Notes on Intuitionistic Fuzzy Sets
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Characterizing intuitionistic fuzzy Γ-Ideals of ordered Γ-semigroups by means of intuitionistic fuzzy points

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Abstract: The notion of Γ-semigroups was introduced by Sen in 1981 and that of intuitionistic fuzzy sets by Atanassov in 1986. Any semigroup can be reduced to a Γ-semigroup but a Γ-semigroup does not necessarily reduce to a semigroup.

In this paper, we find some relations between the intuitionistic fuzzy ideals and the set of all intuitionistic fuzzy points of ordered Γ-semigroups, and investigate some properties of intuitionistic fuzzification of the concept of several ideals in term of characteristic mappings.
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1 Introduction and Preliminaries

The fundamental concept of fuzzy sets in a set was first introduced by Zadeh [34] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere.

After the introduction of the concept of fuzzy sets by Zadeh [34], several researches were conducted on the generalizations of the notion of fuzzy sets and application to many algebraic structures such as: In 1971, Rosenfeld [23] was the first who studied fuzzy sets in the structure of groups. Fuzzy semigroups have been first considered by Kuroki [18, 19, 20], and fuzzy ordered groupoids and ordered semigroups, by Kehayopulu and Tsingelis [10, 11]. In 2007, Kehayopulu and Tsingelis [12] characterized the Green's relations $\mathcal{R}, \mathcal{L}, \mathcal{I}$ of ordered groupoids in terms of fuzzy subsets. In 2008, Zhan and Ma [35] studied fuzzy interior ideals in semigroups. In 2009, Majumder and Sardar [22] studied fuzzy ideals and fuzzy ideal extensions in ordered semigroups. Prince Williams, Latha and Chandrasekeran [33] studied fuzzy bi-ideals in Γ-semigroups. In 2010, Chinram and Saelee [6] studied fuzzy ternary subsemigroups (left ideals, right ideals, lateral ideals, ideals) and fuzzy left filters (right filters, lateral filters, filters) of ordered ternary semigroups. Iampan [8] characterized the relationship between the fuzzy ordered ideals (fuzzy ordered filters) and the characteristic mappings of fuzzy ordered ideals (fuzzy ordered filters) in ordered Γ-semigroups. In 2012, Sardar, Davvaz, Majumder and Kayal [26] studied the generalized fuzzy interior ideals in Γ-semigroups. In 2013, Khan, Sarmin and Khan [13] introduced (λ, θ)-fuzzy bi Γ-ideal of ordered Γ-semigroups.

The idea of intuitionistic fuzzy sets was first published by Atanassov in his pioneer papers [3, 4], as generalization of the notion of fuzzy sets. In2001, Kim and Jun [14] introduced the notion of intuitionistic fuzzy interior ideals of semigroups and some fundamental properties were investigated. In 2002, Kim and Jun [15] introduced the notion of intuitionistic fuzzy ideals of semigroups and some basic properties have been investigated. In 2005, Kim and Lee [16] initiated the concept of intuitionistic fuzzy bi-ideals of semigroups and obtained some useful properties. In 2007, Hong, Xu and Fang [7] characterized the completely regular and the strongly regular ordered semigroups by means of intuitionistic fuzzy bi-ideals. In 2011, Sardar, Mandal and Majumder [28] characterized some relations between the intuitionistic fuzzy ideals and the set of all intuitionistic fuzzy points of semigroups. In 2012, Akram [1] studied the intuitionistic fuzzy sets in ternary semigroups and the corresponding sets of intuitionistic fuzzy points.

The concept of Γ-semigroups, a generalization of both the concepts of semigroups and ternary semigroups, was introduced by Sen [29] and the concept of ordered Γ-semigroups was introduced by Sen and Seth [31]. For examples of Γ-semigroups and ordered Γ-semigroups, see [24, 25, 30].

Several researches were conducted on the intuitionistic fuzzy sets in Γ-semigroups and in ordered Γ-semigroups such as: In 2007, Uçkun, Öztürk and Jun [32] introduced the notion of intuitionistic fuzzy Γ-ideals in Γ-semigroups. In 2011, Sardar, Majumder and Mandal [27] introduced the concept of intuitionistic fuzzy prime, semiprime ideal and also intuitionistic fuzzy ideal extensions in Γ-semigroups. In 2012, Lekkoksung [21] defined intuitionistic fuzzy bi-Γ-ideals of Γ-semigroups.

In 2013, Akram [2] introduced the notion of prime, semiprime, strongly prime, irreducible and strongly irreducible intuitionistic fuzzy bi-Γ-ideals in Γ-semigroups and investigated some of the properties related to these Γ-ideals. In 2014, Kanlaya and Iampan [9] studied the coincidences of fuzzy generalized bi-ideals, fuzzy bi-ideals, fuzzy interior ideals and fuzzy ideals in regular, left regular, right regular, intra-regular, semisimple ordered Γ-semigroups.

The intuitionistic fuzzification of the concept of several ideals play an important role in studying the structure of in Γ-semigroups and in ordered Γ-semigroups.

Therefore, we will find some relations between the intuitionistic fuzzy ideals and the set of all intuitionistic fuzzy points of ordered Γ-semigroups, and investigate some properties of intuitionistic fuzzification of the concept of several ideals in term of characteristic mappings.

Before going to prove the main results we need the following definitions that we use later.

Definition 1.1. Let M and Γ be any two nonempty sets. Then (M, Γ) is called a Γ-*semigroup* [29] if there exists a mapping $M \times \Gamma \times M \to M$, written as $(a, \gamma, b) \mapsto a\gamma b$, satisfying the following identity $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. A nonempty subset K of M is called a Γ-*subsemigroup* of M if $a\gamma b \in K$ for all $a, b \in K$ and $\gamma \in \Gamma$.

For subsets A and B of a Γ-semigroup (M, Γ) and Γ' of Γ, let $A\Gamma' B := \{a\gamma b \mid a \in A, b \in B$ and $\gamma \in \Gamma'\}$. If $A = \{a\}$, then we also write $\{a\}\Gamma' B$ as $a\Gamma' B$, and similarly if $B = \{b\}$ or $\Gamma' = \{\gamma\}$.

Definition 1.2. A partially ordered Γ-semigroup $(M, \Gamma, \leq)$ is called an *ordered* Γ-*semigroup* [31] if for any $a, b, c \in M$ and $\gamma \in \Gamma$, $a \leq b$ implies $a\gamma c \leq b\gamma c$ and $c\gamma a \leq c\gamma b$. For convenience, we simply write M instead of an ordered Γ-semigroup $(M, \Gamma, \leq)$. An ordered Γ-semigroup M is called an *anti-chain* [17] if for any $a, b \in M$, $a \leq b$ implies $a = b$.

Definition 1.3. Let M be an ordered Γ-semigroup. For $A \subseteq M$, we define $(A] := \{t \in M \mid t \leq a$ for some $a \in A\}$.

Definition 1.4. Let M be an ordered Γ-semigroup. A nonempty subset A of M is called a *left (resp. right)* Γ-*ideal* of M if (1) $M\Gamma A \subseteq A$ (resp. $A\Gamma M \subseteq A$), and (2) $(A] \subseteq A$. A nonempty subset A of M is called a Γ-*ideal* (some authors called a *two-sided* Γ-*ideal*) of M if it is both a left Γ-ideal and a right Γ-ideal of M. That is, (1) $M\Gamma A \subseteq A$ and $A\Gamma M \subseteq A$, and (2) $(A] \subseteq A$.

Definition 1.5. Let M be an ordered Γ-semigroup. A Γ-subsemigroup A of M is called an *interior* Γ-*ideal* of M if (1) $M\Gamma A\Gamma M \subseteq A$, and (2) $(A] \subseteq A$. A Γ-subsemigroup A of M is called a *bi*-Γ-*ideal* of M if (1) $A\Gamma M\Gamma A \subseteq A$, and (2) $(A] \subseteq A$.

