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# Edge regular properties of truncations of intuitionistic fuzzy graphs

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**Abstract:** In this paper, the adjacency sequence of intuitionistic fuzzy graph is defined. Also the degree of an edge in truncations of intuitionistic fuzzy graphs are obtained and edge regular properties of truncations of intuitionistic fuzzy graphs are discussed.

**Keywords:** Adjacency sequence, Edge regular intuitionistic fuzzy graph, Truncation of intuitionistic fuzzy graph, Upper and lower truncations of intuitionistic fuzzy graph.

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## 1 Introduction

Intuitionistic Fuzzy Graph theory was introduced by A. Shannon and K. Atanassov. In [8] R. Parvathi and M. G. Karunambigai introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In [3] M. G. Karunambigai and Palanivel introduce the edge regular intuitionistic fuzzy graph. In [9] K. Radha and N. Kumaravel introduced the edge degree and the edge regular properties of truncations of fuzzy graphs. In [6] Akram and Alshehri introduces the intuitionistic fuzzy cycle intuitionistic fuzzy trees. A. Nagoor Gani and H. Sheik Mujibur Rahman introduced the lower and upper truncations of intuitionistic fuzzy graphs in [7]. In this paper we introduce the degree of an edge in truncations of intuitionistic fuzzy graphs and edge regular properties of truncations of intuitionistic fuzzy graphs.

## 2 Preliminaries

Let $G : (V, E)$ be an intuitionistic fuzzy graph. Then the *degree of a vertex* $v$ is defined by $(v) = (d_\mu(v), d_\nu(v))$, where $d_\mu(v) = \sum_{u \neq v} \mu_2\,(v, u)$ and $d_\nu(v) = \sum_{u \neq v} \nu_2\,(v, u)$.

Let $G : (V, E)$ be an IFG and let $e_{ij} \in E$ be an edge in $G$. Then the *degree of an edge* $e_{ij} \in E$ is defined as $d_\mu(e_{ij}) = d_\mu(v_i) + d_\mu(v_j) - 2\mu_2(v_i, v_j)$ (or)
$d_\mu(e_{ij}) = \sum_{v_i v_k \in E, k \neq j} \mu_2(v_i, v_k) + \sum_{v_k v_j \in E, k \neq i} \mu_2(v_k, v_j)$
$d_\nu(e_{ij}) = d_\nu(v_j) + d_\nu(v_j) - 2\nu_2(v_i, v_j)$(or)
$d_\nu(e_{ij}) = \sum_{v_i v_k \in E, k \neq j} \nu_2(v_i, v_k) + \sum_{v_k v_j \in E, k \neq i} \nu_2(v_k, v_j)$, and the edge degree of $G$ is defined by $d_G(e_{ij}) = (d_\mu(e_{ij}), d_\nu(e_{ij}))$. Let $G = \langle V, E \rangle$ be an intuitionistic fuzzy graph. If each edge in $G$ has the same degree $(l_1, l_2)$, then $G$ is said to be an *edge regular intuitionistic fuzzy graph*.

Let $(\mu_1, \nu_1)$ be an intuitionistic fuzzy subset of a set $V$. The *lower and upper truncations* of the intuitionistic fuzzy set $(\mu_1, \nu_1)$ at level $t, 0 < t \leq 1$, are the intuitionistic fuzzy subsets $(\mu_1, \nu_1)_{(t)}$ *and* $(\mu_1, \nu_1)^{(t)}$ defined respectively by

$$(\mu_1, \nu_1)_{(t)}(u) = \begin{cases} (\mu_1, \nu_1)(u), if\ u \in {\mu_1}^t and\ u \in {\nu_1}^t \\ 0 \quad , if\ u \notin {\mu_1}^t (or)\ u \notin {\nu_1}^t \end{cases},$$

$$(\mu_1, \nu_1)^{(t)}(u) = \begin{cases} t \quad , if\ u \in {\mu_1}^t and\ u \in {\nu_1}^t \\ (\mu_1, \nu_1)(u), if\ u \notin {\mu_1}^t (or)\ u \notin {\nu_1}^t \end{cases}$$

