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# Weak and Strong Forms of Intuitionistic Fuzzy sg-Irresolute Mappings

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**Abstract**
In this paper we consider new weak and strong forms of intuitionistic fuzzy irresolute and intuitionistic fuzzy semi-closed maps via the concept of intuitionistic fuzzy sg-closed sets, which we call intuitionistic fuzzy ap-irresolute maps, intuitionistic fuzzy ap-semi closed maps, intuitionistic fuzzy contra-irresolute maps and intuitionistic fuzzy contra sg-irresolute maps and use them to obtain a characterization of intuitionistic fuzzy semi-$T_{1/2}$ spaces.
**Keywords:** Intuitionistic fuzzy topology, Intuitionistic fuzzy semi-generalized closed set, Intuitionistic fuzzy semi-open sets, Intuitionistic fuzzy SG-continuous, Intuitionistic fuzzy contra SG-continuous intuitionistic fuzzy irresolute and intuitionistic fuzzy sg-irresolute maps.
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## 1 Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [16] in 1965. Using the concept of fuzzy sets Chang [4] introduced the concept of fuzzy topological space. In [1], Atanassov introduced the notion of intuitionistic fuzzy sets in 1986. Using the notion of intuitionistic fuzzy sets, Coker [5] defined the notion of intuitionistic fuzzy topological spaces in 1997. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of the directions is related to the properties of intuitionistic fuzzy sets introduced by Gurcay [7] in 1997. After the introduction of intuitionistic fuzzy sets, various notions in classical and fuzzy topology have been extended to intuitionistic fuzzy topological space. The concept of intuitionistic fuzzy semi-closed sets was introduced by H.Gurcay and D.Coker in [7]. Later in 2008 R.Santhi and K.Arun Prakash introduced intuitionistic fuzzy sg-closed sets in [11].In 2000 M.Caldas [2], defined and studied weak and strong forms of irresolute maps in General Topology and in 2008 R.K.Saraf, Seema Mishra and M.Caldas studied the same concept in fuzzy topology.

In this paper we extend the same concept in intuitionistic fuzzy topological spaces. We introduce the concept of irresoluteness called intuitionistic fuzzy ap-irresolute maps and intuitionistic fuzzy ap-semi closed maps by using intuitionistic fuzzy sg-closed sets and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve intuitionistic fuzzy sg-closed sets. Also, in this paper we present a new generalization of irresoluteness called intuitionistic fuzzy contra-irresolute map, we define this class of maps by the requirement that the inverse image of each intuitionistic fuzzy semi open set in the co-domain is intuitionistic fuzzy semi-closed in the domain. This notion is a stronger form of intuitionistic fuzzy ap-irresoluteness. Finally, we characterize the class of intuitionistic fuzzy semi-$T_{1/2}$ spaces in terms of intuitionistic fuzzy ap-irresolute and intuitionistic fuzzy ap-semi closed maps.

Throughout this paper $(X, \tau)$ and $(Y,\kappa)$ are intuitionistic fuzzy topological spaces and $f : (X, \tau) \to (Y,\kappa)$ is a mapping from an IFTS $(X, \tau)$ into an IFTS $(Y,\kappa)$.

# 2 Preliminaries

**Definition 2.1.[1]** An intuitionistic fuzzy set( IFS, for short ) A in X is an object having the form

$$A= \{< x, |\mu_A (x), \gamma_A (x) > | x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\gamma_A: X \to [0,1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non- membership (namely $\gamma_A(x)$ ) of each element $x \in X$ to the set A, respectively, $0 \leq \mu_A (x) + \gamma_A (x) \leq 1$ for each $x \in X$.

**Definition 2.2.[1]** Let A and B be IFS's of the forms

$A = \{< x, |\mu_A(x) , \gamma_A(x) > | x \in X\}$ and $B = \{< x , |\mu_B(x) , \gamma_B(x) > | x \in X\}$.

Then,

(a) $A \leq B$ if and only if $\mu_A (x) \leq \mu_B (x)$ and $\gamma_A (x) \geq \gamma_B (x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \leq B$ and $B \leq A$,
(c) $\bar{A} = \{< x, |\gamma_A (x), \mu_A (x) > / x \in X\}$,
(d) $A \cap B = \{< x, |\mu_A (x) \wedge| \mu_B(x), \gamma_A (x) \vee \gamma_B (x) / x \in X\}$,
(e) $A \cup B = \{< x, |\mu_A (x) \vee \mu_B (x), \gamma_A (x) \wedge \gamma_B (x) / x \in X\}$,
(f) $0_\sim = \{<x, 0, 1>, x \in X\}$ and $1_\sim = \{<x, 1, 0>, x \in X\}$,
(g) $\overline{\overline{A}} = A$ , $\overline{1_\sim} = 0_\sim$ and $\overline{0_\sim} = 1_\sim$ .

**Definition 2.3.[4]** An intuitionistic fuzzy topology ( IFT for short ) on X is a family $\tau$ of IFS's in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $\bar{A}$ of an IFOS A in IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS for short) in X.

**Definition 2.4.[1]** Let f be a mapping from a set X to a set Y. If

$$B = \{\langle y, |\mu_B (y) , \gamma_B (y)\rangle ; y \in Y \}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}|(\mu_B (x)), f^{-1}(\gamma_B (x))\rangle ; x \in X \}.$$

**Definition 2.5.[4]** Let (X, $\tau$) be an IFTS and A = < x, $\mu_A$ , $\gamma_A$> be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

int (A) = $\cup$ {G | G is an IFOS in X and G $\subseteq$ A},

cl (A) = $\cap$ {K | K is an IFCS in X and A $\subseteq$ K}.