Definition 1.6. Let M be an ordered Γ-semigroup. A Γ-ideal A of M is called a *semiprime* Γ-*ideal* of M if for any $x \in M$ and $\gamma \in \Gamma$, $x\gamma x \in A$ implies $x \in A$. A Γ-ideal A of M is called a *prime* Γ-*ideal* of M if for any $x, y \in M$ and $\gamma \in \Gamma$, $x\gamma y \in A$ implies $x \in A$ or $y \in A$.

Definition 1.7. A *fuzzy subset* [34] of a nonempty set X (or a fuzzy set in a nonempty set X) is an arbitrary mapping $\mu\colon X \to [0, 1]$ where $[0, 1]$ is the unit segment of the real line. The mapping $\overline{\mu}\colon X \to [0, 1]$ defined via $\overline{\mu}(x) = 1 - \mu(x)$ is a fuzzy subset of X called the *complement* of μ in X.

Definition 1.8. Let X be a set and $A \subseteq X$. The *characteristic mapping* $f_A\colon X \to [0, 1]$ defined via

$$x \mapsto f_A(x) := \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

By the definition of characteristic mapping, f_A is a mapping of X into $\{0, 1\} \subset [0, 1]$. Hence, f_A is a fuzzy subset of X.

Definition 1.9. Let M be an ordered Γ-semigroup. A nonempty fuzzy subset f of M is called a *fuzzy* Γ-*subsemigroup* of M if for any $x, y \in M$ and $\gamma \in \Gamma$, $f(x\gamma y) \geq \min\{f(x), f(y)\}$. A nonempty fuzzy subset f of M is called a *fuzzy left (resp. right)* Γ-*ideal* of M if (1) for any $x, y \in M, x \leq y$ implies $f(x) \geq f(y)$, and (2) $f(x\gamma y) \geq f(y)$ (resp. $f(x\gamma y) \geq f(x)$) for all

$x, y \in M$ and $\gamma \in \Gamma$. A nonempty fuzzy subset f of M is called a *fuzzy* Γ-*ideal* (some authors called a *fuzzy two-sided* Γ-*ideal*) of M if it is both a fuzzy left Γ-ideal and a fuzzy right Γ-ideal of M. That is, (1) for any $x, y \in M, x \leq y$ implies $f(x) \geq f(y)$, and (2) $f(x\gamma y) \geq f(y)$ and $f(x\gamma y) \geq f(x)$ for all $x, y \in M$ and $\gamma \in \Gamma$.

Definition 1.10. Let M be an ordered Γ-semigroup. A fuzzy Γ-subsemigroup f of M is called a *fuzzy interior* Γ-*ideal* of M if (1) for any $x, y \in M, x \leq y$ implies $f(x) \geq f(y)$, and (2) $f(x\alpha y\beta z) \geq f(y)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. A fuzzy Γ-subsemigroup f of M is called a *fuzzy bi*-Γ-*ideal* of M if (1) for any $x, y \in M, x \leq y$ implies $f(x) \geq f(y)$, and (2) $f(x\alpha y\beta z) \geq \min\{f(x), f(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 1.11. Let M be an ordered Γ-semigroup. M is called *regular* if for each $a \in M$, there exist $x \in M$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$. M is called *left (resp. right) regular* if for each $a \in M$, there exist $x \in M$ and $\alpha, \beta \in \Gamma$ such that $a \leq x\alpha a\beta a$ (resp. $a \leq a\alpha a\beta x$). M is called *intra-regular* if for each $a \in M$, there exist $x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a \leq x\alpha a\beta a\gamma y$. M is called *semisimple* if for each $a \in M$, there exist $x, b, y \in M$ and $\gamma, \alpha, \beta, \delta \in \Gamma$ such that $a \leq x\gamma a\alpha b\beta a\delta y$.

Definition 1.12. An *intuitionistic fuzzy set* (briefly, IFS) [3] A in a nonempty set X is an object having the form, $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, where the functions, $\mu_A \colon X \to [0, 1]$ and $\lambda_A \colon X \to [0, 1]$ denotes the degree of membership and the degree of nonmembership, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$ in X can be identified by an ordered pair (μ_A, λ_A) in $I^X \times I^X$ where $I = [0, 1]$. For simplicity, we shall use IFS for intuitionistic fuzzy set and $A = (\mu_A, \lambda_A)$ for IFS $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, and let $IFS(M)$ be the set of all intuitionistic fuzzy subsets of an ordered Γ-semigroup M.

For examples of intuitionistic fuzzy sets, see [14, 2].

Definition 1.13. Let c be a point of a nonempty set X and let $a \in (0, 1]$ and $b \in [0, 1)$ be two real numbers such that $a + b \leq 1$. Then an *intuitionistic fuzzy point* [5] written as $c_{(a,b)}$ is defined to be an intuitionistic fuzzy subset of X, given by

$$c_{(a,b)}(x) = \begin{cases} (a, b) & \text{if } x = c, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Definition 1.14. Let M be an ordered Γ-semigroup. An IFS $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy* Γ-*subsemigroup* [32] of M if (1) $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$, and (2) $\lambda_A(x\gamma y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$. An IFS $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy left (resp. right)* Γ-*ideal* [32] of M if (1) for any $x, y \in M, x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$, (2) $\mu_A(x\gamma y) \geq \mu_A(y)$ (resp. $\mu_A(x\gamma y) \geq \mu_A(x)$) for all $x, y \in M$ and $\gamma \in \Gamma$, and (3) $\lambda_A(x\gamma y) \leq \lambda_A(y)$ (resp. $\lambda_A(x\gamma y) \leq \lambda_A(x)$) for all $x, y \in M$ and $\gamma \in \Gamma$. An IFS $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy* Γ-*ideal* [32] (some authors called an *intuitionistic fuzzy two-sided* Γ-*ideal*) of M if it is both an intuitionistic fuzzy left Γ-ideal and an intuitionistic fuzzy right Γ-ideal of M.

That is, (1) for any $x, y \in M, x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$, (2) $\mu_A(x\gamma y) \geq \mu_A(y)$ and $\mu_A(x\gamma y) \geq \mu_A(x)$ for all $x, y \in M$ and $\gamma \in \Gamma$, and (3) $\lambda_A(x\gamma y) \leq \lambda_A(y)$ and $\lambda_A(x\gamma y) \leq \lambda_A(x)$ for all $x, y \in M$ and $\gamma \in \Gamma$.

Definition 1.15. Let M be an ordered Γ-semigroup. An intuitionistic fuzzy Γ-subsemigroup $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy interior* Γ*-ideal* [32] of M if (1) for any $x, y \in M, x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$, (2) $\mu_A(x\alpha y\beta z) \geq \mu_A(y)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, and (3) $\lambda_A(x\alpha y\beta z) \leq \lambda_A(y)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

An intuitionistic fuzzy Γ-subsemigroup $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy bi-*Γ*-ideal* [32] of M if (1) for any $x, y \in M, x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$, (2) $\mu_A(x\alpha y\beta z) \geq \min\{\mu_A(x), \mu_A(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, and (3) $\lambda_A(x\alpha y\beta z) \leq \max\{\lambda_A(x), \lambda_A(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Remark 1.16. It is clear that every intuitionistic fuzzy Γ-ideal of an ordered Γ-semigroup M is an intuitionistic fuzzy interior Γ-ideal of M, and every intuitionistic fuzzy Γ-ideal of an ordered Γ-semigroup M is an intuitionistic fuzzy bi-Γ-ideal of M.

Definition 1.17. Let M be an ordered Γ-semigroup. An intuitionistic fuzzy Γ-ideal $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy semiprime* Γ*-ideal* of M if (1) $\mu_A(x) \geq \mu_A(x\gamma x)$ for all $x \in M$ and $\gamma \in \Gamma$, and (2) $\lambda_A(x) \leq \lambda_A(x\gamma x)$ for all $x \in M$ and $\gamma \in \Gamma$. An intuitionistic fuzzy Γ-ideal $A = (\mu_A, \lambda_A)$ in M is called an *intuitionistic fuzzy prime* Γ*-ideal* of M if (1) $\mu_A(x\gamma y) = \max\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$, and (2) $\lambda_A(x\gamma y) = \min\{\lambda_A(x), \lambda_A(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$.

Remark 1.18. It is clear that every intuitionistic fuzzy prime Γ-ideal of an ordered Γ-semigroup M is an intuitionistic fuzzy semiprime Γ-ideal of M.