where ${\mu_1}^t = \{u \in V / \mu_1(u) \geq t\}$ and ${\nu_1}^t = \{u \in V / \nu_1(u) \leq t\}$. Take $\mathrm{V}_{(t)} = (\mu_1, \nu_1)^t$, $\mathrm{E}_{(t)} = (\mu_2, \nu_2)^t$. $\mathrm{G}_{(t)} : (\mu_{(t)}, \nu_{(t)})$ is an intuitionistic fuzzy graph with underlying crisp graph $G^*_{(t)} : (\mathrm{V}_{(t)}, \mathrm{E}_{(t)})$. This is called the lower truncation of the intuitionistic fuzzy graph $G$ at level $t$. Here $\mathrm{V}_{(t)}$ *and* $\mathrm{E}_{(t)}$ may be proper subsets of $V$ and $E$, respectively. Now take $V^{(t)} = V$, $E^{(t)} = E$, then $G^{(t)} : (\mu^{(t)}, \nu^{(t)})$ is an intuitionistic fuzzy graph with underlying crisp graph $G^{(t)*} : (V^{(t)}, \mathrm{E}^{(t)})$. This is called the upper truncation of the intuitionistic fuzzy graph $G$ at level $t$.

# 3 Degree of an edge in truncations of intuitionistic fuzzy graph

## 3.1 Degree of an edge in lower truncation of intuitionistic fuzzy graph

$$d_{G_{(t)}}(uv) = (d_{\mu(G)_{(t)}}(uv), d_{\nu(G)_{(t)}}(uv))$$

$$d_{\mu(G)_{(t)}}(uv) = \sum_{uw \in E_{(t)}, w \neq v} \mu_{2_{(t)}}(uw) + \sum_{wv \in E_{(t)}, w \neq u} \mu_{2_{(t)}}(wv), \forall uv \in E_{(t)}$$

$$= \sum_{uw \in E, w \neq v} \mu_2(uw) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) < t}} \mu_2(uw) + \sum_{wv \in E, w \neq u} \mu_2(wv) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) < t}} \mu_2(wv), \forall uv \in E_{(t)}$$

$$= d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) < t}} \mu_2(uw) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) < t}} \mu_2(wv), \forall uv \in E_{(t)} \tag{1}$$

$$d_{\nu(G)_{(t)}}(uv) = \sum_{uw \in E_{(t)}, w \neq v} \nu_{2_{(t)}}(uw) + \sum_{wv \in E_{(t)}, w \neq u} \nu_{2_{(t)}}(wv), \forall uv \in E_{(t)}$$

$$= \sum_{uw \in E, w \neq v} \nu_2(uw) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) > t}} \nu_2(uw) + \sum_{wv \in E, w \neq u} \nu_2(wv) - \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) > t}} \nu_2(wv), \forall uv \in E_{(t)}$$

$$= d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) > t}} \nu_2(uw) - \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) > t}} \nu_2(wv)\,, \forall uv \in E_{(t)} \quad (2)$$

## 3.2 Degree of an edge in upper truncation of intuitionistic fuzzy graph

$$d_G{}^{(t)}(uv) = (d_{\mu(G)}{}^{(t)}(uv), d_{\nu(G)}{}^{(t)}(uv))$$

$$d_{\mu(G)}{}^{(t)}(uv) = \sum_{uw \in E^{(t)}, w \neq v} \mu_2{}^{(t)}(uw) + \sum_{wv \in E^{(t)}, w \neq u} \mu_2{}^{(t)}(wv)\,, \forall uv \in E^{(t)}$$

$$= \sum_{uw \in E, w \neq v} \mu_2(uw) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) \geq t}} (\mu_2(uw) - t) + \sum_{wv \in E, w \neq u} \mu_2(wv) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) \geq t}} (\mu_2(wv) - t)\,, \forall uv \in E^{(t)}$$

$$= d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) > t}} (\mu_2(uw) - t) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) > t}} (\mu_2(wv) - t)\,, \forall uv \in E^{(t)} \quad (3)$$

$$d_{\nu(G)}{}^{(t)}(uv) = \sum_{uw \in E^{(t)}, w \neq v} \nu_2{}^{(t)}(uw) + \sum_{wv \in E^{(t)}, w \neq u} \nu_2{}^{(t)}(wv)\,, \forall uv \in E^{(t)}$$

$$= \sum_{uw \in E, w \neq v} \nu_2(uw) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) \leq t}} (\nu_2(uw) - t) + \sum_{wv \in E, w \neq u} \nu_2(wv) - \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) \leq t}} (\nu_2(wv) - t)\,, \forall uv \in E^{(t)}$$

$$= d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) < t}} (\nu_2(uw) - t) + \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) < t}} (\nu_2(wv) - t)\,, \forall uv \in E^{(t)} \quad (4)$$