Note that, for any IFS A in (X, $\tau$), we have

$$\mathrm{cl}(\overline{A}) = \overline{\mathrm{int}(A)} \text{ and } \mathrm{int}(\overline{A}) = \overline{\mathrm{cl}(A)}$$

**Definition 2.6.[6]** An IFS A = {<x, |$\mu_A(x)$, $\gamma_A(x)$> | x $\in$ X} in an IFTS (X, $\tau$) is called intuitionistic fuzzy semi open set (IFSOS) if A $\subseteq$ cl(int(A)).

An IFS A is called intuitionistic fuzzy semi closed set, (IFSCS), if the complement Ā is an IFSOS. The family of all intuitionistic fuzzy semi open sets of an IFTS (X,$\tau$ ) is denoted by IFSOS(X).

**Definition 2.7.[15]** Let (X, $\tau$) be an IFTS and A = < x, $\mu_A$ , $\gamma_A$> be an IFS in X. Then the intuitionistic fuzzy semi-interior and intuitionistic fuzzy semi-closure of A are defined by

sint (A) = $\vee$ {B | B is an IFSOS in X and B $\leq$ A},

scl (A) = $\wedge$ {C | C is an IFSCS in X and A $\leq$ C}.

**Definition 2.8.[12]** An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, $\tau$) is called an intuitionistic fuzzy semi-generalized closed set (intuitionistic fuzzy sg-closed, IFSGCS in short) if scl (A) $\subseteq$ U , whenever A $\subseteq$ U and U is intuitionistic fuzzy semi-open set.

The complement Ā of intuitionistic fuzzy semi-generalized closed set A is called intuitionistic fuzzy semi-generalized open set (IFSGOS).

**Definition 2.9.[12]** An intuitionistic fuzzy topological space (X,$\tau$) is said to be intuitionistic fuzzy semi-$T_{1/2}$ space, if every intuitionistic fuzzy sg-closed set in X is intuitionistic fuzzy semi-closed in X.

**Definition 2.10** Let f: X$\rightarrow$Y be a mapping from an IFTS X into an IFTS Y. The mapping f is called

(i) *intuitionistic fuzzy semi-continuous*, if $f^{-1}$(B) is an IFSOS in X for each IFOS B in Y.**[6]**

(ii) *intuitionistic fuzzy sg-continuous*, if $f^{-1}$(B) is an IFSGOS in X for each IFOS B in Y.**[12]**

(iii) *intuitionistic fuzzy irresolute*, if $f^{-1}$(B) is an IFSOS in X for each IFSOS B in Y.**[9]**

(iv) *intuitionistic fuzzy sg-irresolute*, if $f^{-1}$(B) is an IFSGOS in X for each IFSGOS B in Y.**[12]**

(v) *intuitionistic fuzzy contra sg-continuous*, if $f^{-1}$(B) is an IFSGCS in X for each IFOS in Y.**[13]**

(vi) *intuitionistic fuzzy pre-semi-close*d, if f (B) is an IFSCS in Y for every IFSCS B in X.**[9]**

# 3 Intuitionistic Fuzzy ap-irresolute, Intutitionistic fuzzy ap-semi-closed and intuitionistic fuzzy contra irresolute maps

**Definition 3.1** A mapping f: X→ Y is said to be intuitionistic fuzzy approximately irresolute (intuitionistic fuzzy ap-irresolute) if scl(A) ≤ $f^{-1}$(B), whenever B is an IFSOS of Y, A is an IFSGCS of X and A ≤ $f^{-1}$(B).

**Definition 3.2** A mapping f: X→ Y is said to be intuitionistic fuzzy approximately semi-closed (intuitionistic fuzzy ap-semi-closed) if f(B) ≤ sint(A) whenever A is an IFSGOS of Y, B is an IFSCS of X and f(B) ≤ A.

**Theorem 3.3** Every intuitionistic fuzzy irresolute mapping is an intuitionistic fuzzy ap-irresolute mapping.
**Proof:** Let f: X→Y is intuitionistic fuzzy irresolute mapping and B is an IFSOS in Y, A is an IFSGCS in X such that A ≤ $f^{-1}$(B). By our assumption $f^{-1}$(B) is an IFSOS in X. Then scl(A) ≤ $f^{-1}$(B). Hence f is intuitionistic fuzzy ap-irresolute mapping.
The converse of the above theorem is not true as seen from the following example.
**Example 3.4.** Let X= {a, b}, Y= {u, v}.
Let $A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right) \right\rangle$, $B = \left\langle y, \left(\frac{u}{0.4}, \frac{v}{0.7}\right), \left(\frac{u}{0.1}, \frac{v}{0.3}\right) \right\rangle$
Then τ = {$0_\sim$, A, $1_\sim$} and κ = {$0_\sim$, B, $1_\sim$ } are IFTS on X and Y respectively. Define a mapping f:(X, τ) → (Y, κ) by f(a) = u, f(b) = v .Clearly f is intuitionistic fuzzy ap-irresolute map. Now B is an IFSOS in Y and
$f^{-1}(B) = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.3}\right) \right\rangle$

$\mathrm{int}(f^{-1}(B)) = A$.

$\mathrm{cl}(\mathrm{int}(f^{-1}(B))) = \mathrm{cl}(A) = \overline{A}$ .

Hence $f^{-1}(B) \not\subseteq \mathrm{cl}(\mathrm{int}(f^{-1}(B)))$ which shows that $f^{-1}$(B) is not an IFSOS in X.
Hence f is not intuitionistic fuzzy irresolute mapping.