2 Relation between intuitionistic fuzzification of the concept of several ideals and characteristic mappings

In this section, we find the relation between intuitionistic fuzzification of the concept of several ideals and characteristic mappings of ordered Γ-semigroups.

Proposition 2.1. *[8] Let M be an ordered Γ-semigroup and $\emptyset \neq K \subseteq M$. Then K is a Γ-subsemigroup of M if and only if the fuzzy subset f_K is a fuzzy Γ-subsemigroup of M.*

Proposition 2.2. *[8] Let M be an ordered Γ-semigroup and $\emptyset \neq L \subseteq M$. Then L is a left (resp. right, two-sided) Γ-ideal of M if and only if the fuzzy subset f_L is a fuzzy left (resp. right, two-sided) Γ-ideal of M.*

Proposition 2.3. *[9] Let M be an ordered Γ-semigroup and $\emptyset \neq A \subseteq M$. Then A is an interior Γ-ideal of M if and only if the fuzzy subset f_A is a fuzzy interior Γ-ideal of M.*

Proposition 2.4. *[9] Let M be an ordered Γ-semigroup and $\emptyset \neq A \subseteq M$. Then A is a bi-Γ-ideal of M if and only if the fuzzy subset f_A is a fuzzy bi-Γ-ideal of M.*

Proposition 2.5. *Let M be an ordered Γ-semigroup and let f_A be the characteristic mapping of a nonempty subset A of M. Then the following statements are equivalent.*

(i) A is a Γ-subsemigroup of M.

(ii) $(f_A, \overline{f_A})$ is an intuitionistic fuzzy Γ-subsemigroup of M.

Proof. Assume that A is a Γ-subsemigroup of M. Clearly, $f_A\colon M \to [0,1]$ and $\overline{f_A}\colon M \to [0,1]$. For $x \in M$, we have $f_A(x) + \overline{f_A}(x) = f_A(x) + 1 - f_A(x) = 1$. Thus $0 \leq f_A(x) + \overline{f_A}(x) \leq 1$, so $(f_A, \overline{f_A})$ is an intuitionistic fuzzy subset of M. We shall show that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy Γ-subsemigroup of M. By Proposition 2.1, we have for any $x, y \in M$ and $\gamma \in \Gamma$, $\overline{f_A}(x\gamma y) \geq \min\{\overline{f_A}(x), \overline{f_A}(y)\}$. Thus $\overline{f_A}(x\gamma y) \leq \max\{\overline{f_A}(x), \overline{f_A}(y)\}$. Hence, $(f_A, \overline{f_A})$ is an intuitionistic fuzzy Γ-subsemigroup of M.

Conversely, assume that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy Γ-subsemigroup of M. Then f_A is a fuzzy Γ-subsemigroup of M. By Proposition 2.1, we have A is a Γ-subsemigroup of M. $\square$

Proposition 2.6. *Let M be an ordered Γ-semigroup and let f_A be the characteristic mapping of a nonempty subset A of M. Then the following statements are equivalent.*

(i) A is a left (resp. right, two-sided) Γ-ideal of M.

(ii) $(f_A, \overline{f_A})$ is an intuitionistic fuzzy left (resp. right, two-sided) Γ-ideal of M.

Proof. Assume that A is a left Γ-ideal of M. Clearly, $f_A\colon M \to [0,1]$ and $\overline{f_A}\colon M \to [0,1]$. For $x \in M$, we have $f_A(x) + \overline{f_A}(x) = f_A(x) + 1 - f_A(x) = 1$. Thus $0 \leq f_A(x) + \overline{f_A}(x) \leq 1$, so $(f_A, \overline{f_A})$ is an intuitionistic fuzzy subset of M. We shall show that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy left Γ-ideal of M. By Proposition 2.2, we have (1) for any $x, y \in M, x \leq y$ implies $f_A(x) \geq f_A(y)$, and (2) $f_A(x\gamma y) \geq f_A(y)$ for all $x, y \in M$ and $\gamma \in \Gamma$. Thus (1) for any $x, y \in M, x \leq y$ implies $\overline{f_A}(x) \leq \overline{f_A}(y)$, and (2) $\overline{f_A}(x\gamma y) \leq \overline{f_A}(y)$ for all $x, y \in M$ and $\gamma \in \Gamma$. Hence, $(f_A, \overline{f_A})$ is an intuitionistic fuzzy left Γ-ideal of M.

Conversely, assume that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy left Γ-ideal of M. Then f_A is a fuzzy left Γ-ideal of M. By Proposition 2.2, we have A is a left Γ-ideal of M. $\square$

Proposition 2.7. *Let M be an ordered Γ-semigroup and let f_A be the characteristic mapping of a nonempty subset A of M. Then the following statements are equivalent.*

(i) A is an interior Γ-ideal of M.

(ii) $(f_A, \overline{f_A})$ is an intuitionistic fuzzy interior Γ-ideal of M.

Proof. Assume that A is an interior Γ-ideal of M. Clearly, $f_A\colon M \to [0,1]$ and $\overline{f_A}\colon M \to [0,1]$. For $x \in M$, we have $f_A(x) + \overline{f_A}(x) = f_A(x) + 1 - f_A(x) = 1$. Thus $0 \leq f_A(x) + \overline{f_A}(x) \leq 1$, so $(f_A, \overline{f_A})$ is an intuitionistic fuzzy subset of M. We shall show that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy interior Γ-ideal of M. By Proposition 2.3, we have (1) $f_A(x\gamma y) \geq \min\{f_A(x), f_A(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$, (2) for any $x, y \in M, x \leq y$ implies $f_A(x) \geq f_A(y)$, and (3) $f_A(x\alpha y\beta z) \geq f_A(y)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Thus (1) $\overline{f_A}(x\gamma y) \leq \max\{\overline{f_A}(x), \overline{f_A}(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$, (2) for any $x, y \in M, x \leq y$ implies $\overline{f_A}(x) \leq \overline{f_A}(y)$, and (3)

$\overline{f_A}(x\alpha y\beta z) \leq \overline{f_A}(y)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Hence, $(f_A, \overline{f_A})$ is an intuitionistic fuzzy interior Γ-ideal of M.

Conversely, assume that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy interior Γ-ideal of M. Then f_A is a fuzzy interior Γ-ideal of M. By Proposition 2.3, we have A is an interior Γ-ideal of M. $\square$

Proposition 2.8. *Let M be an ordered Γ-semigroup and let f_A be the characteristic mapping of a nonempty subset A of M. Then the following statements are equivalent.*

(i) A is a bi-Γ-ideal of M.

(ii) $(f_A, \overline{f_A})$ is an intuitionistic fuzzy bi-Γ-ideal of M.

Proof. Assume that A is a bi-Γ-ideal of M. Clearly, $f_A\colon M \to [0,1]$ and $\overline{f_A}\colon M \to [0,1]$. For $x \in M$, we have $f_A(x) + \overline{f_A}(x) = f_A(x) + 1 - f_A(x) = 1$. Thus $0 \leq f_A(x) + \overline{f_A}(x) \leq 1$, so $(f_A, \overline{f_A})$ is an intuitionistic fuzzy subset of M. We shall show that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy bi-Γ-ideal of M. By Proposition 2.4, we have (1) $f_A(x\gamma y) \geq \min\{f_A(x), f_A(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$, (2) for any $x, y \in M, x \leq y$ implies $f_A(x) \geq f_A(y)$, and (3) $f_A(x\alpha y\beta z) \geq \min\{f_A(x), f_A(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Thus (1) $\overline{f_A}(x\gamma y) \leq \max\{\overline{f_A}(x), \overline{f_A}(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$, (2) for any $x, y \in M, x \leq y$ implies $\overline{f_A}(x) \leq \overline{f_A}(y)$, and (3) $\overline{f_A}(x\alpha y\beta z) \leq \max\{\overline{f_A}(x), \overline{f_A}(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Hence, $(f_A, \overline{f_A})$ is an intuitionistic fuzzy bi-Γ-ideal of M.