**Theorem 3.3:** Let $G : (V, E)$ be an intuitionistic fuzzy graph such that $\mu_2(uv) \geq t$ and $\nu_2(uv) \leq t, \forall\, uv \in E$, where $0 < t \leq 1$. Then for any $uv \in E_{(t)}$, $d_{G_{(t)}}(uv) = d_G(uv)$

*Proof:* From (1), for any $uv \in E_{(t)}$

$$d_{\mu(G)_{(t)}}(uv) = d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) < t}} \mu_2(uw) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) < t}} \mu_2(wv)$$

$\Rightarrow d_{\mu(G)_{(t)}}(uv) = d_{\mu(G)}(uv)$. From (2), for any $uv \in E_{(t)}$

$$d_{\nu(G)_{(t)}}(uv) = d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) > t}} \nu_2(uw) - \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) > t}} \nu_2(wv)$$

$$\Rightarrow d_{\nu(G)_{(t)}}(uv) = d_{\nu(G)}(uv)$$

$$d_{G_{(t)}}(uv) = \left(d_{\mu(G)_{(t)}}(uv), d_{\nu(G)_{(t)}}(uv)\right) = (d_{\mu(G)}(uv), d_{\nu(G)}(uv)) = d_G(uv). \qquad \square$$

**Theorem 3.4:** Let $G : (V, E)$ be an intuitionistic fuzzy graph such that $\mu_2(uv) = c_1$ and $\nu_2(uv) = c_2, \forall\, uv \in E$, where $c_1$and $c_2$ are constants. Then for any $uv \in E^{(t)}$,

$$d_G{}^{(t)}(uv) = \begin{cases} d_G(uv), if\, c_1 < t \; and \; c_2 > t \\ d_G(uv) - (c-t)d_{G^*}(uv), if\, c_1 \geq t \; and \; c_2 \leq t \end{cases}$$

*Proof:* Let $c_1 < t$,

$$d_{\mu(G)}{}^{(t)}(uv) = d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) > t}} (\mu_2(uw) - t) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) > t}} (\mu_2(wv) - t)$$

$$= d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ c_1 > t}} (c_1 - t) - \sum_{\substack{wv \in E, w \neq u \\ c_1 > t}} (c_1 - t)$$

$\Rightarrow d_{\mu(G)}{}^{(t)}(uv) = d_{\mu(G)}(uv)$. Let $c_2 > t$,

$$d_{\nu(G)}{}^{(t)}(uv) = d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) < t}} (\nu_2(uw) - t) - \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) < t}} (\nu_2(wv) - t)$$

$$= d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ c_2 < t}} (c_2 - t) - \sum_{\substack{wv \in E, w \neq u \\ c_2 < t}} (c_2 - t)$$

$d_{\nu(G)}{}^{(t)}(uv) = d_{\nu(G)}(uv)$, Hence $d_G{}^{(t)}(uv) = d_G(uv)$. Similarly, when $c_1 \geq t$,

$$d_{\mu(G)}{}^{(t)}(uv) = d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \mu_2(uw) > t}} (\mu_2(uw) - t) - \sum_{\substack{wv \in E, w \neq u \\ \mu_2(vw) > t}} (\mu_2(wv) - t)$$

$$= d_{\mu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ c_1 > t}} (\, c_1 - t) - \sum_{\substack{wv \in E, w \neq u \\ c_1 > t}} (\, c_1 - t)$$

$$= d_{\mu(G)}(uv) - (\, c_1 - t)(d_{G^*}(u) - 1) - (\, c_1 - t)(d_{G^*}(v) - 1)$$

$$= d_{\mu(G)}(uv) - (\, c_1 - t)(d_{G^*}(u) + d_{G^*}(v) + 2) = d_{\mu(G)}(uv) - (\, c_1 - t)d_{G^*}(uv).$$