**Theorem 3.5** If f: X→Y is an intuitionistic fuzzy pre-semi-closed mapping, then f is intuitionistic fuzzy ap-semi-closed mapping.
**Proof:** Assume that f: X→Y is intuitionistic fuzzy pre-semi-closed mapping and B is an IFSCS in X, A is an IFSGOS in Y such that f(B) ≤ A. By our assumption f(B) is an IFSCS in Y. Since A is an IFSGOS in Y, by definition f(B)≤ sint(A).Hence f is intuitionistic fuzzy ap-irresolute mapping.
**Example 3.6** Let X= {a, b}, Y={u, v}.
Let $A = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right) \right\rangle$, $B = \left\langle y, \left(\frac{u}{0.6}, \frac{v}{0.4}\right), \left(\frac{u}{0.2}, \frac{v}{0.4}\right) \right\rangle$
Then τ = {$0_\sim$, A, $1_\sim$} and κ = {$0_\sim$,B, $1_\sim$ } are IFTS on X and Y respectively. Define a mapping f:(X, τ) → (Y, κ) by f(a) = u, f(b) = v . Clearly f is intuitionistic fuzzy ap-semi-closed map.
Let $M = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.5}\right) \right\rangle \in$ IFSCS(X).

$$f(M) = \left\langle y, \left(\frac{u}{0.5}, \frac{v}{0.4}\right), \left(\frac{u}{0.2}, \frac{v}{0.5}\right) \right\rangle$$

cl(f(M)) = $1_\sim$ ,

int[cl(f(M))] = int($1_\sim$) = $1_\sim \not\subseteq$ M.

Hence f (M) is not an IFSCS in Y.

Hence f is not intuitionistic fuzzy pre-semi-closed mapping.

**Theorem 3.7** A mapping f: X→Y is:

(i) intuitionistic fuzzy ap-irresolute if $f^{-1}$(A) is an IFSCS in X for every IFSOS A in Y.

(ii) intuitionistic fuzzy ap-semi-closed if f (B) is an IFSOS in Y for every IFSCS B in X.

**Proof:**

(i) Let A be an IFSOS in Y and B be an IFSGCS in X such that B $\leq f^{-1}$(A).Then scl(B) $\leq$ scl($f^{-1}$(A)).Since $f^{-1}$(A) is an IFSCS in X, scl(B) $\leq f^{-1}$(A).Thus f is intuitionistic fuzzy ap-irresolute.

(ii) Let A be an IFSGOS of Y and B be an IFSCS of X such that f(B) $\leq$ A. Then, sint(f(B)) $\leq$ sint(A).Since f(B) is an IFSOS in X, we have f(B) $\leq$ sint(A).Thus f is intuitionistic fuzzy ap-semi-closed.

The converse of the above theorem is not true in general as seen from the following examples.

**Example 3.8** Let X= {a, b}, Y= {u, v}.

Let $A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right) \right\rangle$, $B = \left\langle y, \left(\frac{u}{0.4}, \frac{v}{0.7}\right), \left(\frac{u}{0.1}, \frac{v}{0.3}\right) \right\rangle$

Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\kappa = \{0_\sim, B, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping f:(X, τ) → (Y, κ) by f(a) = u, f(b) = v . Clearly f is intuitionistic fuzzy ap-irresolute map.

Now B is an IFSOS in Y and

$$f^{-1}(B) = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.3}\right) \right\rangle$$

cl($f^{-1}$(B)) = $1_\sim$.

int[cl($f^{-1}$(B) )] = $1_\sim \not\subseteq f^{-1}$(B) .

Hence $f^{-1}$(B) is not an IFSCS in X.

**Example 3.9** Let X= {a}, Y= {b}.

Let $A = \left\langle x, \left(\frac{a}{0.2}\right), \left(\frac{a}{0.4}\right) \right\rangle$, $B = \left\langle y, \left(\frac{b}{0.6}\right), \left(\frac{b}{0.2}\right) \right\rangle$

Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\kappa = \{0_\sim, B, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping f: (X, τ) → (Y, κ) by f(a) = b. Clearly f is intuitionistic fuzzy ap-semi-closed map.

Let $Z = \left\langle x, \left(\frac{a}{0.3}\right), \left(\frac{a}{0.4}\right) \right\rangle \in$ IFSCS(X).

$$f(Z) = \left\langle x, \left(\frac{b}{0.3}\right), \left(\frac{b}{0.4}\right) \right\rangle$$

int (f (Z)) = $0_\sim$, cl[int(f (Z) )] = $0_\sim$

f (Z) $\not\subseteq$ cl[int(f (Z) )] = $0_\sim$ .

Hence f (Z) is not an IFSOS in X.

In the following theorem we impose certain conditions, which make the converse of Theorem.3.7 true.

**Theorem 3.10** Let f: X→Y is a mapping from an IFTS X into an IFTS Y.

(i) If IFSOS and IFSCS of (X,τ) coincide, then f is intuitionistic fuzzy ap-irresolute if and only if $f^{-1}(B)$ is an IFSCS of X for every IFSOS B of Y.

(ii) If IFSOS and IFSCS of (Y,κ) coincide, then f is intuitionistic fuzzy ap-semi-closed if and only if f(B) is an IFSOS of Y for every IFSCS B of X.

