuitionistic mathematics.

The same will be valid (even in more powerful form) for the situation from Fig. 4, where there are three separate processes and in some moment we cannot determine the observed event to which of the processes belongs. For the observer from Fig. 4, there are five time-intervals – A, B, C, D and E. He observes the first process in all time-intervals, but now, in time-intervals B, C and D, it is combined with other processes, the second process in time-intervals B and C, the third process in time-intervals C and D, the three processes (first, second and third) in time-interval C.

We can illustrate the above construction, e.g., by the following example. Each person has immune system that functions continuously (the first process), but its status can be different in the separate time moments with respect to the status of the human body.

By this reason we must use a granule for representing the status of the immune system. In some moment, a viral illness starts (the second process). It is not clear exactly when did the illness start: when the first virus entered the human body, or when the viral load on the human body was great enough to start the illness. For representation of the three processes we need granules again. In some (exactly determined) moment the patient starts taking medicines (the third process) which influence their health status. The end-point of the third process is also unpredictable. We can decide that the second process stopped, but we cannot exactly determine the status of the patient's immune system. The medicines influence the patient's immune system and change its parameters. That is why it struggles not only with the viruses, but at a subsequent stage – with the medicines, too.

We can increase the complexity of the constructed examples even more, if, for example, we detailize the parameters of the medicines, if we would like to account the influences of each medicine over the others, etc.

All these and a lot of other examples illustrate that parallelism generates fuzziness.

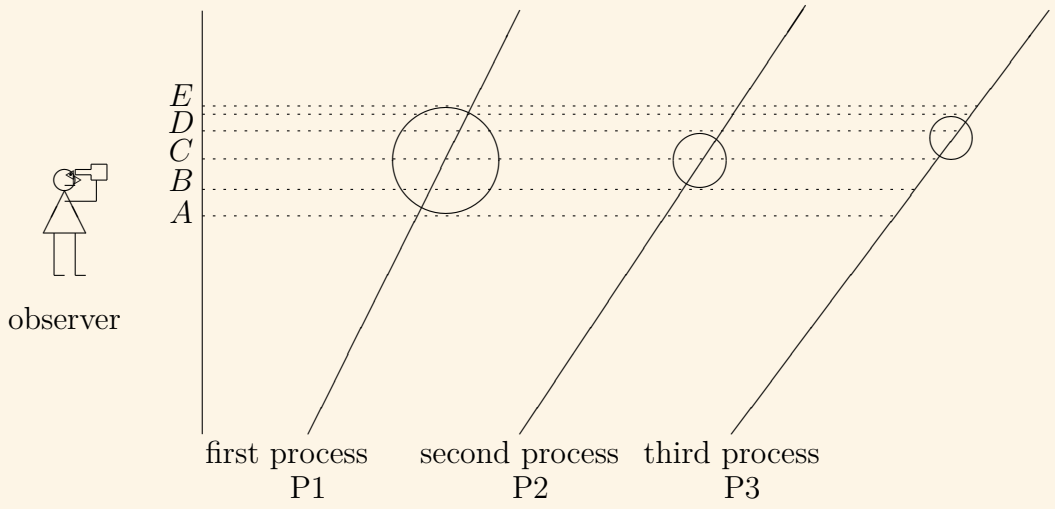

Fig. 4.