Notes on Intuitionistic Fuzzy Sets
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Effects of fuzzy and advanced fuzzy sets in mammogram image clustering

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Abstract: Breast cancer is one of the leading causes of death in the world among women. Early detection of breast masses may increase the survival rate and so automated techniques are suggested by researchers across the globe. Mammogram images are vague and many of the patterns are not visible clearly, thereby making it difficult to distinguish them using traditional crisp classification methods. So, segmentation of mammogram images remains a complex task and is necessary before moving for classification. Fuzzy/advanced fuzzy methods consider uncertainty in an image and so these sets can deal with uncertainty in medical images in an efficient manner. This paper presents the effect of using fuzzy, intuitionistic fuzzy set, Interval-type2 fuzzy set and intuitionistic fuzzy set of second type on segmenting lesions in mammogram images. Quantitative and qualitative analysis reveals that segmentation error is more on using fuzzy sets where only one uncertainty, which is membership function, is considered. Experimental results show that though intuitionistic fuzzy set, Interval-type2 fuzzy set performs well, intuitionistic fuzzy set of second type performs better than the other sets.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy set of second type, Interval valued type-2 fuzzy set, Intuitionistic fuzzy entropy, Clustering.
2020 Mathematics Subject Classification: 03E72, 03B20, 68U10, 62H30.

1 Introduction

Breast cancer is the second most common cause of mortality for women diagnosed with malignancy, and it accounts for around 10–12% of female cancer. As mammogram images are good at detecting breast masses, analyzing mammographic images can help discover breast cancer early. A crucial step in the diagnosis of breast cancer is the segmentation of breast mass. After segmentation, regions of interest are selected, and classification is done. Diagnosis becomes challenging when the size of the mass is not uniform. Clustering is the step where different regions are segmented. It partitions the image into several regions. Fuzzy C-means clustering (FCM) by Bezdek *et al.* [6] is the first and widely used clustering algorithm. Later, many researchers proposed modified FCM algorithm that uses kernel functions like Gaussian, radial basis, hyper tangent kernels [12, 14]. However, accuracy remains still a challenging task, particularly for mammographic images, which are not crisp and have uneven illumination. Researchers attempted to consider ambiguity and vagueness in these mammogram images as these images lack distinct borders or regions that are difficult to distinguish from one another. Therefore, there is some degree of ambiguity in the regions. Since then, research has been carried out for better segmentation and detection. Fuzzy sets, intuitionistic fuzzy sets, and intuitionistic fuzzy set of second type (IFSST) are applied for processing of mammogram images. As these sets consider vagueness and medical images have unclear boundaries, these sets are better suited for processing. If the segmentation is done properly, region selection will be accurate, thereby classification accuracy will be better.

Fuzzy set considers only the membership function, i.e., the degree of belongingness that considers only one type of uncertainty. But accuracy is still a problem. Atanassov [5] introduced the concept of intuitionistic fuzzy set where two degrees – membership degree and non-membership degree are considered.

Chaira [7, 8] suggested intuitionistic fuzzy C-means clustering algorithm for the first time and worked on medical images. Later, many researchers modified intuitionistic fuzzy clustering algorithms and obtained better accuracy. Verma *et al.* [20] suggested a modified intuitionistic fuzzy clustering algorithm where they considered hesitation degree, while Aruna Kumar and Harish [1] used Hausdorff distance measure in place of Euclidean distance and an intuitionistic fuzzy generator to compute the non-membership degree, but didn’t consider hesitation degree.

There is another form of fuzzy set called type-2 fuzzy set. It consists of primary and secondary membership functions and secondary membership functions are the degree to which the primary membership function is considered to be fuzzy. Thus, it considers two levels of membership function – upper and lower membership. Cherif *et al.* [11] suggested few intuitionistic based interval-type 2 fuzzy similarity measure and applied those measures on clustering. Few researchers used intuitionistic fuzzy set of second type (IFSST), suggested by Atanassov [3–5], which is another concept of IFS that allows users greater flexibility in expressing their level of membership and non-membership degree as compared to IFS in image processing. In 2006, Parvathi in her dissertation thesis [18] applied IFSST operators in image processing problems like enhancement and segmentation. Premalatha and Dhanalakshmi [17] suggested a Pythagorean fuzzy method, where they enhanced lung images of COVID-affected patients and segmented. Notably, the Pythagorean fuzzy set (PFS) is exactly the intuitionistic fuzzy set of second type (IFSST), which had already been introduced in 1993 by Atanassov [5]. Chaira and Sarkar [10] suggested an IFSST method for the enhancement of mammogram images.