**Proof:**

**(i)** Assume that f is intuitionistic fuzzy ap-irresolute. Let B be any IFS of X such that $B \leq G$ where G is an IFSOS of X. Then by hypothesis $scl(B) \leq scl(G) = G$. Therefore all subsets of X are IFSGCS (and hence all are IFSGOS). So, for any IFSOS B of Y, $f^{1}(B)$ is an IFSGCS in X. Since f is intuitionistic fuzzy ap-irresolute, $scl[f^{1}(B)] \leq f^{1}(B)$. But always $f^{1}(B) \leq scl[f^{1}(B)]$. Therefore, $scl[f^{1}(B)] = f^{1}(B)$. Hence, $f^{-1}(B)$ is an IFSCS in X.
The converse part follows from the Theorem-3.7.

**(ii)** Assume that f is intuitionistic fuzzy ap-semi-closed. Let B be any IFS of X such that $G \leq f(B)$ where G is an IFSCS of Y. Then by hypothesis $G = sint(G) \leq sint(f(B))$. Therefore all sets of Y are IFSGOS. Therefore for any IFSCS B of X, f(B) is an IFSGOS in Y. Since f is intuitionistic fuzzy ap-semi-closed, $f(B) \leq sint(f(B))$. But always $sint[f(B)] \leq f(B)$. Hence, $f(B) = sint[f(B)]$. Therefore f(B) is an IFSOS in Y.
The converse follows from the Theorem.3.7.
As an immediate consequence of Theorem.3.10, we have the following corollary.

**Corollary 3.11** Let f: X →Y be a mapping from an IFTS X into an IFTS Y.

(i) If the IFSCS and IFSOS of (X,τ) coincide, then f is intuitionistic fuzzy ap irresolute if and only if f is intuitionistic fuzzy irresolute.

(ii) If the IFSCS and IFSOS of (Y,κ) coincide, then f is intuitionistic fuzzy ap-semi-closed if and only if f is intuitionistic fuzzy pre-semi-closed.

**Proof:**

**(i)** Assume that f is intutionistic fuzzy ap-irresolute. Let B be an IFSOS of Y. Then by Theorem.3.10 $f^{-1}(B)$ is an IFSCS in X. Since IFSCS and IFSOS of X coincide $f^{-1}(B)$ is an IFSOS. Hence f is intuitionistic fuzzy irresolute mapping.
The converse follows from the Theorem.3.3**.**

**(ii)** Similar to (i).

**Definition 3.12** A mapping f: X→Y is said to be intuitionistic fuzzy contra-irresolute if $f^{-1}(B)$ is an IFSCS in X for every IFSOS B in Y.

**Definition 3.13** A mapping f: X→Y is said to be intuitionistic fuzzy contra-pre-semi-closed if f (B) is an IFSOS in Y for every IFSCS B in X.

**Remark 3.14** Intuitionistic fuzzy contra-irresoluteness and fuzzy irresoluteness are independent of each other.

**Example 3.15** Let X= {u, v}.

Let M = $\left\langle x, \left(\frac{u}{0.7}, \frac{v}{0.3}\right), \left(\frac{u}{0.2}, \frac{v}{0.5}\right) \right\rangle$

Then $\tau = \{0_\sim, M, 1_\sim\}$ is an IFTS on X. Define a mapping $f:(X,\tau) \to (X,\tau)$ by $f(\gamma_A(u)) = |\mu_A(u)$, $f(\gamma_A(v)) = |\mu_A(v)$. Clearly f is intuitionistic fuzzy contra-irresolute mapping. M is an IFSOS in X and

$$f^{-1}(M) = \left\langle x, \left(\frac{u}{0.2}, \frac{v}{0.5}\right), \left(\frac{u}{0.7}, \frac{v}{0.3}\right) \right\rangle$$

$int(f^{-1}(M)) = 0_\sim$ .

$cl(int\,(f^{-1}(M)\,) = cl\,(0_\sim)$.

Clearly $f^{-1}(M)\,) \not\subseteq 0_\sim$. Therefore $f^{-1}(M)$ is not an IFSOS in X.

Hence f is not an intuitionistic fuzzy irresolute mapping.

**Example 3.16** Let X= {a, b}.

Let $A = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.1}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right) \right\rangle$

Then $\tau = \{0_\sim, A, 1_\sim\}$ is an IFTS on X. Define a mapping $f:(X,\tau) \to (X,\tau)$ by f(a) = a, f(b) = b. Clearly f is intuitionistic fuzzy irresolute mapping.

$$B = \left\langle x, \left(\frac{a}{0.7}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.5}\right) \right\rangle \in \text{IFSOS}(X)$$

$$f^{-1}(B) = \left\langle x, \left(\frac{a}{0.7}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.5}\right) \right\rangle$$

$cl(f^{-1}(B)) = 1_\sim$.

$int(cl\,(f^{-1}(B)\,) = int\,(1_\sim) = 1_\sim$.

Clearly, $int(cl\,(f^{-1}(B)\,) \not\subseteq f^{-1}(B)$.

$\therefore$ $f^{-1}(B)$ is not an IFSCS in X.

Hence, f is not intuitionistic fuzzy contra-irresolute mapping.

In the similar manner, one can prove that intuitionistic fuzzy contra-pre-semi-closed maps and intuitionistic fuzzy pre-semi-closed maps are independent notions.

**Theorem 3.17** Let f: X→Y is a mapping from IFTS X into an IFTS Y. Then the following conditions are equivalent:

(i) f is intuitionistic fuzzy contra-irresolute.

(ii) the inverse image of each IFSCS in Y is an IFSOS in X.