When, $c_2 \leq t$,

$$d_{\nu(G)}{}^{(t)}(uv) = d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ \nu_2(uw) < t}} (\nu_2(uw) - t) - \sum_{\substack{wv \in E, w \neq u \\ \nu_2(vw) < t}} (\nu_2(wv) - t)$$

$$= d_{\nu(G)}(uv) - \sum_{\substack{uw \in E, w \neq v \\ c_2 < t}} (\, c_2 - t) - \sum_{\substack{wv \in E, w \neq u \\ c_2 < t}} (\, c_2 - t)$$

$$= d_{\nu(G)}(uv) - (\, c_2 - t)(d_{G^*}(u) - 1) - (\, c_2 - t)(d_{G^*}(v) - 1)$$

$$= d_{\nu(G)}(uv) - (\, c_2 - t)(d_{G^*}(u) + d_{G^*}(v) - 2) = d_{\nu(G)}(uv) - (\, c_2 - t)d_{G^*}(uv)$$

Hence, $d_G{}^{(t)}(uv) = d_G(uv) - (c - t)d_{G^*}(uv)$. □

## 4 Edge regular property of truncations of intuitionistic fuzzy graph

**Remark 4.1:** Let $G : (V, E)$ is an edge regular intuitionistic fuzzy graph, then $G_{(\mathrm{t})}: (\mu_{(t)}, \nu_{(t)})$ and $G^{(t)}: (\mu^{(t)}, \nu^{(t)})$ need not be edge regular intuitionistic fuzzy graphs. For example in figure 4.1 $G : (\mu, \nu)$ is (1.4, 1.2) edge regular intuitionistic fuzzy graph, but $G_{(\mathrm{t})}: (\mu_{(t)}, \nu_{(t)})$ and $G^{(t)}: (\mu^{(t)}, \nu^{(t)})$, $t$ = (.2, .6) are not edge regular intuitionistic fuzzy graphs.

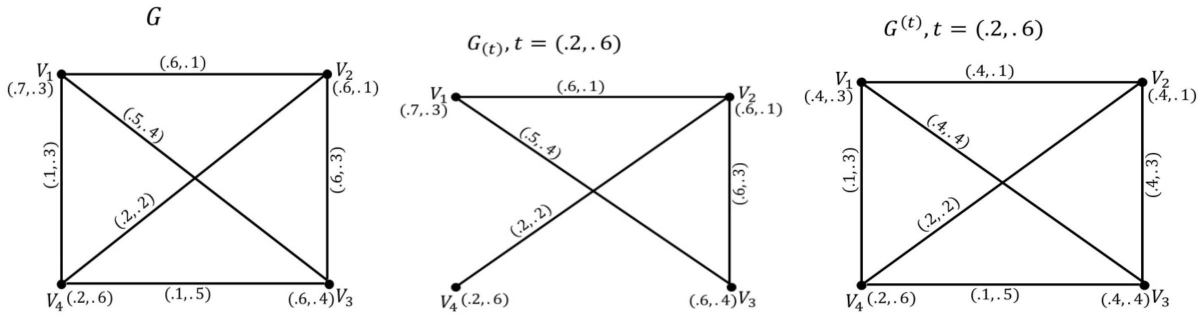


Figure 4.1

**Remark 4.2:** If $G_{(\mathrm{t})}: (\mu_{(t)}, \nu_{(t)})$ and $G^{(t)}: (\mu^{(t)}, \nu^{(t)})$ are edge regular intuitionistic fuzzy graphs, then $G : (\mu, \nu)$ need not be edge regular intuitionistic fuzzy graph. For example in Figure 4.2 $G_{(t)}: (\mu_{(t)}, \nu_{(t)})$, $t$ = (.5, .3) is (.6, .3) edge regular and $G^{(t)}: (\mu^{(t)}, \nu^{(t)})$, $t$ = (.4, .2) is (.8, .4) edge regular, but Let $G : (\mu, \nu)$ is not an edge regular.