There are very few works on mammogram image processing. In this work, as mammogram image segmentation is our main objective, the effect of clustering abnormal regions/lesions in mammogram images using intuitionistic fuzzy set, Interval-type 2 fuzzy set, IFSST is explored. The authors have implemented the existing clustering algorithms using the above-mentioned fuzzy sets on mammogram images to see the segmentation accuracy using diverse types of fuzzy/advanced fuzzy sets where uncertainty/vagueness are accounted in a different manner.

The paper is structured as follows. Section 2 highlights the preliminaries of fuzzy set, intuitionistic fuzzy sets, Interval-type 2 fuzzy set, and IFSST. Section 3 details the clustering method using all the fuzzy sets. Section 4 discussed the results and a thorough discussion. Section 5 concludes the paper.

2 Preliminaries

Fuzzy set theory was suggested by Zadeh [21], where each element z in a set A is assigned a membership degree, and the non-membership degree is a complement of the membership degree, i.e., $\mu_A(z)+\nu_A(z)=1$.

Atanassov's intuitionistic fuzzy set (IFS) theory is an enhancement of classical fuzzy set theory. It introduces two key independent parameters: the membership degree $\alpha_A(z),$ and the non-membership degree $\beta_A(z)$, traditionally denoted by $\mu_A(z), \nu_A(z),$ respectively. Unlike in standard fuzzy sets, where non-membership degree is 1 minus the membership degree, IFS separates the two, considering the hesitation involved when defining the degree of belongingness to a set. Because of this, the non-membership degree may not be the exact complement of the membership degree, but rather a value that satisfies the condition: $\alpha_A(z)+\beta_A(z)\le 1$. This 'less than or equal to' is due to the degree of indeterminacy or hesitation, thereby allowing for a more flexible and realistic modeling of uncertain information.

An intuitionistic fuzzy set A may be represented mathematically as:

$$A=\left\{(z,\alpha_A(z),\beta_A(z),\gamma_A(z)\;\middle|\; z\in\mathcal{I}\right\}$$

where $\alpha_A(z),$ is a membership degree, $\beta_A(z)$ is a non-membership degree, and $\gamma_A(z)$ is the hesitation degree. These three degrees follow the condition: $\alpha_A(z)+\beta_A(z)+\gamma_A(z)=1$.

Type-2 fuzzy set is called a "fuzzy fuzzy set". It considers the membership function in an ordinary fuzzy set to be fuzzy, thereby accounting for a better handling of uncertainty than an ordinary fuzzy set. It is described by primary and secondary membership grades. It considers two membership levels on the ordinary fuzzy set – an upper and a lower membership level. An interval type-2 fuzzy set is represented as:

$$\tilde{A}=\{z,\alpha_U(z),\alpha_L(z)\mid z\in\mathcal{I}\}$$

where $\alpha_U(x),\ \alpha_L(x)$ are the upper and lower membership functions with the following condition:

$$\alpha_U(z)\le\alpha(z)\le\alpha_L(z)\in\left[0,1\right].$$

The upper and lower membership functions are written as in [15]:

$$\alpha^{lower}=\left[\alpha(z)\right]^{\frac{1}{k}},\quad \alpha^{upper}=\left[\alpha(z)\right]^{k}\text{, where } k\in[0,1].$$

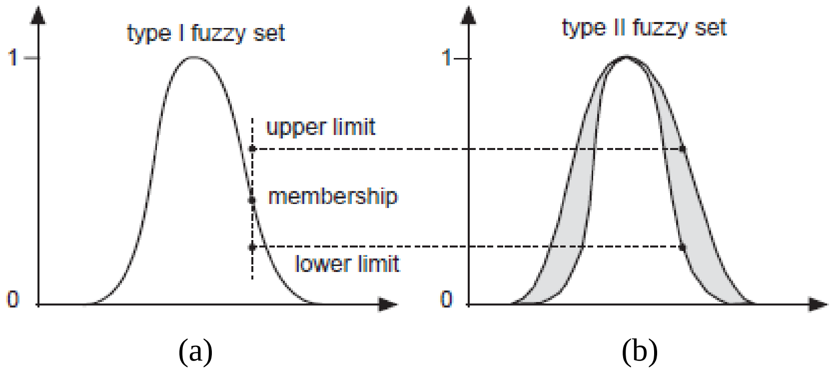

Figure 1. (a) Type-1 fuzzy membership function; (b) Type-2 fuzzy membership function.

