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ON INTUITIONISTIC FUZZY INTERPRETATIONS OF ELEMENTS OF UTILITY THEORY.
Part 1

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ABSTRACT: Some intuitionistic fuzzy interpretations of elements of the utility theory are discussed.

KEYWORDS: intuitionistic fuzzy logic, intuitionistic fuzzy tautology, utility theory

The basis of the present remark is the Fishburn's book [1], where the basic elements of the utility theory are introduced. Here we shall show haw some of the elements of this book can be interpreted in an intuitionistic fuzzy sense.

First, we shall introduce some remarks on the Intuitionistic Fuzzy Logic (IFL) (see [2-5]). In it, to each propositional form p (cf. [6]) we assign two real numbers, $\mu(p)$ and $\nu(p)$, called truth- and falsity-degrees, respectively, with the following constraint:

$$\mu(p) + \nu(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositional forms S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

Here we shall interpret some concepts of [1] using the above idea.

First, we shall note that if we have a set X whose elements can be called *alternatives,* or *consequences,* or *commodity bundles,* or *cash flows,* or *systems,* or *allocations,* or *inventory policies,* or *strategies,* or so forth, then we can assign to these elements a pair of real numbers in the interval $[0, 1]$ whose sum does not exceed 1. These two numbers can represent, respectively the degree of our *consent,* or *assurance,* or *preference,* etc. and the degree of our *dissent,* or *disagreement,* or *reluctance,* or so forth.

Let us assume that the function V juxtaposes for the alternative x (everywhere below we shall only use this word, but all is the same if we use some other, e.g., from the below list) the couple

$$V(x) = \langle \mu(x),\ \nu(x) \rangle$$

for which

$$\mu(x) + \nu(x) \leq 1.$$

In all above cases, the number

$$\pi(x) = 1 - \mu(x) - \nu(x)$$

will represent the degree of *indifference,* or *nonchalance,* or *languor,* etc.

Second, for two alternatives $x, y \in X$ we shall introduce the notions of *intuitionistic fuzzy preference (IFP)* through:

$$x \prec y \text{ if and only if } \mu(x) \leq \mu(y) \text{ and } \nu(x) \geq \nu(y)$$

and of *intuitionistic fuzzy indifference (IFI)* through:

$$x \bowtie y \text{ if and only if } (\mu(x) < \mu(y) \text{ and } \nu(x) < \nu(y)) \text{ or } (\mu(x) > \mu(y) \text{ and } \nu(x) > \nu(y))$$

It can be seen easily that

$$x \bowtie y \text{ if and only if } \neg(x \prec y) \text{ and } \neg(y \prec x).$$

Now, we shall demonstrate a possibility for an application of the IFL within utility theory, giving intuitionistic fuzzy interpretations and modifications of two important assertions of this theory.

Following [1] (cf. [7,8]) we shall prove the following

THEOREM. For every four alternatives $x, y, z, w \in X$:

$$\text{if } x \prec y \text{ and } z \prec w \text{ then } x \prec w \text{ or } z \prec y \text{ or } x \bowtie z \text{ or } y \bowtie w.$$

We must note immediately, that if the interpretation were plain fuzzy, but not intuitionistic fuzzy one, then the latter two expressions would have to be reduced to the former two and the Theorem 1 would coincide with the theorem from [1,7,8]. In the case of an intuitionistic fuzzy interpretation the assertion is not valid without the last two members. Indeed, if

$$V(x) = \langle 0.4, 0.6 \rangle,\ V(y) = \langle 0.5, 0.5 \rangle,\ V(z) = \langle 0.2, 0.1 \rangle,\ V(w) = \langle 0.3, 0.1 \rangle,$$

then $x \prec y$ and $z \prec w$ are true, but $x \prec w$ and $z \prec y$ are false.