Conversely, assume that $(f_A, \overline{f_A})$ is an intuitionistic fuzzy bi-Γ-ideal of M. Then f_A is a fuzzy bi-Γ-ideal of M. By Proposition 2.4, we have A is a bi-Γ-ideal of M. $\square$

Lemma 2.9. *Let M be an ordered Γ-semigroup and let $\{A_i\}_{i\in I}$ be a family of intuitionistic fuzzy Γ-subsemigroups of M. Then $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy Γ-subsemigroup of M, provided it is nonempty.*

Proof. Clearly, $\bigcap\limits_{i\in I} A_i \in IFS(M)$. Let $x, y \in M$ and $\gamma \in \Gamma$. Then $\mu_{A_i}(x\gamma y) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ for all $i \in I$ and $\lambda_{A_i}(x\gamma y) \leq \max\{\lambda_{A_i}(x), \lambda_{A_i}(y)\}$ for all $i \in I$. Thus

$$\begin{aligned}&\bigwedge_{i\in I} \mu_{A_i}(x\gamma y) = \inf\{\mu_{A_i}(x\gamma y)\}_{i\in I} \geq \inf\{\min\{\mu_{A_i}(x), \mu_{A_i}(y)\}\}_{i\in I} =\\ &\min\{\inf\{\mu_{A_i}(x)\}_{i\in I}, \inf\{\mu_{A_i}(y)\}_{i\in I}\} = \min\{\bigwedge_{i\in I} \mu_{A_i}(x), \bigwedge_{i\in I} \mu_{A_i}(y)\}\end{aligned}$$

and

$$\begin{aligned}&\bigvee_{i\in I} \lambda_{A_i}(x\gamma y) = \sup\{\lambda_{A_i}(x\gamma y)\}_{i\in I} \leq \sup\{\max\{\lambda_{A_i}(x), \lambda_{A_i}(y)\}\}_{i\in I} =\\ &\max\{\sup\{\lambda_{A_i}(x)\}_{i\in I}, \sup\{\lambda_{A_i}(y)\}_{i\in I}\} = \max\{\bigvee_{i\in I} \lambda_{A_i}(x), \bigvee_{i\in I} \lambda_{A_i}(y)\}.\end{aligned}$$

Hence, $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy Γ-subsemigroup of M. $\square$

Lemma 2.10. *Let M be an ordered Γ-semigroup and let $\{A_i\}_{i\in I}$ be a family of intuitionistic fuzzy left (resp. right, two-sided) Γ-ideals of M. Then $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy left (resp. right, two-sided) Γ-ideal of M, provided it is nonempty.*

Proof. Clearly, $\bigcap\limits_{i\in I} A_i \in IFS(M)$. Let $x, y \in M$ be such that $x \le y$. Then $\mu_{A_i}(x) \ge \mu_{A_i}(y)$ for all $i \in I$ and $\lambda_{A_i}(x) \le \lambda_{A_i}(y)$ for all $i \in I$. Thus $\bigwedge\limits_{i\in I} \mu_{A_i}(x) = \inf\{\mu_{A_i}(x)\}_{i\in I} \ge \inf\{\mu_{A_i}(y)\}_{i\in I} = \bigwedge\limits_{i\in I} \mu_{A_i}(y)$ and $\bigvee\limits_{i\in I} \lambda_{A_i}(x) = \sup\{\lambda_{A_i}(x)\}_{i\in I} \le \sup\{\lambda_{A_i}(y)\}_{i\in I} = \bigvee\limits_{i\in I} \lambda_{A_i}(y)$. Let $x, y \in M$ and $\gamma \in \Gamma$. Then $\mu_{A_i}(x\gamma y) \ge \mu_{A_i}(y)$ for all $i \in I$ and $\lambda_{A_i}(x\gamma y) \le \lambda_{A_i}(y)$ for all $i \in I$. Thus

$$\bigwedge_{i\in I} \mu_{A_i}(x\gamma y) = \inf\{\mu_{A_i}(x\gamma y)\}_{i\in I} \ge \inf\{\mu_{A_i}(y)\}_{i\in I} = \bigwedge_{i\in I} \mu_{A_i}(y)$$

and

$$\bigvee_{i\in I} \lambda_{A_i}(x\gamma y) = \sup\{\lambda_{A_i}(x\gamma y)\}_{i\in I} \le \sup\{\lambda_{A_i}(y)\}_{i\in I} = \bigvee_{i\in I} \lambda_{A_i}(y).$$

Hence, $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy left Γ-ideal of M. □

Lemma 2.11. *Let M be an ordered Γ-semigroup and let $\{A_i\}_{i\in I}$ be a family of intuitionistic fuzzy interior Γ-ideals of M. Then $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy interior Γ-ideal of M, provided it is nonempty.*

Proof. By Lemma 2.9, $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy Γ-subsemigroup of M. Let $x, y \in M$ be such that $x \le y$. Then $\mu_{A_i}(x) \ge \mu_{A_i}(y)$ for all $i \in I$ and $\lambda_{A_i}(x) \le \lambda_{A_i}(y)$ for all $i \in I$. Thus $\bigwedge\limits_{i\in I} \mu_{A_i}(x) = \inf\{\mu_{A_i}(x)\}_{i\in I} \ge \inf\{\mu_{A_i}(y)\}_{i\in I} = \bigwedge\limits_{i\in I} \mu_{A_i}(y)$ and $\bigvee\limits_{i\in I} \lambda_{A_i}(x) = \sup\{\lambda_{A_i}(x)\}_{i\in I} \le \sup\{\lambda_{A_i}(y)\}_{i\in I} = \bigvee\limits_{i\in I} \lambda_{A_i}(y)$. Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Then $\mu_{A_i}(x\alpha y\beta z) \ge \mu_{A_i}(y)$ for all $i \in I$ and $\lambda_{A_i}(x\alpha y\beta z) \le \lambda_{A_i}(y)$ for all $i \in I$. Thus

$$\bigwedge_{i\in I} \mu_{A_i}(x\alpha y\beta z) = \inf\{\mu_{A_i}(x\alpha y\beta z)\}_{i\in I} \ge \inf\{\mu_{A_i}(y)\}_{i\in I} = \bigwedge_{i\in I} \mu_{A_i}(y)$$

and

$$\bigvee_{i\in I} \lambda_{A_i}(x\alpha y\beta z) = \sup\{\lambda_{A_i}(x\alpha y\beta z)\}_{i\in I} \le \sup\{\lambda_{A_i}(y)\}_{i\in I} = \bigvee_{i\in I} \lambda_{A_i}(y).$$

Hence, $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy interior Γ-ideal of M. □

Lemma 2.12. *Let M be an ordered Γ-semigroup and let $\{A_i\}_{i\in I}$ be a family of intuitionistic fuzzy bi-Γ-ideals of M. Then $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy bi-Γ-ideal of M, provided it is nonempty.*

Proof. By Lemma 2.9, $\bigcap\limits_{i\in I} A_i$ is an intuitionistic fuzzy Γ-subsemigroup of M. Let $x, y \in M$ be such that $x \le y$. Then $\mu_{A_i}(x) \ge \mu_{A_i}(y)$ for all $i \in I$ and $\lambda_{A_i}(x) \le \lambda_{A_i}(y)$ for all $i \in I$. Thus $\bigwedge\limits_{i\in I} \mu_{A_i}(x) = \inf\{\mu_{A_i}(x)\}_{i\in I} \ge \inf\{\mu_{A_i}(y)\}_{i\in I} = \bigwedge\limits_{i\in I} \mu_{A_i}(y)$ and $\bigvee\limits_{i\in I} \lambda_{A_i}(x) = \sup\{\lambda_{A_i}(x)\}_{i\in I} \le \sup\{\lambda_{A_i}(y)\}_{i\in I} = \bigvee\limits_{i\in I} \lambda_{A_i}(y)$. Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Then $\mu_{A_i}(x\alpha y\beta z) \ge \min\{\mu_{A_i}(x), \mu_{A_i}(z)\}$ for all $i \in I$ and $\lambda_{A_i}(x\alpha y\beta z) \le \max\{\lambda_{A_i}(x), \lambda_{A_i}(z)\}$ for all $i \in I$. Thus

$$\bigwedge_{i\in I} \mu_{A_i}(x\alpha y\beta z) = \inf\{\mu_{A_i}(x\alpha y\beta z)\}_{i\in I} \ge \inf\{\min\{\mu_{A_i}(x), \mu_{A_i}(z)\}\}_{i\in I} = \min\{\inf\{\mu_{A_i}(x)\}_{i\in I}, \inf\{\mu_{A_i}(z)\}_{i\in I}\} = \min\{\bigwedge_{i\in I} \mu_{A_i}(x), \bigwedge_{i\in I} \mu_{A_i}(z)\}$$

and

$$\bigvee_{i\in I}\lambda_{A_i}(x\alpha y\beta z)=\sup\{\lambda_{A_i}(x\alpha y\beta z)\}_{i\in I}\leq\sup\{\max\{\lambda_{A_i}(x),\lambda_{A_i}(z)\}\}_{i\in I}=$$
$$\max\{\sup\{\lambda_{A_i}(x)\}_{i\in I},\sup\{\lambda_{A_i}(z)\}_{i\in I}\}=\max\{\bigvee_{i\in I}\lambda_{A_i}(x),\bigvee_{i\in I}\lambda_{A_i}(z)\}.$$