The intuitionistic fuzzy set of second type (IFSST) is another concept of IFS, introduced by Atanassov to address certain limitations of the IFS. Rangasamy and Palaniappan [19] introduced few operations: concentration, dilation on IFSST. Ejegwa and Adamu [13] suggested distance measures: Hamming distance, Euclidean distance on IFSST, and applied to diagnostic medicine. It offers a more flexible and accurate framework for representing uncertainty. Unlike IFS, where the sum of the membership, $\alpha_A(z)$, and non-membership, $\beta_A(z)$, degrees must not be greater than 1, IFSST allows cases where $\alpha_A(z)+\beta_A(z)\geq 1$. This relaxed constraint enables a broader and more expressive representation of uncertain information, giving users more freedom in assigning degrees of membership and non-membership thereby satisfying the conditions:

$$\alpha_A(z)+\beta_A(z)\leq 1 \text{ or } \alpha_A(z)+\beta_A(z)\geq 1$$
$$\text{and } 0\leq \alpha_A(z)^2+\beta_A(z)^2\leq 1.$$

Figure 2 shows the geometrical interpretation of IFSST, and the idea is adapted from [2], where Atanassov had given the geometrical interpretation of intuitionistic fuzzy set, intuitionistic fuzzy propositional calculus, and modal logic.

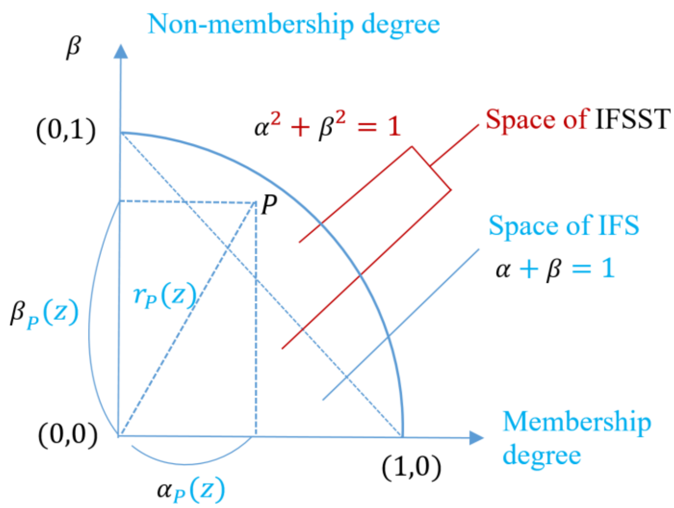

Figure 2. Geometrical interpretation of IFSST

From Figure 2, it is seen that the admissible region of an intuitionistic fuzzy number of second type is larger than that of each intuitionistic fuzzy number. Therefore, IFSST provides a broader framework for representing uncertainty in medical images.

In the presented approach, we compare the performance of fuzzy sets, intuitionistic fuzzy sets, interval type-2 fuzzy sets, and IFSSTs in the analysis of mammogram images.

i.1 Description of different methods

i) Fuzzy c means clustering (FCM) [6] – This is the first fuzzy clustering method proposed by Bezdek. Unlike hard clustering methods like K-Means, where each data point belongs exclusively to one cluster, FCM allows a data point to belong to multiple clusters with different membership degrees. The method is explained in other methods mentioned below, where the proceedure given different authors using other advanced fuzzy methods are mentioned to improve FCM.

ii) Intuitionistic fuzzy clustering method (IFCM-1) [20]: The authors worked on CT brain image segmentation where they proposed a modified intuitionistic fuzzy C-means (mIFCM) algorithm incorporating hesitation degree, which considers the hesitation in the computation of centroids. It assigns a membership value to each data point, which is based on the similarity between the data point and the centroid of a specific cluster. Hesitation degree is incorporated in the computation of cluster centroids, addressing the uncertainty inherent in assigning data points to clusters. Membership degree depends on the distance in clustering. Sugeno-type intuitionistic fuzzy generator was used to compute the non- membership function for finding the hesitation degree. In their experiment, λ in the Sugeno type intuitionistic fuzzy generator was tried using different values, but at $\lambda = 1.5$ the results were better.

