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A GENERALIZATION OF SOME OPERATIONS DEFINED OVER INTUITIONISTIC FUZZY SETS

Stephan Danchev
Center of Biomedical Engineering, Bulgarian Academy of Sciences,
Acad. G. Bonchev street, bl. 105, 1113 Sofia, Bulgaria
e-mai: CLBME@BGEARN.BITNET

The Intuitionistic Fuzzy Sets (IFS) are introduced in [1] as nontrivial extension of the Fuzzy Sets. Many operations over these sets are defined and examined in [2] - [7]. In the present work we shall define a two-parametric operation $\otimes_{(x,y)}$ that generalizes the known operations $\cup$, $\cap$, @, \$ and $\triangleright\triangleleft$.

For this purpose first we define a function $f : \mathbb{R} \times [0,1] \times [0,1] \rightarrow \mathbb{R}$ as follows:

$$f(x,a,b) = \begin{cases} \left(\dfrac{a^x + b^x}{2}\right)^{1/x} & \text{for } x > 0 \text{ or } (x < 0 \text{ and } a.b > 0) \\ \sqrt{ab} & \text{for } x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

Further we shall use the properties (1) - (3) of this function.

(1) $f(x,a,b)$ is continuous in $\mathbb{R} \times [0,1] \times [0,1]$.

(2) $f(x,a,b)$ monotonically increases with respect to x. If $0 \neq a \neq b \neq 0$ then f strongly increases.

(3) $\lim\limits_{x \to +\infty} f(x,a,b) = \max\{a,b\}$, $\lim\limits_{x \to -\infty} f(x,a,b) = \min\{a,b\}$. Therefore we can extend the definition of f over $(\mathbb{R} \cup \{-\infty, +\infty\}) \times [0,1] \times [0,1]$:

$$f(+\infty,a,b) = \max\{a,b\}$$
$$f(-\infty,a,b) = \min\{a,b\}$$

Let A and B are IFS over fixed (normal) set E. We define

$$A \otimes_{(x,y)} B = \{\langle e, f(x,\mu_A(e),\mu_B(e)), f(y,\nu_A(e),\nu_B(e))\rangle : e \in E\}$$

The operation $\otimes_{(x,y)}$ is not defined correctly for all $x, y \in \mathbb{R} \cup \{-\infty, +\infty\}$. Proposition (4) gives the necessary and sufficient conditions.

(4) $\otimes_{(x,y)}$ is defined correctly only in the following three cases:
- $x \in (-\infty, 1], \quad y \in (-\infty, 1]$
- $x = -\infty, \quad y \in \mathbb{R} \cup \{-\infty, +\infty\}$
- $x \in \mathbb{R} \cup \{-\infty, +\infty\}, \quad y = -\infty$

It is easy to see for which values of the parameters we can obtain the known operations $\cup$, $\cap$, @, \$ and $\rhd\!\lhd$ by the new one:

$$\begin{aligned} A \cup B &= A \otimes_{(+\infty, -\infty)} B \\ A \cap B &= A \otimes_{(-\infty, +\infty)} B \\ A @ B &= A \otimes_{(1,1)} B \\ A \$ B &= A \otimes_{(0,0)} B \\ A \rhd\!\lhd B &= A \otimes_{(-1,-1)} B \end{aligned}$$

$\otimes_{(x,y)}$ is binary operation, but we can define it as n - ary one:

$$\bigotimes_{1 \le j \le n}^{(x,y)} A_j = \{\langle e, F_n(x, \mu_{A_1}(e), \mu_{A_2}(e), \dots \mu_{A_n}(e)), F_n(y, \nu_{A_1}(e), \nu_{A_2}(e), \dots \nu_{A_n}(e))\rangle : e \in E\},$$

where the functions $F_n : (\mathbb{R} \cup \{-\infty, +\infty\}) \times [0,1]^n \to \mathbb{R}$ are defined as follows:

$$F_n(x, a_1, a_2, \dots a_n) = \begin{cases} \left(\frac{1}{n}\sum_{j=1}^{n} a_j^{\,x}\right)^{1/x} & \text{for } x \in (0, +\infty) \text{ or } (x \in (-\infty, 0) \text{ and } \prod_{j=1}^{n} a_j > 0) \\ \sqrt[n]{\prod_{j=1}^{n} a_j} & \text{for } x = 0 \\ 0 & \text{for } x \in (-\infty, 0) \text{ and } \prod_{j=1}^{n} a_j = 0 \\ \min\{a_1, a_2, \dots a_n\} & \text{for } x = -\infty \\ \max\{a_1, a_2, \dots a_n\} & \text{for } x = +\infty \end{cases}$$

It is not hard to see that propositions (1) - (4) remain true in this case.

Finally we shall give some elementary properties of the operation $\otimes_{(x,y)}$ (we assume x and y are such that $\otimes_{(x,y)}$ is correctly defined). Their proofs are straightforward having in mind (1) - (3).

$$A \otimes_{(x,y)} A = A$$

$$A \otimes_{(x,y)} B = B \otimes_{(x,y)} A$$

$$\overline{A \otimes_{(x,y)} B} = \overline{A} \otimes_{(y,x)} \overline{B}$$

$$A \otimes_{(x,y)} B \subseteq A \otimes_{(z,t)} B \text{ for all } A, B \quad \text{iff} \quad x \leq z \text{ and } y \geq t$$

$$A \subseteq B \quad \Rightarrow \quad A \subseteq A \otimes_{(x,y)} B \subseteq B$$

$$AB \subseteq A \otimes_{(x,y)} B \subseteq A + B$$

$$(A \otimes_{(x,y)} B) \otimes_{(x,y)} (C \otimes_{(x,y)} D) = (A \otimes_{(x,y)} C) \otimes_{(x,y)} (B \otimes_{(x,y)} D) = (A \otimes_{(x,y)} D) \otimes_{(x,y)} (B \otimes_{(x,y)} C)$$

$$(A \otimes_{(x,y)} B) \otimes_{(x,y)} C = (A \otimes_{(x,y)} C) \otimes_{(x,y)} (B \otimes_{(x,y)} C)$$

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