Hence, $\bigcap_{i\in I} A_i$ is an intuitionistic fuzzy bi-Γ-ideal of M. $\square$

3 Main Results

Let M be an ordered Γ-semigroup. Then for any two intuitionistic fuzzy subsets $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ of M and $\gamma\in\Gamma$, we define the order relation "$\subseteq$" as follows: $A\subseteq B$ if and only if $\mu_A\preceq\mu_B$ and $\lambda_A\succeq\lambda_B$ where $\mu_A\preceq\mu_B$ if and only if $\mu_A(x)\leq\mu_B(x)$ for all $x\in M$, and $\lambda_A\succeq\lambda_B$ if and only if $\lambda_A(x)\geq\lambda_B(x)$ for all $x\in M$. The union $A\cup B$ and intersection $A\cap B$ are intuitionistic fuzzy subsets defined as follows: $A\cup B=\{(x,\max\{\mu_A(x),\mu_B(x)\},\min\{\lambda_A(x),\lambda_B(x)\})\mid x\in M\}$, and $A\cap B=\{(x,\min\{\mu_A(x),\mu_B(x)\},\max\{\lambda_A(x),\lambda_B(x)\})\mid x\in M\}$. Also, the product $A\gamma B$ is an intuitionistic fuzzy subsets defined as follows: $A\gamma B=\{(x,\mu_{A\gamma B}(x),\lambda_{A\gamma B}(x))\mid x\in M\}$ where

$$\mu_{A\gamma B}(x)=\begin{cases}\sup_{x=a\gamma b}\{\min\{\mu_A(a),\mu_B(b)\}\} & \text{if } x \text{ is expressible as } x=a\gamma b,\\ 0 & \text{otherwise}\end{cases}$$

and

$$\lambda_{A\gamma B}(x)=\begin{cases}\inf_{x=a\gamma b}\{\max\{\lambda_A(a),\lambda_B(b)\}\} & \text{if } x \text{ is expressible as } x=a\gamma b,\\ 0 & \text{otherwise.}\end{cases}$$

It is easy to verify that $(IFS(M),\Gamma,\subseteq)$ is an ordered Γ-semigroup.

If $\{A_i\}_{i\in I}$ is a family of intuitionistic fuzzy subsets of M, then their intersection $\bigcap_{i\in I}A_i=\{(x,\bigwedge_{i\in I}\mu_{A_i}(x),\bigvee_{i\in I}\lambda_{A_i}(x))\mid x\in M\}$ is an intuitionistic fuzzy subset of M where $\bigwedge_{i\in I}\mu_{A_i}(x)=\inf\{\mu_{A_i}(x)\}_{i\in I}$ and $\bigvee_{i\in I}\lambda_{A_i}(x)=\sup\{\lambda_{A_i}(x)\}_{i\in I}$.

Now, let $\underline{M}$ be the set of all intuitionistic fuzzy points of an ordered Γ-semigroup M. Then clearly $\underline{M}$ is a Γ-subsemigroup of $IFS(M)$, as for any $x_{(a,b)},y_{(c,d)},z_{(e,f)}\in\underline{M}$ and $\alpha,\beta,\gamma\in\Gamma$, we have $x_{(a,b)}\gamma y_{(c,d)}=(x\gamma y)_{(a\wedge c,b\vee d)}\in\underline{M}$ where $a\wedge c=\min\{a,c\}$ and $b\vee d=\max\{b,d\}$, and $(x_{(a,b)}\alpha y_{(c,d)})\beta z_{(e,f)}=x_{(a,b)}\alpha(y_{(c,d)}\beta z_{(e,f)})$. If $A=(\mu_A,\lambda_A)$ is an intuitionistic fuzzy subset of M, then $\underline{A}$ denotes the set of all intuitionistic fuzzy points contained in A [5], i.e. an intuitionistic fuzzy point $x_{(a,b)}$ is said to be contained in A if $x_{(a,b)}\subseteq A$. That is, $\underline{A}=\{x_{(a,b)}\in\underline{M}\mid\mu_A(x)\geq a$ and $\lambda_A(x)\leq b\}$.

Proposition 3.1. *Let M be an ordered Γ-semigroup and let $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ be two intuitionistic fuzzy subsets of M. Then the following statements hold.*

(i) $\underline{A\cup B}\supseteq\underline{A}\cup\underline{B}$.

(ii) $\underline{A\cap B}=\underline{A}\cap\underline{B}$.

(iii) $\underline{A\gamma B}\supseteq\underline{A}\gamma\underline{B}$ *for all* $\gamma\in\Gamma$.

Proof. (i) For $x_{(a,b)} \in \underline{M}$, we have

$$\begin{aligned} x_{(a,b)} \in \underline{A \cup B} &\Leftrightarrow \mu_{A\cup B}(x) \geq a \text{ and } \lambda_{A\cup B}(x) \leq b \\ &\Leftrightarrow \max\{\mu_A(x), \mu_B(x)\} \geq a \text{ and } \min\{\lambda_A(x), \lambda_B(y)\} \leq b \\ &\Leftrightarrow (\mu_A(x) \geq a \text{ or } \mu_B(x) \geq a) \text{ and } (\lambda_A(x) \leq b \text{ or } \lambda_B(y) \leq b) \\ &\Leftarrow (\mu_A(x) \geq a \text{ and } \lambda_A(x) \leq b) \text{ or } (\mu_B(x) \geq a \text{ and } \lambda_B(y) \leq b) \\ &\Leftrightarrow x_{(a,b)} \in \underline{A} \text{ or } x_{(a,b)} \in \underline{B} \\ &\Leftrightarrow x_{(a,b)} \in \underline{A} \cup \underline{B}. \end{aligned}$$

Hence, $\underline{A} \cup \underline{B} \subseteq \underline{A \cup B}$.
(ii) For $x_{(a,b)} \in \underline{M}$, we have

$$\begin{aligned} x_{(a,b)} \in \underline{A \cap B} &\Leftrightarrow \mu_{A\cap B}(x) \geq a \text{ and } \lambda_{A\cap B}(x) \leq b \\ &\Leftrightarrow \min\{\mu_A(x), \mu_B(x)\} \geq a \text{ and } \max\{\lambda_A(x), \lambda_B(y)\} \leq b \\ &\Leftrightarrow (\mu_A(x) \geq a \text{ and } \mu_B(x) \geq a) \text{ and } (\lambda_A(x) \leq b \text{ and } \lambda_B(y) \leq b) \\ &\Leftrightarrow (\mu_A(x) \geq a \text{ and } \lambda_A(x) \leq b) \text{ and } (\mu_B(x) \geq a \text{ and } \lambda_B(y) \leq b) \\ &\Leftrightarrow x_{(a,b)} \in \underline{A} \text{ and } x_{(a,b)} \in \underline{B} \\ &\Leftrightarrow x_{(a,b)} \in \underline{A} \cap \underline{B}. \end{aligned}$$