Just like fuzzy C-means clustering (FCM), intuitionistic fuzzy C-means clustering is based on minimizing the objective function and is defined as

$$J_m(U, v; Z) = \sum_{i=1}^{c} \sum_{k=1}^{n} (u_{ik})^m d^2(z_k, v_i).$$

Here, the term $u_{ik} \subset U$ $(c \times n)$ denotes the membership value of data z_k to the *i*-th cluster, where $u_{ik} \in [0, 1]$ and satisfies the condition $\sum_{i=1}^{c} u_{ik} = 1 \quad (1 < k < 1)$.

As FCM does not consider hesitation, the centroid of clusters is inappropriate, so due to the introduction of hesitation degree, the new membership function is updated as: $u'_{ik} = u_{ik} + \gamma_{ik}$. With this new membership degree, u'_{ik}, intuitionistic fuzzy cluster centroids, $V'_i = (\alpha'(v_i), \beta'(v_i), \gamma'(v_i))$, are computed as follows:

$$\alpha'(v_i) = \frac{\sum_{k=1}^{n} (u'_{ik})^m \,.\, \alpha(z_k)}{\sum_{k=1}^{n} (u'_{ik})^m}, \quad \beta'(v_i) = \frac{\sum_{k=1}^{n} (u'_{ik})^m \,.\, \beta(z_k)}{\sum_{k=1}^{n} (u'_{ik})^m}, \quad \gamma'(v_i) = \frac{\sum_{k=1}^{n} (u'_{ik})^m \,.\, \gamma(z_k)}{\sum_{k=1}^{n} (u'_{ik})^m},$$

for any $i = 1, 2, 3, \ldots, c$ and $k = 1, 2, 3, \ldots, n$.

The membership function, u_{ik}, is written as:

$$u_{ik} = \frac{1}{\sum_{j=1}^{c}\left[\dfrac{d^2\left(z_k, V_i'\right)}{d^2\left(z_k, V_j'\right)}\right]^{\frac{1}{m-1}}},$$

where $d^2\left(z_k, V_i'\right) = \left(\alpha\left(z_k\right) - \alpha'(v_i)\right)^2 + \left(\beta\left(z_k\right) - \beta'(v_i)\right)^2 + \left(\gamma\left(z_k\right) - \gamma'(v_i)\right)^2$.

The membership values are updated at each iteration and are continued until

$$\max_{i,k}\left|U_{ik}'^{new} - U_{ik}'^{prev}\right| < \varepsilon,$$

where $\varepsilon = 0.01$, which is user defined.

iii) Intuitionistic fuzzy clustering method (IFCM-2) [9]: The work is based on clustering of abnormal regions in mammogram images using intuitionistic fuzzy set.

The image is fuzzified with a membership function as:

$$\alpha_{mn} = \frac{g_{mn} - g_{min}}{g_{max} - g_{min}},$$

where α_{mn} is the membership degree at the (m, n)-th point of the image.

Using the novel intuitionistic fuzzy generator from [9], the membership function of the intuitionistic fuzzy image is given as:

$$\alpha_{mn}' = \frac{\left[1 + \left(\delta - 1\right)^2\right].\alpha_{mn}}{1 + .\alpha_{mn}\left(\delta - 1\right)^2},$$

where $\delta > 1$ and is a constant term.

The non-membership function of the intuitionistic fuzzy image is given as:

$$\beta_{mn}' = \frac{1 - \alpha_{mn}}{1 + 2.\alpha_{mn}.\left(\delta - 1\right)^2 + \alpha_{mn}.\left(\delta - 1\right)^4},$$

where δ is computed as: $\delta_{opt} = \max_{\delta} IFE\left(A, \delta\right)$, where

$$IFE\left(A, \delta\right) = \frac{1}{N}\sum_{n=1}^{N}\sum_{m=1}^{M}\gamma_{mn}e^{1-\gamma_{mn}},$$

and γ_{mn} is the hesitation degree at the (m, n)-th point.