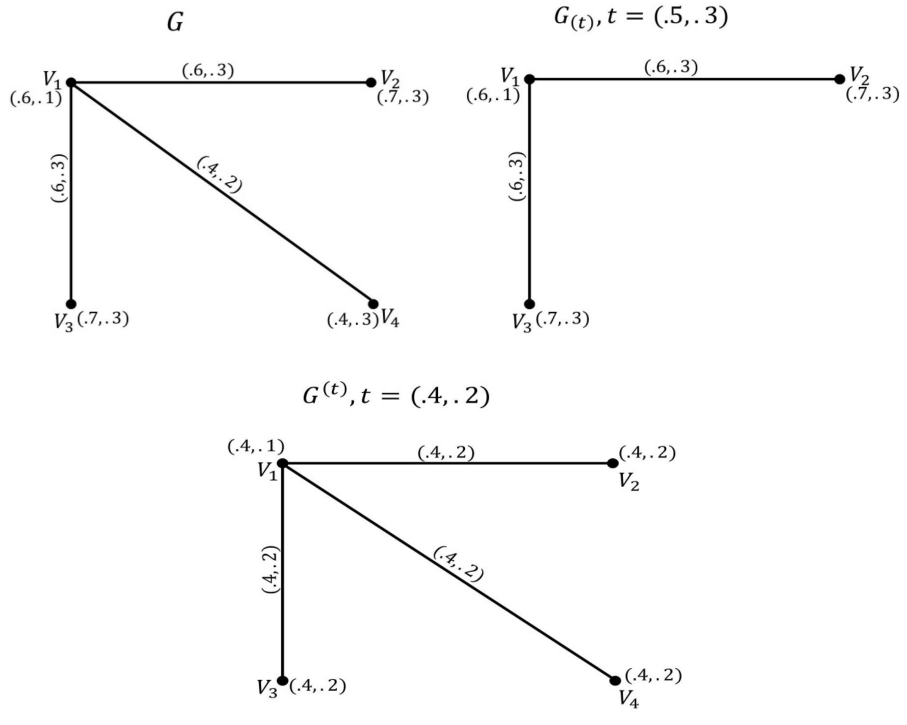


Figure 4.2

In the following Theorems, we obtain some conditions under which $G_{(t)}$ and $G^{(t)}$ are edge regular.

**Theorem 4.3:** Let $G : (V, E)$ be an intuitionistic fuzzy graph such that $\mu_2$ and $\nu_2$ are constant functions with constant values $c_1$ and $c_2$. For every $0 < t \leq c_1 \,\&\, t \geq c_2$, $G_{(\mathrm{t})}$ is edge regular if and only if $G$ is edge regular.

*Proof:* If $0 < t \leq c_1 \,\&\, t \geq c_2$ then $V_{(\mathrm{t})} = V$, because $\mu_1(v) \geq t, \nu_1(v) \leq t, \forall v \in V$ and $E_{(\mathrm{t})} = E$, because $\mu_2(e) \geq t, \nu_2(e) \leq t, \forall e \in E$. From Theorem 3.3,

$d_{\mu(G)}(e) = d_{\mu(G)_{(t)}}(e)$ and $d_{\nu(G)}(e) = d_{\nu(G)_{(t)}}(e), \forall e \in E$

$\Rightarrow d_G(e) = (d_{\mu(G)}(e), d_{\nu(G)}(e)) = (d_{\mu(G)_{(t)}}(e), d_{\nu(G)_{(t)}}(e)) = d_{G_{(t)}}(e), \forall e \in E$

Hence $G$ is edge regular if and only if $G_{(\mathrm{t})}$ is edge regular. □

**Remark 4.4:** If $c_1 < t,\ c_2 > t$, then $G_{(\mathrm{t})}$ is an empty graph.

**Theorem 4.5:** Let $G : (V, E)$ be an intuitionistic fuzzy graph such that $\mu_2$ and $\nu_2$ are constant functions. Then $G : (\mu, \nu)$ is an edge regular intuitionistic fuzzy graph if and only if $G^{(t)}$ is an edge regular intuitionistic fuzzy graph, where $0 < t \leq 1$.

*Proof:* Assume that $G : (V, E)$ is an $(m_1, m_2)-$ edge regular intuitionistic fuzzy graph. Since $\mu_2, \nu_2$ are constant functions, by Theorem (2.9), $G^*: (V, E)$ is an edge regular graph. Let $G^*$ be $k-$edge regular. By Theorem 3.4, When $t > c_1 \,\&\, t < c_2$, $d_{G^{(t)}}(uv) = d_G(uv) = (m_1, m_2)$, $\forall uv \in E^{(t)}$. Therefore $G^{(t)}$ is $(m_1, m_2)-$ edge regular. When $t \leq c_1 \,\&\, t \geq c_2$, $d_{G^{(t)}}(uv) = (m_1, m_2) - (c - t)k\ \forall uv \in E^{(t)}$. Therefore $G^{(t)}$ is $((m_1, m_2) - (c - t)k) -$ edge regular.