Hence, $\underline{A \cap B} = \underline{A} \cap \underline{B}$.
(iii) For $x_{(a,b)} \in \underline{A}, y_{(c,d)} \in \underline{B}$ and $\gamma \in \Gamma$, we have $x_{(a,b)}\gamma y_{(c,d)} = (x\gamma y)_{(a\wedge c, b\vee d)}$ and $(\mu_A(x) \geq a$ and $\lambda_A(x) \leq b)$ and $(\mu_B(y) \geq c$ and $\lambda_B(y) \leq d)$. Thus $(\mu_A(x) \geq a$ and $\mu_B(y) \geq c)$ and $(\lambda_A(x) \leq b$ and $\lambda_B(y) \leq d)$. This implies that $\min\{\mu_A(x), \mu_B(y)\} \geq a \wedge c$ and $\max\{\lambda_A(x), \lambda_B(y)\} \leq b \vee d$. Consider

$$\begin{aligned} \mu_{A\gamma B}(x\gamma y) &= \begin{cases} \sup_{x\gamma y = a\gamma b}\{\min\{\mu_A(a), \mu_B(b)\}\} & \text{if } x\gamma y \text{ is expressible as } x\gamma y = a\gamma b, \\ 0 & \text{otherwise} \end{cases} \\ &= \sup_{x\gamma y = a\gamma b}\{\min\{\mu_A(a), \mu_B(b)\}\} \text{ if } x\gamma y \text{ is expressible as } x\gamma y = a\gamma b \\ &\geq \min\{\mu_A(x), \mu_B(y)\} \\ &\geq a \wedge c \end{aligned}$$

and

$$\begin{aligned} \lambda_{A\gamma B}(x\gamma y) &= \begin{cases} \inf_{x\gamma y = a\gamma b}\{\max\{\lambda_A(a), \lambda_B(b)\}\} & \text{if } x\gamma y \text{ is expressible as } x\gamma y = a\gamma b, \\ 0 & \text{otherwise} \end{cases} \\ &= \inf_{x\gamma y = a\gamma b}\{\max\{\lambda_A(a), \lambda_B(b)\}\} \text{ if } x\gamma y \text{ is expressible as } x\gamma y = a\gamma b \\ &\leq \max\{\lambda_A(x), \lambda_B(y)\} \\ &\leq b \vee d. \end{aligned}$$

Hence, $x_{(a,b)}\gamma y_{(c,d)} = (x\gamma y)_{(a\wedge c, b\vee d)} \in \underline{A\gamma B}$ and so $\underline{A}\gamma\underline{B} \subseteq \underline{A\gamma B}$ for all $\gamma \in \Gamma$. □

From Proposition 3.1, we have Corollary 3.2.

Corollary 3.2. *Let M be an ordered Γ-semigroup and let $\{A_i\}_{i=1}^{n}$ be a family of intuitionistic fuzzy subsets of M. Then the following statements hold.*

(i) $\underline{\bigcup_{i=1}^{n} A_i} \supseteq \bigcup_{i=1}^{n} \underline{A_i}$.

(ii) $\underline{\bigcap_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \underline{A_i}$.

(iii) $\underline{A_1\gamma_1 A_2 \ldots A_{n-1}\gamma_{n-1}A_n} \supseteq \underline{A_1}\gamma_1\underline{A_2} \ldots \underline{A_{n-1}}\gamma_{n-1}\underline{A_n}$ *for all* $\gamma_1, \gamma_2, \ldots, \gamma_{n-1} \in \Gamma$.

Proposition 3.3. *Let M be an ordered Γ-semigroup and let $A = (\mu_A, \lambda_A)$ be a nonempty intuitionistic fuzzy subset of M. Then the following statements are equivalent.*

(i) *A is an intuitionistic fuzzy Γ-subsemigroup of M.*

(ii) *$\underline{A}$ is a Γ-subsemigroup of $\underline{M}$.*

Proof. Assume that A is an intuitionistic fuzzy Γ-subsemigroup of M. Let $x_{(a,b)}, y_{(c,d)} \in \underline{A}$ and $\gamma \in \Gamma$. Then $x_{(a,b)}\gamma y_{(c,d)} = (x\gamma y)_{(a\wedge c, b\vee d)}$ and $\mu_A(x) \geq a$ and $\lambda_A(x) \leq b$, and $\mu_A(y) \geq c$ and $\lambda_A(y) \leq d$. Thus $\mu_A(x) \geq a$ and $\mu_A(y) \geq c$, and $\lambda_A(x) \leq b$ and $\lambda_A(y) \leq d$. This implies that $\min\{\mu_A(x), \mu_A(y)\} \geq a \wedge c$ and $\max\{\lambda_A(x), \lambda_A(y)\} \leq b \vee d$. Since A is an intuitionistic fuzzy Γ-subsemigroup of M, we have $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq a \wedge c$ and $\lambda_A(x\gamma y) \leq \max\{\lambda_A(x), \lambda_A(y)\} \leq b \vee d$. Hence, $x_{(a,b)}\gamma y_{(c,d)} = (x\gamma y)_{(a\wedge c, b\vee d)} \in \underline{A}$ and so $\underline{A}$ is a Γ-subsemigroup of $\underline{M}$.

Conversely, assume that $\underline{A}$ is a Γ-subsemigroup of $\underline{M}$. Let $x, y \in M$ and $\gamma \in \Gamma$. If $\mu_A(x) = 0$ or $\mu_A(y) = 0$, and $\lambda_A(x) = 1$ or $\lambda_A(y) = 1$, then $\mu_A(x\gamma y) \geq \min\{\mu_A(x), \mu_A(y)\} = 0$ and $\lambda_A(x\gamma y) \leq \max\{\lambda_A(x), \lambda_A(y)\} = 1$. Suppose that $\mu_A(x), \mu_A(y) \neq 0$ and $\lambda_A(x), \lambda_A(y) \neq 1$. Since $\mu_A(x) \geq \mu_A(x)$ and $\lambda_A(x) \leq \lambda_A(x)$, we have $x_{(\mu_A(x), \lambda_A(x))} \in \underline{A}$. Similarly, $y_{(\mu_A(y), \lambda_A(y))} \in \underline{A}$. Since $\underline{A}$ is a Γ-subsemigroup of $\underline{M}$, we have

$$(x\gamma y)_{(\mu_A(x)\wedge\mu_A(y), \lambda_A(x)\vee\lambda_A(y))} = x_{(\mu_A(x), \lambda_A(x))}\gamma y_{(\mu_A(y), \lambda_A(y))} \in \underline{A}.$$

Thus $\mu_A(x\gamma y) \geq \mu_A(x) \wedge \mu_A(y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\lambda_A(x\gamma y) \leq \lambda_A(x) \vee \lambda_A(y) = \max\{\lambda_A(x), \lambda_A(y)\}$. Hence, A is an intuitionistic fuzzy Γ-subsemigroup of M. $\square$

Lemma 3.4. *Let M be an ordered Γ-semigroup. If $x_{(a,b)}, y_{(c,d)} \in \underline{M}$ is such that $\mu_{x_{(a,b)}} \preceq \mu_{y_{(c,d)}}$ (resp. $\lambda_{x_{(a,b)}} \succeq \lambda_{y_{(c,d)}}$), then $x = y$. Moreover, $a \leq c$ (resp. $b \geq d$).*

Proof. Assume that $x_{(a,b)}, y_{(c,d)} \in \underline{M}$ is such that $\mu_{x_{(a,b)}} \preceq \mu_{y_{(c,d)}}$ and suppose that $x \neq y$. Then $\mu_{x_{(a,b)}}(z) \leq \mu_{y_{(c,d)}}(z)$ for all $z \in M$, so $a = \mu_{x_{(a,b)}}(x) \leq \mu_{y_{(c,d)}}(x) = 0$. Thus $a = 0$ which is a contradiction. Hence, $x = y$ and so $a = \mu_{x_{(a,b)}}(x) \leq \mu_{y_{(c,d)}}(x) = \mu_{y_{(c,d)}}(y) = c$. $\square$