After obtaining the value δ, an intuitionistic fuzzy image is formed and then converted to an interval type-2 fuzzy image. The two levels – the upper and the lower level of membership, are computed as:

$$\mu_{lower} = \left[abs\left(\alpha_{mn}' - \gamma_{mn}\right)\right]^r,$$

$$\mu_{upper} = \left(\alpha_{mn}' + \gamma_{mn}\right)^r,$$

where $\gamma_{mn} = 1 - \alpha_{mn}' - \beta_{mn}'$ and r is a fuzzy hedge.

Using Zadeh's fuzzy t-conorm, the interval type-2 fuzzy membership function is computed as:

$$T\left(\alpha_{lower},\alpha_{upper}\right)=\min\left(\alpha_{lower},\alpha_{upper}\right).$$

Intuitionistic fuzzy clustering was applied, where four fuzzy features – pixel energy, mean, standard deviation, and edge were used. Intuitionistic fuzzy C-means algorithm clusters the feature vectors by searching for local minima of the following objective function [7]:

$$J=\sum_{i=1}^{c}\sum_{k=1}^{n}u_{ik}'^{m}d\left(z_k,v_i\right)^2+\sum_{i=1}^{c}\gamma_i e^{1-\gamma_i},$$

for $k\in[1,n]$ and $m=2$, where $d\left(z_k,v_i\right)$ is any distance measure between the cluster center v_i and data point z_k in the pattern, α_{ik} is the membership value of the *k*-th data (z_k) in the *i*-th cluster, *c* is the number of clusters and *n* is the number of data points.

The membership matrix is given as:

$$u_{ik}=\frac{1}{\sum_{j=1}^{c}\left[\frac{d_{ik}^2}{d_{jk}^2}\right]^{\frac{1}{m-1}}}.$$

The membership function u_{ik} is updated at each iteration on incorporating the hesitation degree, which is given as: $u_{ik}'=u_{ik}+\gamma_{ik}$, where γ_{ik} is the hesitation degree of the *k*-th element in cluster *i*.

Likewise, the cluster center is also updated in each iteration as:

$$v_i'=\frac{\sum_{k=1}^{n}u_{ik}'.z_k}{\sum_{k=1}^{n}u_{ik}'}$$

for any $i=1,2,3,\ldots,c$ and $k=1,2,3,\ldots,n$.

iv) **Interval Type-2 fuzzy clustering** [11], where the authors proposed three Type-2 fuzzy similarity measures. In [16], the author mentioned that IFSs are almost like interval valued fuzzy sets, and this can be considered as special cases of Type-2 fuzzy sets where both hesitance and uncertainty are handled. They used different types of image databases like wine, linguistics, and a mammography database.

$$S1(A,B)=1-\frac{s^u+s^l}{2},$$

where

$$s^u=\frac{1}{2n}\sum_{i=1}^{n}\frac{\left|\alpha_A{}^u\left(z_i\right)-\alpha_B{}^u\left(z_i\right)\right|}{\alpha_A{}^u\left(z_i\right)+\alpha_B{}^u\left(z_i\right)}+\frac{\left|\alpha_A{}^u\left(z_i\right)-\alpha_B{}^u\left(z_i\right)\right|}{2-\alpha_A{}^u\left(z_i\right)-\alpha_B{}^u\left(z_i\right)},$$

$$s^l=\frac{1}{2n}\sum_{i=1}^{n}\frac{\left|\alpha_A{}^l\left(z_i\right)-\alpha_B{}^l\left(z_i\right)\right|}{\alpha_A{}^l\left(z_i\right)+\mu_B{}^l\left(z_i\right)}+\frac{\left|\alpha_A{}^l\left(z_i\right)-\alpha_B{}^l\left(z_i\right)\right|}{2-\alpha_A{}^l\left(z_i\right)-\alpha_B{}^l\left(z_i\right)}$$

The conversion from intuitionistic fuzzy set to Type 2 fuzzy set is given as follows [16]:

$$A_{IFS}=\left\{z,\alpha_A(z),\beta_A(z)\right\}.$$

Then the mapping from IFS to Type-2 fuzzy set is given as:

$$A_{IFS} = \left[1/(\alpha_A(z) + p\gamma_A(z)) + 0/(1 - \beta_A(z) + p\gamma_A(z))\right]/z,$$

where $p \in [0,1]$. Here, '1' in secondary grades means the certainty on membership and '0' in secondary grade means uncertainty in non-membership degrees. Said in another way, the true and false membership functions are represented by the secondary grade. With the new distance measures, clustering is performed.

iv) **IFSST clustering – 1** [10]: In this work, the authors enhanced the mammogram images using an intuitionistic fuzzy image of second type. The image is initially converted to an intuitionistic fuzzy image of second type using the IFSST generator.