Let us assume that $x \prec y$ and $z \prec w$. Therefore the following four inequalities are valid simultaneously

$$\mu(x) \leq \mu(y), \tag{1}$$

$$\nu(x) \geq \nu(y), \tag{2}$$

$$\mu(z) \leq \mu(w), \tag{3}$$

$$\nu(z) \geq \nu(w). \tag{4}$$

Let us assume that none of the relations $x \prec w$, and $z \prec y$, and $x \bowtie z$, and $y \bowtie w$ is valid. Therefore,

$$\left\{\begin{array}{lcr} & \mu(x) > \mu(w) & (5)\\ \text{or} & & \\ & \nu(x) < \nu(w) & (6)\end{array}\right.$$

and

$$\left\{\begin{array}{lcr} & \mu(z) > \mu(y) & (7)\\ \text{or} & & \\ & \nu(z) < \nu(y) & (8)\end{array}\right.$$

and

$$\left\{\begin{array}{lcr} \left\{\begin{array}{l} \\ \text{or} \\ \\ \end{array}\right. & \begin{array}{c} \mu(x) \geq \mu(z) \\ \\ \nu(x) \geq \nu(z) \end{array} & \begin{array}{r} (9) \\ \\ (10) \end{array} \\ \text{and} & & \\ \left\{\begin{array}{l} \\ \text{or} \\ \\ \end{array}\right. & \begin{array}{c} \mu(x) \leq \mu(z) \\ \\ \nu(x) \leq \nu(z) \end{array} & \begin{array}{r} (11) \\ \\ (12) \end{array} \end{array}\right.$$

and

$$\left\{\begin{array}{lcr} \left\{\begin{array}{l} \\ \text{or} \\ \\ \end{array}\right. & \begin{array}{c} \mu(y) \geq \mu(w) \\ \\ \nu(y) \geq \nu(w) \end{array} & \begin{array}{r} (13) \\ \\ (14) \end{array} \\ \text{and} & & \\ \left\{\begin{array}{l} \\ \text{or} \\ \\ \end{array}\right. & \begin{array}{c} \mu(y) \leq \mu(w) \\ \\ \nu(y) \leq \nu(w) \end{array} & \begin{array}{r} (15) \\ \\ (16) \end{array} \end{array}\right.$$

Let us assume (5). Therefore from (1) and (5) it follows that

$$\mu(y) \geq \mu(x) > \mu(w),$$

i.e., (15) is not valid. Therefore, (16) is valid and from it and (4) it follows that

$$\nu(y) \leq \nu(w) \leq \nu(z),$$

i.e., (8) is not valid. Therefore (7) is valid and hence from (5), (3), (7) and (1) we obtain:

$$\mu(x) > \mu(w) \geq \mu(z) > \mu(y) \geq \mu(x),$$

which is a contradiction.

Let us assume that (5) is not valid. Therefore (6) is valid and from (2), (6) and (4) it follows that

$$\nu(y) \leq \nu(x) < \nu(w) \leq \nu(z),$$

i.e., (8) and (10) are not valid and therefore, (7) and (9) are valid and from (3), (7), (1) and (4) if follows that

$$\mu(w) \geq \mu(z) > \mu(y) \geq \mu(x) \geq \mu(z),$$

which is a contradiction.

Therefore, our assumption that no one of the four alternatives is valid, is wrong, that proves the Theorem.

REFERENCES:

[1] Fishburn P., Utility Theory for Decision Making, Jonh Wiley & Sons, New York, 1970.

[2] Atanassov K., Two variants of intuitonistc fuzzy propositional calculus. Preprint IM-MFAIS-5-88, Sofia, 1988.

[3] Atanassov K., Two variants of intuitionistic fuzzy modal logic Preprint IM-MFAIS-3-89, Sofia, 1989.

[4] Atanassov K., Gargov G., Intuitionistic fuzzy logic. Comptes Rendus de l'Academie bulgare des Sciences, Tome 43, 1990, No. 3, 9-12.

[5] Gargov G., Atanassov K., Two results in intuitionistic fuzzy logic, Comptes Rendus de l'Academie bulgare des Sciences, Tome 45, 1992, No. 12, 29-31.

[6] Mendelson E., Introduction to mathematical logic, Princeton, NJ: D. Van Nostrand, 1964.

[7] Luce R., Semiorders and theory of utility discrimination. Econometrica, Vol. 24, 1956, 178-191.

[8] Scott D., Suppes P., Fundational aspects of theories of measurement. Journal of Symbolic Logic, Vol. 23, 1958, 113-128.