**Proof:**

**(i) ⇒ (ii)** Let f be intuitionistic fuzzy contra-irresolute. Let B be an IFSCS in Y. Then $f^{-1}(\overline{B})$ is an IFSCS in X for every IFSOS $\overline{B}$ in Y. Since $f^{1}(\overline{B}) = \overline{f^{-1}(B)}$, we have $f^{-1}(B)$ is an IFSOS in X.

**(ii) ⇒ (i)** Proof similar to **(i)**.

**Theorem 3.18** Every intuitionistic fuzzy contra-irresolute map is intuitionistic fuzzy ap-irresolute mapping.

**Proof:** Let f: X→Y be intuitionistic fuzzy contra-irresolute mapping and B be an IFSOS in Y. By our assumption $f^{-1}(B)$ is an IFSCS in X. By Theorem.3.7, f is intuitionistic fuzzy ap-irresolute mapping.

The converse of the above theorem is not true as seen from the following example.

**Example 3.19** Let X= {t, u}.

Let F = $\left\langle x,\left(\frac{t}{0.7},\frac{u}{0.3}\right),\left(\frac{t}{0.3},\frac{u}{0.6}\right)\right\rangle$

Then $\tau$ = {$0_{\sim}$, F,$1_{\sim}$} is an IFTS on X. Define a mapping f :(X, $\tau$) → (X, $\tau$) by f(t) = t, f(u) = |u. Clearly f is intuitionistic fuzzy ap-irresolute mapping.

Let G = $\left\langle x,\left(\frac{t}{0.9},\frac{u}{0.5}\right),\left(\frac{t}{0.1},\frac{u}{0.5}\right)\right\rangle$ ∈ IFSOS(X).

Then $f^{-1}(G)$ = $\left\langle x,\left(\frac{t}{0.9},\frac{u}{0.5}\right),\left(\frac{t}{0.1},\frac{u}{0.5}\right)\right\rangle$

cl( ($f^{-1}$(G) )= $1_{\sim}$.

int[cl( ($f^{-1}$(G) )] = int($1_{\sim}$) = $1_{\sim}$.

int[cl( ($f^{-1}$(G) )] $\not\subseteq$ $f^{-1}$(G)

Therefore ($f^{-1}$(G) is not an IFSCS in X.
Hence f is not intuitionistic fuzzy contra-irresolute mapping.

**Definition 3.20** A mapping f: X→Y is said to be intuitionistic fuzzy perfectly contra-irresolute if the inverse image of every IFSOS in Y is intuitionistic fuzzy semi clopen in X.

**Theorem 3.21** Every intuitionistic fuzzy perfectly contra irresolute mapping is an intuitionistic fuzzy contra-irresolute mapping.

**Proof**: Let f: X→Y is intuitionistic fuzzy perfectly contra-irresolute mapping and let B be an IFSOS in Y. Then by our assumption $f^{-1}$(B) is intuitionistic fuzzy semi clopen set in X. Thus $f^{-1}$(B) is an IFSCS in X. Hence f is intuitionistic fuzzy contra-irresolute mapping.
The converse of the above theorem is not true in general as seen from following example.

**Example 3.22** Let X= {u, v}.

Let M = $\left\langle x,\left(\frac{u}{0.7},\frac{v}{0.3}\right),\left(\frac{u}{0.2},\frac{v}{0.5}\right)\right\rangle$

Then $\tau$ = {$0_{\sim}$, M, $1_{\sim}$} is an IFTS on X. Define a mapping f:(X, $\tau$) → (X,$\tau$) by f($\gamma_A$(u)) = |$\mu_A$(u), f($\gamma_A$(v)) = |$\mu_A$(v). Clearly f is intuitionistic fuzzy contra-irresolute mapping.
M is an IFSOS in X and

$f^{-1}$(M) = $\left\langle x,\left(\frac{u}{0.2},\frac{v}{0.5}\right),\left(\frac{u}{0.7},\frac{v}{0.3}\right)\right\rangle$

int ($f^{-1}$(M)) = $0_{\sim}$ .

cl(int ($f^{-1}$(M) ) = cl ($0_{\sim}$).

Clearly, $f^{-1}$(M))$\not\subseteq$ $0_{\sim}$.

∴ $f^{-1}$(M) is not an IFSOS in X .

Hence f is not an intuitionistic fuzzy perfectly contra-irresolute mapping.

**Theorem 3.23** Every intuitionistic fuzzy perfectly contra irresolute mapping is intuitionistic fuzzy irresolute mapping.

**Proof:** Obvious.

The reverse implication of the above theorem is not true as seen from following example.

**Example 3.24** Let X= {a, b}.

Let A = $\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.1}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right)\right\rangle$

Then $\tau = \{0_\sim, A, 1_\sim\}$ is an IFTS on X. Define a mapping f:(X, τ) → (X,τ) by f(a) = a, f(|b) = b. Clearly f is intuitionistic fuzzy irresolute mapping.