As mammogram images are not well illuminated, a fuzzy membership function is initially computed as:

$$\alpha_{mn} = \left(\frac{g_{mn} - g_{min}}{g_{max} - g_{min}}\right)^{1.25}.$$

The IFSST membership function is written as:

$$\alpha_{\tilde{P}}(z) = \frac{\alpha(z).\left(1 + e^{\delta}\right)}{1 + \alpha(z).e^{\delta}}.$$

The IFSST non-membership function is written as:

$$\beta_{\tilde{P}}(z) = \frac{1 - \alpha(z)}{1 + 2.\alpha(z).e^{\delta} + \alpha(z).e^{2\delta}}, \quad \delta > 0.$$

Using a graphical approach, the optimum value of δ is computed. The enhanced image obtained using IFSST was then segmented using a fuzzy clustering algorithm.

(v) **IFSST clustering – 2** [17]: The authors worked on COVID lung images. They initially normalized the image and then computed the IFSST fuzzy membership function, which is written as:

$$\alpha^{PFS}(z) = 1 - \frac{1 - \alpha(z)}{1 + \left(e^{\delta} - 1\right)\alpha(z)}.$$

The IFSST non-membership function is written as:

$$\beta^{PFS}(z) = \frac{1 - \alpha^{PFS}(z)}{1 + \left(e^{\delta+1} - 1\right)\alpha^{PFS}(z)}, \quad \delta > 0.$$

The degree of indeterminacy is written as:

$$\gamma^{PFS}(z)^2 = \sqrt{1 - \alpha^{PFS}(z)^2 - \beta^{PFS}(z)^2}.$$

The optimum value of δ in the membership function is obtained by maximizing their proposed IFSST entropy (*PFE*), which is given as:

$$\delta_{opt} = \max_{\delta} PFE(A;\delta),$$

where *PFE* is the IFSST entropy. With the value of δ_{opt}, IFSST images are obtained and enhanced using intensification operator. The enhanced images are segmented into different regions.

2.2 Results and discussion

Experiments are performed on several mammogram images. Qualitative analysis has been done on the images. Sample results on six images are shown using fuzzy [6], intuitionistic fuzzy clustering methods – 1, 2 [20, 9], interval type-2 fuzzy sets [11], and IFSST methods – 1, 2 [10, 17]. All the methods are applied on the mammogram images to see the segmentation accuracy of different types of fuzzy and variants of fuzzy sets on mammogram images.

Quantitative analysis is shown in Table 1. The table shows segmentation errors on the six methods, with segmentation errors computed with respect to the ground truth images. It is observed that FCM (fuzzy C-means clustering) gives 2.46% segmentation error, intuitionistic fuzzy C-means clustering method (IFCM-1) gives 2.25% segmentation error, and IFCM-2 gives 2.06% segmentation error. Interval type-2 fuzzy set shows 2.29% segmentation error. The IFCM-2 method used a different type of intuitionistic fuzzy generator other than the Sugeno fuzzy generator in IFCM-1, and used the Interval type-2 fuzzy method for clustering.

The IFSST clustering method-1 and IFSST method-2 exhibit, respectively, 1.34% and 1.72% segmentation error.

Table 1. Segmentation accuracy on mammogram image using different fuzzy techniques