Lemma 3.5. *Let M be an ordered Γ-semigroup and let $A = (\mu_A, \lambda_A)$ be a nonempty intuitionistic fuzzy subset of M. If $x \in M$ is such that $\mu_A(x) > 0$ and $\lambda_A(x) < 1$, then $x_{(\mu_A(x), \lambda_A(x))} \in \underline{A}$.*

Proof. Assume that $x \in M$ is such that $\mu_A(x) > 0$ and $\lambda_A(x) < 1$. Then $\mu_A(x) \in (0, 1]$, $\lambda_A(x) \in [0, 1)$ and $\mu_A(x) + \lambda_A(x) \leq 1$, so $x_{(\mu_A(x), \lambda_A(x))} \in \underline{M}$. Since $\mu_A(x) \geq \mu_A(x)$ and $\lambda_A(x) \leq \lambda_A(x)$, we have $x_{(\mu_A(x), \lambda_A(x))} \in \underline{A}$. $\square$

Proposition 3.6. *Let M be an ordered Γ-semigroup and let $A = (\mu_A, \lambda_A)$ be a nonempty intuitionistic fuzzy subset of M. Then if A is an intuitionistic fuzzy left (resp. right, two-sided) Γ-ideal of M, then $\underline{A}$ is a left (resp. right, two-sided) Γ-ideal of $\underline{M}$.*

Proof. Assume that A is an intuitionistic fuzzy left Γ-ideal of M. Let $x_{(a,b)} \in \underline{M}, y_{(c,d)} \in \underline{A}$ and $\gamma \in \Gamma$. Then $x_{(a,b)}\gamma y_{(c,d)} = (x\gamma y)_{(a\wedge c, b\vee d)}$ and $\mu_A(y) \geq c$ and $\lambda_A(y) \leq d$. Since A is an intuitionistic fuzzy left Γ-ideal of M, we have $\mu_A(x\gamma y) \geq \mu_A(y) \geq c \geq a \wedge c$ and $\lambda_A(x\gamma y) \leq \lambda_A(y) \leq d \leq b \vee d$. Hence, $x_{(a,b)}\gamma y_{(c,d)} = (x\gamma y)_{(a\wedge c, b\vee d)} \in \underline{A}$. Let $x_{(a,b)} \in \underline{M}$ and $y_{(c,d)} \in \underline{A}$ be such that $x_{(a,b)} \subseteq y_{(c,d)}$. By Lemma 3.4, we have $x = y, a \leq c$ and $b \geq d$. Since $y_{(c,d)} \in \underline{A}$, we have $\mu_A(x) = \mu_A(y) \geq c \geq a$ and $\lambda_A(x) = \lambda_A(y) \leq d \leq b$. Thus $x_{(a,b)} \in \underline{A}$. Hence, $\underline{A}$ is a left Γ-ideal of $\underline{M}$. □

Proposition 3.7. *Let M be an ordered Γ-semigroup and let $A = (\mu_A, \lambda_A)$ be a nonempty intuitionistic fuzzy subset of M. Then if A is an intuitionistic fuzzy interior Γ-ideal of M, then $\underline{A}$ is an interior Γ-ideal of $\underline{M}$.*

Proof. Assume that A is an intuitionistic fuzzy interior Γ-ideal of M. By Proposition 3.3, we have $\underline{A}$ is a Γ-subsemigroup of $\underline{M}$. Let $x_{(a,b)}, z_{(e,f)} \in \underline{M}, y_{(c,d)} \in \underline{A}$ and $\alpha, \beta \in \Gamma$. Then $x_{(a,b)}\alpha y_{(c,d)}\beta z_{(e,f)} = (x\alpha y\beta z)_{(a\wedge c\wedge e, b\vee d\vee f)}$ and $\mu_A(y) \geq c$ and $\lambda_A(y) \leq d$. Since A is an intuitionistic fuzzy interior Γ-ideal of M, we have $\mu_A(x\alpha y\beta z) \geq \mu_A(y) \geq c \geq a \wedge c \wedge e$ and $\lambda_A(x\alpha y\beta z) \leq \lambda_A(y) \leq d \leq b \vee d \vee f$. Hence, $x_{(a,b)}\alpha y_{(c,d)}\beta z_{(e,f)} = (x\alpha y\beta z)_{(a\wedge c\wedge e, b\vee d\vee f)} \in \underline{A}$. Let $x_{(a,b)} \in \underline{M}$ and $y_{(c,d)} \in \underline{A}$ be such that $x_{(a,b)} \subseteq y_{(c,d)}$. By Lemma 3.4, we have $x = y, a \leq c$ and $b \geq d$. Since $y_{(c,d)} \in \underline{A}$, we have $\mu_A(x) = \mu_A(y) \geq c \geq a$ and $\lambda_A(x) = \lambda_A(y) \leq d \leq b$. Thus $x_{(a,b)} \in \underline{A}$. Hence, $\underline{A}$ is an interior Γ-ideal of $\underline{M}$. □

Proposition 3.8. *Let M be an ordered Γ-semigroup and let $A = (\mu_A, \lambda_A)$ be a nonempty intuitionistic fuzzy subset of M. Then if A is an intuitionistic fuzzy bi-Γ-ideal of M, then $\underline{A}$ is a bi-Γ-ideal of $\underline{M}$.*

Proof. Assume that A is an intuitionistic fuzzy bi-Γ-ideal of M. By Proposition 3.3, we have $\underline{A}$ is a Γ-subsemigroup of $\underline{M}$. Let $y_{(c,d)} \in \underline{M}, x_{(a,b)}, z_{(e,f)} \in \underline{A}$ and $\alpha, \beta \in \Gamma$. Then $x_{(a,b)}\alpha y_{(c,d)}\beta z_{(e,f)} = (x\alpha y\beta z)_{(a\wedge c\wedge e, b\vee d\vee f)}$ and $\mu_A(x) \geq a$ and $\lambda_A(x) \leq b$, and $\mu_A(z) \geq e$ and $\lambda_A(z) \leq f$. Since A is an intuitionistic fuzzy bi-Γ-ideal of M, we have $\mu_A(x\alpha y\beta z) \geq \min\{\mu_A(x), \mu_A(z)\} \geq a \wedge e \geq a \wedge c \wedge e$ and $\lambda_A(x\alpha y\beta z) \leq \max\{\lambda_A(x), \lambda_A(z)\} \leq b \vee f \leq b \vee d \vee f$. Hence, $x_{(a,b)}\alpha y_{(c,d)}\beta z_{(e,f)} = (x\alpha y\beta z)_{(a\wedge c\wedge e, b\vee d\vee f)} \in \underline{A}$. Let $x_{(a,b)} \in \underline{M}$ and $y_{(c,d)} \in \underline{A}$ be such that $x_{(a,b)} \subseteq y_{(c,d)}$. By Lemma 3.4, we have $x = y, a \leq c$ and $b \geq d$. Since $y_{(c,d)} \in \underline{A}$, we have $\mu_A(x) = \mu_A(y) \geq c \geq a$ and $\lambda_A(x) = \lambda_A(y) \leq d \leq b$. Thus $x_{(a,b)} \in \underline{A}$. Hence, $\underline{A}$ is a bi-Γ-ideal of $\underline{M}$. □

Proposition 3.9. *Let M be an ordered Γ-semigroup and let $A = (\mu_A, \lambda_A)$ be a nonempty intuitionistic fuzzy subset of M. Then if A is an intuitionistic fuzzy semiprime Γ-ideal of M, then $\underline{A}$ is a semiprime Γ-ideal of $\underline{M}$.*

Proof. Assume that A is an intuitionistic fuzzy semiprime Γ-ideal of M. Then A is an intuitionistic fuzzy Γ-ideal of M. By Proposition 3.6, we have $\underline{A}$ is a Γ-ideal of $\underline{M}$. Let $x_{(a,b)} \in \underline{M}$ and

$\gamma \in \Gamma$ be such that $x_{(a,b)}\gamma x_{(a,b)} \in \underline{A}$. Then $(x\gamma x)_{(a,b)} = (x\gamma x)_{(a\wedge a, b\vee b)} = x_{(a,b)}\gamma x_{(a,b)} \in \underline{A}$, so $\mu_A(x\gamma x) \geq a$ and $\lambda_A(x\gamma x) \leq b$. Since A is an intuitionistic fuzzy semiprime Γ-ideal of M, we have for all $x \in M$ and $\gamma \in \Gamma$, $\mu_A(x) \geq \mu_A(x\gamma x)$ and $\lambda_A(x) \leq \lambda_A(x\gamma x)$. This implies that $\mu_A(x) \geq \mu_A(x\gamma x) \geq a$ and $\lambda_A(x) \leq \lambda_A(x\gamma x) \leq b$. Thus $x_{(a,b)} \in \underline{A}$. Hence, $\underline{A}$ is a semiprime Γ-ideal of $\underline{M}$. $\square$

Theorem 3.10. *Let M be an ordered Γ-semigroup. Then the following statements hold.*

(i) If M is an anti-chain and is intra-regular, then $\underline{M}$ is intra-regular.