B = $\left\langle x, \left(\frac{a}{0.7}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.5}\right)\right\rangle \in$ IFSOS(X)

$f^{-1}(B) = \left\langle x, \left(\frac{a}{0.7}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.5}\right)\right\rangle$

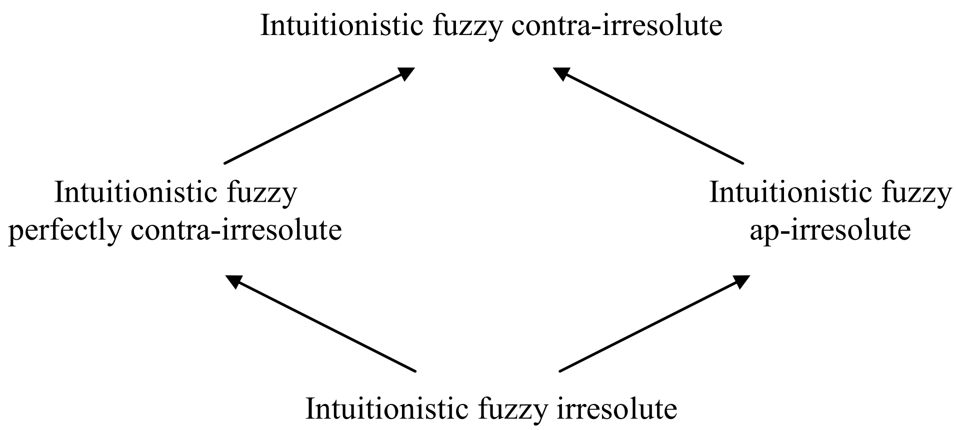
$cl(f^{-1}(B)) = 1_\sim$.

$int(cl\,(f^{-1}(B)\,) = int\,(1_\sim) = 1_\sim$.Clearly $int(cl\,(f^{-1}(B)\,) \not\subseteq f^{-1}(B)$.

$\therefore$ $f^{-1}(B)$ is not an IFSCS in X.

Hence f is not intuitionistic fuzzy perfectly contra-irresolute mapping.

Thus we have the following diagram.




**Theorem 3.25** Let f: X→Y be a mapping from IFTS (X, τ) into an I. Then the following conditions are equivalent:

(i) f is intuitionistic fuzzy perfectly contra-irresolute;
(ii) f is intuitionistic fuzzy contra-irresolute and intuitionistic fuzzy irresolute.

**Proof: (i) ⇒ (ii):** Let f be intuitionistic fuzzy perfectly contra-irresolute mapping. Let B be an IFSOS in Y. By our assumption $f^{-1}(B)$ is intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is both IFSOS and IFSCS in X. Hence f is intuitionistic fuzzy contra-irresolute and intuitionistic fuzzy irresolute.

**(ii)⇒ (i):** Let f be both intuitionistic fuzzy contra-irresolute and intuitionistic fuzzy irresolute and let B be an IFSOS in Y. By our assumption $f^{-1}(B)$ is both IFSOS and IFSCS in X. (i.e) $f^{-1}(B)$ is intuitionistic fuzzy semi clopen set in X. Hence f is intuitionistic fuzzy perfectly contra-irresolute.

The next two theorems establish conditions under which maps and inverse maps preserve intuitionistic fuzzy sg-closed sets.

**Theorem 3.26** If a map f: X→Y is intuitionistic fuzzy irresolute and intuitionistic fuzzy ap-semi-closed, then $f^{-1}(A)$ is intuitionistic fuzzy sg-closed whenever A is intuitionistic fuzzy sg-closed set in Y.

**Proof:** Let A be an IFSGCS in Y. Suppose that $f^{-1}(A) \leq B$, where B is an IFSOS of X. Taking complements we obtain $\overline{B} \leq f^{1}(\overline{A})$ or $f(\overline{B}) \leq \overline{A}$. Since f is intuitionistic fuzzy ap-semi-closed $f(\overline{B}) \leq \mathrm{sint}(\overline{A}) = \overline{\mathrm{scl}A}$. It follows that $\overline{B} \leq f^{-1}(\overline{\mathrm{scl}(A)}) = \overline{f^{-1}\mathrm{scl}(A)}$ and hence $f^{1}(\mathrm{scl}A) \leq B$. Since f is intuitionistic fuzzy irresolute $f^{1}(\mathrm{scl}(A))$ is an IFSCS.

Thus we have $\mathrm{scl}(f^{-1}(A)) \leq \mathrm{scl}(f^{-1}(\mathrm{scl}(A))) = f^{-1}(\mathrm{scl}(A)) \leq B$.

This implies $f^{-1}(A)$ is an IFSGCS in X.

A similar argument shows that inverse images of intuitionistic fuzzy sg-open sets are intuitionistic fuzzy sg-open sets.

**Theorem 3.27** If a map f: X→Y is intuitionistic fuzzy ap-irresolute and intuitionistic fuzzy pre-semi-closed, then for every IFSGCS B of X, f(B) is an IFSGCS in Y.

**Proof:** Let B be an IFSGCS in X and f(B) ≤ G, where G is an IFSOS in Y. Then, $B \leq f^{-1}(G)$ holds. Since f is intuitionistic fuzzy ap-irresolute, scl (B) $\leq f^{-1}(G)$ and hence f(sclB) ≤ G. Since f is intuitionistic fuzzy pre-semi-closed f(sclB) is an IFSCS in Y. Therefore we have scl (f(B)) ≤ scl(f(sclB)) = f(sclB) ≤ G. Hence f(B) is an IFSGCS in Y.

**Theorem 3.28** Let f: X→Y and g: Y→Z be two mappings. Then,

(i) g∘f is intuitionistic fuzzy ap-irresolute, if f is intuitionistic fuzzy ap-irresolute and g is intuitionistic fuzzy irresolute.

(ii) g∘f is intuitionistic fuzzy ap-semi-closed, if f is intuitionistic fuzzy pre-semi-closed and g is intuitionistic fuzzy ap-semi-closed.

(iii)g∘f is intuitionistic fuzzy ap-semi-closed, if f is intuitionistic fuzzy ap-semi-closed and g is intuitionistic fuzzy pre-semi-open and $g^{-1}$ preserves intuitionistic fuzzy sg-open sets.