Images	**FCM**	**IFCM-1**	**IFCM-2**	**Interval type-2**	**IFSST-1**	**IFSST-2**
Mam#1	1.30	0.27	0.17	1.77	0.29	0.74
Mam#2	1.69	1.64	0.96	0.23	0.22	0.35
Mam#3	0.70	0.62	0.56	4.99	4.70	1.29
Mam#4	3.69	3.69	0.96	1.95	0.24	3.76
Mam#5	6.18	3.99	4.35	5.96	6.12	2.04
Mam#6	0.20	0.27	0.18	0.80	0.24	0.33
Mam#7	1.35	0.64	0.61	1.71	0.85	0.47
Mam#8	5.63	5.61	9.84	2.42	0.57	3.86
Mam#9	4.80	5.03	4.80	0.85	0.33	2.90
Mam#10	0.40	0.40	0.11	2.50	0.34	0.27
Mam#11	1.25	0.90	0.22	0.23	0.22	0.28
Mam#12	1.67	3.37	2.43	0.37	3.56	2.30
Mam#13	0.24	0.55	2.59	0.70	1.29	0.17
Mam#14	4.74	2.98	2.53	4.49	0.92	4.74
Mam#15	1.33	1.40	0.73	4.43	2.14	1.89
Mam#16	6.10	6.10	3.06	5.43	0.57	3.19
Mam#17	2.25	2.25	2.46	0.21	0.27	0.40
Mam#18	0.73	0.73	0.12	2.26	1.05	1.95
Average	**2.46%**	**2.25%**	**2.06%**	**2.29%**	**1.34%**	**1.72%**

Qualitative analysis is shown in Figure 3. Comparison is done with respect to the ground truth images. Here original image is the ground truth image that is similar to the segmented actual image.