(ii) If $\underline{M}$ is intra-regular, then M is intra-regular.

Proof. (i) Assume that M is an anti-chain and is intra-regular. Let $x_{(a,b)} \in \underline{M}$ where $x \in M$. Then there exist $u, v \in M$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x = u\alpha x\beta x\gamma v$. Thus $u_{(a,b)}, v_{(a,b)} \in \underline{M}$, so $u_{(a,b)}\alpha x_{(a,b)}\beta x_{(a,b)}\gamma v_{(a,b)} = (u\alpha x\beta x\gamma v)_{(a\wedge a\wedge a\wedge a, b\vee b\vee b\vee b)} = (u\alpha x\beta x\gamma v)_{(a,b)} = x_{(a,b)}$. Hence, $x_{(a,b)} \subseteq u_{(a,b)}\alpha x_{(a,b)}\beta x_{(a,b)}\gamma v_{(a,b)}$, so $\underline{M}$ is intra-regular.
(ii) Assume that $\underline{M}$ is intra-regular. Let $x \in M$. Then for any $a \in (0,1]$ and $b \in [0,1)$ such that $a+b \leq 1$, there exist $u_{(c,d)}, v_{(e,f)} \in \underline{M}$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x_{(a,b)} \subseteq u_{(c,d)}\alpha x_{(a,b)}\beta x_{(a,b)}\gamma v_{(e,f)}$ $= (u\alpha x\beta x\gamma v)_{(c\wedge a\wedge a\wedge e, d\vee b\vee b\vee f)} = (u\alpha x\beta x\gamma v)_{(c\wedge a\wedge e, d\vee b\vee f)}$.

By Lemma 3.4, we have $x = u\alpha x\beta x\gamma v$. Hence, M is intra-regular. $\square$

Theorem 3.11. *Let M be an ordered Γ-semigroup. Then the following statements hold.*

(i) If M is an anti-chain and is regular, then $\underline{M}$ is regular.

(ii) If $\underline{M}$ is regular, then M is regular.

Proof. (i) Assume that M is an anti-chain and is regular. Let $x_{(a,b)} \in \underline{M}$ where $x \in M$. Then there exist $u \in M$ and $\alpha, \beta \in \Gamma$ such that $x = x\alpha u\beta x$. Thus $u_{(a,b)} \in \underline{M}$, so $x_{(a,b)}\alpha u_{(a,b)}\beta x_{(a,b)} = (x\alpha u\beta x)_{(a\wedge a\wedge a, b\vee b\vee b)} = (x\alpha u\beta x)_{(a,b)} = x_{(a,b)}$. Hence, $x_{(a,b)} \subseteq x_{(a,b)}\alpha u_{(a,b)}\beta x_{(a,b)}$, so $\underline{M}$ is regular.
(ii) Assume that $\underline{M}$ is regular. Let $x \in M$. Then for any $a \in (0,1]$ and $b \in [0,1)$ such that $a+b \leq 1$, there exist $u_{(c,d)} \in \underline{M}$ and $\alpha, \beta \in \Gamma$ such that $x_{(a,b)} \subseteq x_{(a,b)}\alpha u_{(c,d)}\beta x_{(a,b)} = (x\alpha u\beta x)_{(a\wedge c\wedge a, b\vee d\vee b)} = (x\alpha u\beta x)_{(a\wedge c, b\vee d)}$. By Lemma 3.4, we have $x = x\alpha u\beta x$. Hence, M is regular. $\square$

Theorem 3.12. *Let M be an ordered Γ-semigroup. Then the following statements hold.*

(i) If M is an anti-chain and is left (resp. right) regular, then $\underline{M}$ is left (resp. right) regular.

(ii) If $\underline{M}$ is left (resp. right) regular, then M is left (resp. right) regular.

Proof. (i) Assume that M is an anti-chain and is left regular. Let $x_{(a,b)} \in \underline{M}$ where $x \in M$. Then there exist $u \in M$ and $\alpha, \beta \in \Gamma$ such that $x = u\alpha x\beta x$. Thus $u_{(a,b)} \in \underline{M}$, so $u_{(a,b)}\alpha x_{(a,b)}\beta x_{(a,b)} = (u\alpha x\beta x)_{(a\wedge a\wedge a, b\vee b\vee b)} = (u\alpha x\beta x)_{(a,b)} = x_{(a,b)}$. Hence, $x_{(a,b)} \subseteq u_{(a,b)}\alpha x_{(a,b)}\beta x_{(a,b)}$, so $\underline{M}$ is left regular.

(ii) Assume that $\underline{M}$ is left regular. Let $x \in M$. Then for any $a \in (0,1]$ and $b \in [0,1)$ such that $a+b \leq 1$, there exist $u_{(c,d)} \in \underline{M}$ and $\alpha, \beta \in \Gamma$ such that $x_{(a,b)} \subseteq u_{(c,d)}\alpha x_{(a,b)}\beta x_{(a,b)} = (u\alpha x\beta x)_{(c\wedge a\wedge a, d\vee b\vee b)} = (u\alpha x\beta x)_{(c\wedge a, d\vee b)}$. By Lemma 3.4, we have $x = u\alpha x\beta x$. Hence, M is left regular. □

Theorem 3.13. *Let M be an ordered Γ-semigroup. Then the following statements hold.*

(i) If M is an anti-chain and is semisimple, then $\underline{M}$ is semisimple.

(ii) If $\underline{M}$ is semisimple, then M is semisimple.

Proof. (i) Assume that M is an anti-chain and is semisimple. Let $x_{(a,b)} \in \underline{M}$ where $x \in M$. Then there exist $u, v, w \in M$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $x = u\alpha x\beta v\gamma x\delta w$. Thus $u_{(a,b)}, v_{(a,b)}, w_{(a,b)} \in \underline{M}$, so $u_{(a,b)}\alpha x_{(a,b)}\beta v_{(a,b)}\gamma x_{(a,b)}\delta w_{(a,b)} = (u\alpha x\beta v\gamma x\delta w)_{(a\wedge a\wedge a\wedge a\wedge a, b\vee b\vee b\vee b\vee b)} = (u\alpha x\beta v\gamma x\delta w)_{(a,b)} = x_{(a,b)}$. Hence, $x_{(a,b)} \subseteq u_{(a,b)}\alpha x_{(a,b)}\beta v_{(a,b)}\gamma x_{(a,b)}\delta w_{(a,b)}$, so $\underline{M}$ is semisimple.
(ii) Assume that $\underline{M}$ is semisimple. Let $x \in M$. Then for any $a \in (0,1]$ and $b \in [0,1)$ such that $a+b \leq 1$, there exist $u_{(c,d)}, v_{(e,f)}, w_{(g,h)} \in \underline{M}$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that

$$\begin{aligned} x_{(a,b)} \subseteq u_{(c,d)}\alpha x_{(a,b)}\beta v_{(e,f)}\gamma x_{(a,b)}\delta w_{(g,h)} &= (u\alpha x\beta v\gamma x\delta w)_{(c\wedge a\wedge e\wedge a\wedge g, d\vee b\vee f\vee b\vee h)} \\ &= (u\alpha x\beta v\gamma x\delta w)_{(c\wedge a\wedge e\wedge g, d\vee b\vee f\vee h)}. \end{aligned}$$

By Lemma 3.4, we have $x = u\alpha x\beta v\gamma x\delta w$. Hence, M is semisimple. □

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