**Proof:**

(i) Assume that A is an IFSGCS of X and B be an IFSOS in Z for which $A \leq (g\circ f)^{-1}(B)$. Since g is intuitionistic fuzzy irresolute $g^{-1}(B)$ is an IFSOS in Y. Since f is intuitionistic fuzzy ap-irresolute $\mathrm{scl}(A) \leq f^{-1}[g^{-1}(B)] = (g\circ f)^{-1}(B)$. This proves that g∘f is intuitionistic fuzzy ap-irresolute mapping.

(ii) Suppose that B is as IFSCS in X and A is an IFSGOS in Z for which (g∘f)(B) ≤ A. Then f(B) is an IFSCS in Y, as f is intuitionistic fuzzy pre-semi-closed. Since g is intuitionistic fuzzy ap-semi-closed, g(f(B)) = (g∘f)(B) ≤ sint(A). This implies that g∘f is intuitionistic fuzzy ap-semi-closed.

(iii)Assume that B is an IFSCS in X and A is an IFSGoS in Z for which (g∘f)(B) ≤ A. Hence $f(B) \leq g^{-1}(A)$. Since A is IFSGOS in Z and $g^{-1}$ preserves intuitionistic fuzzy sg-open sets $g^{-1}(A)$ is IFSGOS in Y, which implies $f(B) \leq \mathrm{sint}[g^{-1}(A)]$. Thus

$$(g\circ f)(B) = g(f(B)) \leq g(\mathrm{sint}(g^{-1}(A))) \leq \mathrm{sint}(gg^{-1}(A)) \leq \mathrm{sint}\, A$$

This implies that g∘f is intuitionistic fuzzy ap-semi-closed.

**Theorem 3.29** Let f : X→Y be a mapping from an IFTS (X, τ) into an IFTS (Y,κ), and g: Y→Z be a mapping from an IFTS (Y,κ) into an IFTS (Z,δ) respectively. Then

(i) g∘f intuitionistic fuzzy perfectly contra-irresolute if f and g are intuitionistic fuzzy perfectly contra-irresolute.

(ii) g∘f intuitionistic fuzzy contra-irresolute if f is intuitionistic fuzzy perfectly contra-irresolute and g is intuitionistic fuzzy contra-irresolute.

(iii) $g \circ f$ intuitionistic fuzzy irresolute if f is intuitionistic fuzzy perfectly contra-irresolute and g is intuitionistic fuzzy contra-irresolute.
(iv) $g \circ f$ is intuitionistic fuzzy irresolute if f is intuitionistic fuzzy perfectly contra-irresolute and g is intuitionistic fuzzy irresolute.
(v) $g \circ f$ is intuitionistic fuzzy contra-irresolute if f is intuitionistic fuzzy perfectly contra-irresolute and g is intuitionistic fuzzy irresolute.
(vi) $g \circ f$ is intuitionistic fuzzy irresolute if f is intuitionistic fuzzy contra-irresolute and g is intuitionistic fuzzy contra-irresolute.
(vii) $g \circ f$ is intuitionistic fuzzy contra-irresolute if f is intuitionistic fuzzy contra-irresolute and g is intuitionistic fuzzy irresolute.

**Proof:** Follows from definitions.

**Theorem 3.30** If f: X→Y is a intuitionistic fuzzy contra-irresolute mapping from an IFTS $(X, \tau)$ into an IFTS $(Y,\kappa)$, then f is intuitionistic fuzzy contra semi-continuous mapping.

**Proof:** Let G be an IFOS in Y. Then G is an IFSOS in Y. Since f is intuitionistic fuzzy contra-irresolute, it follows that $f^{-1}(G)$ is an IFSCS in X. Hence f is intuitionistic fuzzy contra semi-continuous mapping.

The converse of Theorem3.30 is not true in general as seen from the following example.

**Example 3.31** Let X= {a, b, c}, Y= {u, v, w}.

Let A = $\left\langle x, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle$ , B = $\left\langle y, \left(\frac{u}{1}, \frac{v}{0.4}, \frac{w}{0.4}\right), \left(\frac{u}{0}, \frac{v}{0.6}, \frac{w}{0.6}\right)\right\rangle$

Define a mapping $f:(X, \tau) \to (Y, \kappa)$ by f(a) = u, f(b) = v , f(c) = w. Then $\tau = \{0_\sim, A, 1_\sim\}$ and $\kappa = \{0_\sim, B, 1_\sim\}$ are IFTS on X and Y respectively

$$f^{-1}(B) = \left\langle x, \left(\frac{a}{1}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle$$

$$Cl\,[f^{-1}(B)] = 1_\sim \cap \overline{A} = \overline{A}\ ,\ int(\overline{A}) = A \subseteq f^{-1}(B)$$

Therefore f is intuitionistic fuzzy contra-semi continuous mapping.

Let C = $\left\langle y, \left(\frac{u}{1}, \frac{v}{0.7}, \frac{w}{0.7}\right), \left(\frac{u}{0}, \frac{v}{0.2}, \frac{w}{0.3}\right)\right\rangle$ be on IFSOS in Y.

$f^{-1}(C) = \left\langle x, \left(\frac{a}{1}, \frac{b}{0.7}, \frac{c}{0.7}\right), \left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0.3}\right)\right\rangle$

$cl(f^{-1}(C)) = 1_\sim.\ int(1_\sim) = 1_\sim \not\subset f^{-1}(C)$

$\therefore$ $f^{-1}(C)$ is not in IFSCS in X.

Hence f is not an intuitionistic fuzzy contra-irresolute mapping.