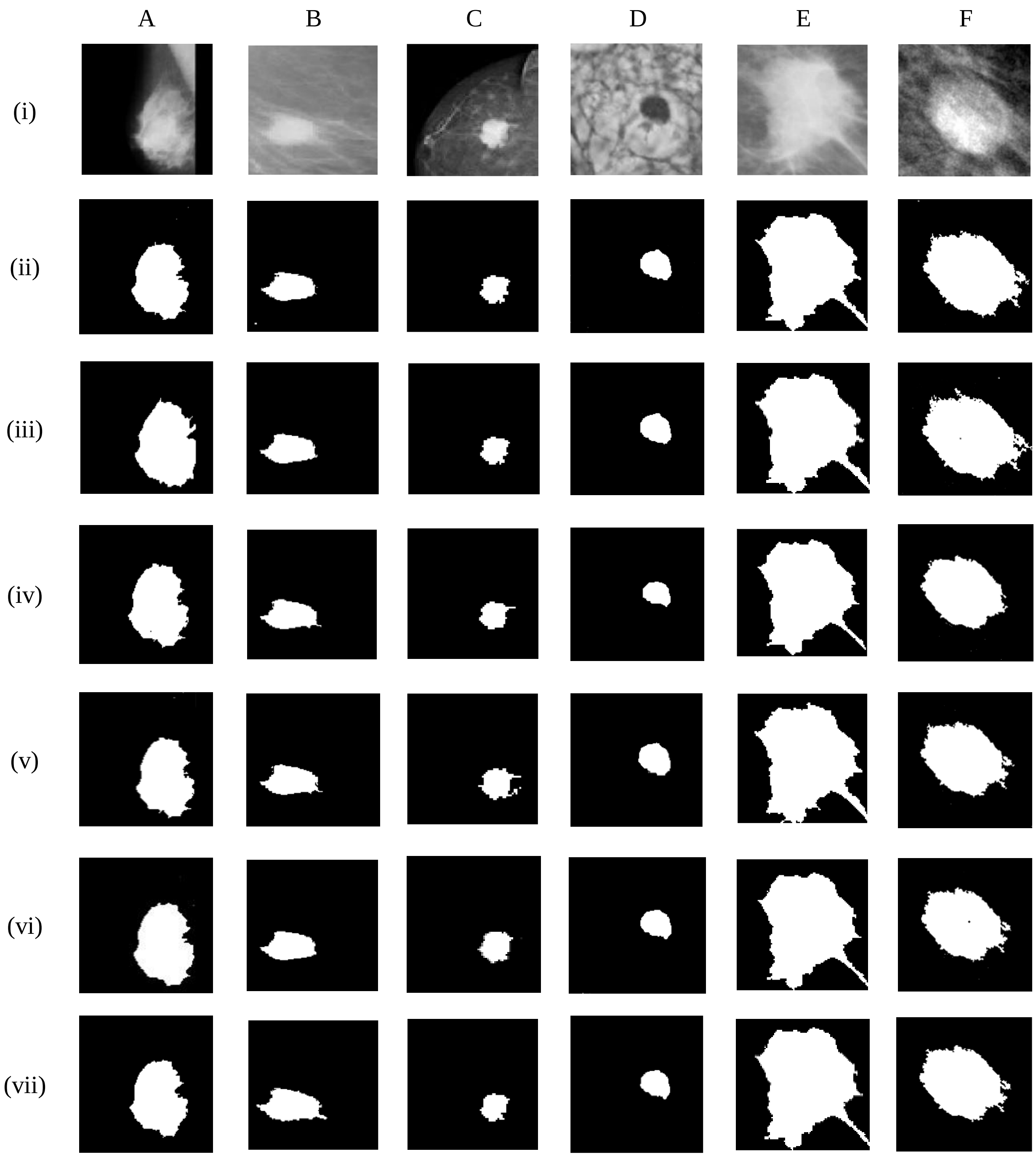

Figure 3. 3A = Mam#1, 3B = Mam#12, 3C = Mam#6, 3D = Mam#10, 3E = Mam#14, 3F = Mam#11. i) Mammogram image, ii) IFCM-1, iii) FCM, iv) IFCM-2, v) Interval type-2 fuzzy, vi) IFSST-1, vii) IFSST-2.

Figure 3A(i) is a mammogram image with a lesion. It is observed that in fuzzy C-means, IFCM-2 and IFSST-2, shown in Figs. 3A(iii, iv, vii), respectively, the size of the mass region is larger than the original image. IFCM-1 in Fig. 3A(ii) is slightly better in terms of size. The segmented lesion in Interval type-2 fuzzy result in Fig. 3A(v) is smaller than the original image. IFSST-1 in Fig. 3A(vi) shows a better result, and the size is almost the same as the original image.

Figure 3B(i) shows a mammogram image with a lesion. The segmented lesion in Figs. 3B(ii–iv) is almost similar, and the size is smaller than the original image. Interval type-2 fuzzy segmented image in Fig. 3B(v) is slightly better. IFSST segmented images in Figs. 3B(vi, vii) are better.

Fig. 3C(i) shows a mammogram image with a lesion. The IFSST-2 segmented image in Fig. 3C(v) is slightly bigger in size. IFSST-1 in Fig. 3C(vi) gives a better representation of the mass region with almost the same size as that of the lesion. The size of images in Figs. 3C(ii, iii, iv, vii) are slightly smaller in size and the outer boundaries are not uniform.

Fig. 3D(i) shows a mammogram image with a lesion. FCM and IFCM-1 segmented images in Figs. 3D(ii, iii) are slightly larger while IFSST-2 segmented image in Fig. 3D(vii) is slightly smaller than the original image. IFSST-1 segmented image in Fig. 3D(vi) is better in terms of size. The size of the Interval type-2 fuzzy segmented image in Fig. 3D(v) is very large. The size of IFCM-2 segmented image in Fig. 3D(iv) is small.

Fig. 3E(i) shows a mammogram image with a lesion. The Interval type-2 fuzzy segmented image in Fig. 3E(v) does not give better results as the size is bigger. FCM, IFCM-1, 1FCM-2 in Figs. 3E(ii, iii, iv, vii) are almost similar with size either slightly less or greater than the ground truth images. IFSST-1 in Fig. 3E(vi) gives a better representation of the mass region with almost the same size as that of the lesion.

Fig. 3F(i) shows a mammogram image with a lesion. The size of the lesion is large in the fuzzy segmented image in Fig. 3F(iii). The size of IFCM-1 in Fig. 3F(ii) is still bigger than the original image. The size of IFCM-2 in Fig. 3F(iv, v, iv, vii) are almost similar to the original image.

3 Conclusion

From the results it is seen that the clustered images using IFCM-2 show slightly better results (2.06%) as this method used a different type of intuitionistic fuzzy generator other than Sugeno fuzzy generator in IFCM-1 and used Interval type-2 fuzzy method for clustering.

In both its variants, IFSST shows very little segmentation error, respectively 1.34% and 1.72% for IFSST-1 and IFSST-2. IFSST gives a better representation and detailed clustering result about mass. Arguably, this is because the admissible region of each IFSST number is larger than that of each intuitionistic fuzzy number. Intuitionistic fuzzy sets and Interval type-2 fuzzy sets give better results than fuzzy sets alone, as a fuzzy set considers only membership function as an uncertainty.

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