**Definition 3.32** A mapping f: X→Y from an IFTS X into an IFTS Y is said to be intuitionistic fuzzy contra sg-irresolute if $f^{-1}(B)$ is an IFSGOS in X for every IFSGCS B in Y.

**Theorem 3.33** Every intuitionistic fuzzy contra sg-irresolute mapping is an intuitionistic fuzzy sg-continuous mapping.

**Proof:** Let f : X→Y be an intuitionistic fuzzy contra sg-irresolute mapping. Let B be an IFCS in Y. Then B is an IFSGCS in X. Hence f is intuitionistic fuzzy contra sg-continuous mapping.

The converse of Theorem.3.22 is not true in general as seen from the following example.

**Example 3.34** Let X= {a, b, c} , Y={u, v, w}.

Let A = $\left\langle x, \left(\frac{a}{0}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle$, B = $\left\langle y, \left(\frac{u}{1}, \frac{v}{0.4}, \frac{w}{0.4}\right), \left(\frac{u}{0}, \frac{v}{0.6}, \frac{w}{0.6}\right)\right\rangle$

Then $\tau$ = {$0_\sim$, A, $1_\sim$} and $\kappa$ = {$0_\sim$, B, $1_\sim$ } are IFTS on X and Y respectively. Define a mapping f : (X, $\tau$) → (Y, $\kappa$) by f(a) = u, f(b) = v , f(c) = w. Clearly f is intuitionistic fuzzy contra sg-continuous map.

Let C = $\left\langle y, \left(\frac{u}{0}, \frac{v}{0.4}, \frac{w}{0.3}\right), \left(\frac{u}{1}, \frac{v}{0.6}, \frac{w}{0.7}\right)\right\rangle$ be an IFSGCS in Y.

$f^{-1}(C) = \left\langle x, \left(\frac{a}{1}, \frac{b}{0.4}, \frac{c}{0.3}\right), \left(\frac{a}{1}, \frac{b}{0.6}, \frac{c}{0.7}\right)\right\rangle$

sint[$f^{-1}$(C)] = $0_\sim$.

Let G = $\left\langle x, \left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{1}, \frac{b}{0.8}, \frac{c}{0.7}\right)\right\rangle$ is an IFSCS in X and G $\subseteq$ $f^{-1}$(C), but

G $\not\subseteq$ sint[$f^{-1}$(C)]

$f^{-1}$(C) is not an IFSGOS in X.

Hence f is not an intuitionistic fuzzy contra sg-irresolute mapping.

**Theorem 3.35** Let f: X→Y be a mapping from IFTS X into an IFTS Y. Then the following statements are equivalent:

(i) f is intuitionistic fuzzy contra sg-irresolute.

(ii) $f^{-1}$(B) is an IFSGCS in X for every IFSGOS in Y.

**Proof:** Obvious.

**Theorem 3.36** Let f: X→Y be a function and let g: X→X×Y be the graph function of f, defined by g(x) = (x, f(x)) for every x∈X. If g is intuitionistic fuzzy contra sg-irresolute, then f is intuitionistic fuzzy contra sg-irresolute.

**Proof:** Let B be an IFSGCS in Y, then $f^{-1}(B) = f^{-1}(1_\sim \times B) = 1_\sim \cap f^{-1}(B) = g^{-1}(1_\sim \times B)$. Since $1_\sim$×B is an IFSGCS and g is contra sg-irresolute $g^{-1}(1_\sim \times B)$ is an IFSGOS in X. Hence $f^{-1}$(B) is an IFSGOS in X. Hence f is intuitionistic contra sg-irresolute.

**Theorem 3.37** If f: X→Y is intuitionistic fuzzy contra sg-irresolute mapping and (X,$\tau$) is intuitionistic fuzzy semi-$T_{1/2}$ space, then f is intuitionistic fuzzy contra irresolute mapping.

**Proof:** Let B be an IFSCS in Y. Then B is an IFSGCS in Y. Since f is intuitionistic fuzzy contra sg-irresolute mapping, $f^{-1}$(B) is an IFSGOS in X. Since (X,$\tau$) is intuitionistic fuzzy semi-$T_{1/2}$ space, $f^{-1}$(B) is an IFSOS in X. Thus f is intuitionistic fuzzy irresolute mapping.

**Theorem 3.38** Let (X,$\tau$) be an IFTS, then the following statements are equivalent:

(i) (X,$\tau$) is intuitionistic fuzzy semi-$T_{1/2}$ space.

(ii) For every IFTS (Y,$\kappa$) and every mapping f: X→Y, f is intuitionistic fuzzy ap-irresolute mapping.

**Proof:**
**(i) ⇒ (ii)** Let B be an IFSGCS in X and suppose that $B \le f^{-1}(A)$ where A is an IFSOS in Y. Since $(X,\tau)$ is intuitionistic fuzzy semi-$T_{1/2}$ space, B is an IFSCS implies $scl(B) = B$. Therefore $scl(B) \le f^{-1}(A)$. Then f is intuitionistic fuzzy ap-irresolute mapping.
**(ii) ⇒ (i)** Let B be an IFSGCS in X and let Y be the set X with the topology

$$\kappa = \{0_{\sim}, B, 1_{\sim}\}.$$

Let f: $(X, \tau) \to (Y, \kappa)$ be the identity mapping. By our assumption f is intuitionistic fuzzy ap-irresolute mapping. Since B is an IFSGCS in X and IFSOS in Y and $B \le f^{-1}(B)$ implies $scl(B) \le f^{-1}(B)$. Hence B is an IFSCS in X and therefore $(X,\tau)$ is intuitionistic fuzzy semi-$T_{1/2}$ space.

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