Conversely, assume that $G^{(t)}$ is an edge regular for every $0 < t \leq 1$. When $t > c_1 \,\&$ $t < c_2$, by Theorem 3.4 $d_G(uv) = d_G{}^{(t)}(uv), \forall uv \in E^{(t)}$. $\Rightarrow G$ is also edge regular. Let $t \leq c_1 \,\&\, t \geq c_2$.

Then $({\mu_2}^{(t)}, {\nu_2}^{(t)})$ is a constant function of constant value $t$. Therefore by Theorem (2.9), $G^{(t)*}$ is edge regular. Since underlying crisp graphs of $G$ and $G^{(t)}$ are same, $G^*$ is edge regular. From Theorem 3.4 $d_G(uv) = d_G{}^{(t)}(uv) + (c - t)\ d_{G^*}(uv) \forall uv \in E$. Hence $G$ is an edge regular intuitionistic fuzzy graph. □

**Theorem 4.6:** Let $G : (V, E)$ be an intuitionistic fuzzy graph on an odd cycle $G^*$. Then $G$ is edge regular if and only if $\mu_2$ and $\nu_2$ are constant functions.

*Proof:* Let $G$ be a $(k_1, k_2)-$ edge regular intuitionistic fuzzy graph on an odd cycle $v_1v_2, \dots, v_nv_1$. Let $\mu_2(v_1v_2) = s_1$ and $\nu_2(v_1v_2) = s_2$.

Here, $d_\mu(v_2v_3) = \mu_2(v_1v_2) + \mu_2(v_3v_4) = k_1 \Rightarrow s_1 + \mu_2(v_3v_4) = k_1 \Rightarrow \mu_2(v_3v_4) = k_1 - s_1$

$d_\nu(v_2v_3) = \nu_2(v_1v_2) + \nu_2(v_3v_4) = k_2 \Rightarrow s_2 + \nu_2(v_3v_4) = k_2 \Rightarrow \nu_2(v_3v_4) = k_2 - s_2$

Similarly, $\mu_2(v_5v_6) = s_1, \mu_2(v_7v_8) = k_1 - s_1$ and so on, $\nu_2(v_5v_6) = s_2, \nu_2(v_7v_8) = k_2 - s_2$ and so on. Proceeding this way,

$$\mu_2(v_nv_1) = \begin{cases} s_1\ if\ n - 1 \equiv 0(mod\ 4) \\ k_1 - s_1\ if\ n - 1 \not\equiv 0(mod\ 4) \end{cases}$$

and

$$\nu_2(v_nv_1) = \begin{cases} s_2\ if\ n - 1 \equiv 0(mod\ 4) \\ k_2 - s_2\ if\ n - 1 \not\equiv 0(mod\ 4) \end{cases}$$

*Case (i)*: $\mu_2(v_nv_1) = s_1$ and $\nu_2(v_nv_1) = s_2$. Therefore $\mu_2(v_2v_3) = k_1 - s_1, \mu_2(v_4v_5) = s_1$, $\mu_2(v_6v_7) = k_1 - s_1$ and so on. Since $n - 1 \equiv 0(mod\ 4)$, $\mu_2(v_{n-1}v_n) = s_1$.

$$d_\mu(v_nv_1) = k \Rightarrow \mu_2(v_nv_1) + \mu_2(v_1v_2) = k_1 \Rightarrow s_1 + s_1 = k_1 \Rightarrow s_1 = \frac{k_1}{2}.$$

Therefore $k_1 - s_1 = k_1 - \frac{k_1}{2} = \frac{k_1}{2}$. $\mu_2(v_i v_{i+1}) = \frac{k_1}{2}$, $\forall\, i = 1,2,3, \dots n$, where $v_{n+1} = v_1$.
Similarly we get $s_2 = \frac{k_2}{2}$. $\nu_2(v_i v_{i+1}) = \frac{k_2}{2}$, $\forall\, i = 1,2,3, \dots n$, Where $v_{n+1} = v_1$.

*Case* (*ii*): $\mu_2(v_n v_1) = k_1 - s_1$ and $\nu_2(v_n v_1) = k_2 - s_2$.
Proceeding as above, $\mu_2(v_2 v_3) = s_1, \mu_2(v_4 v_5) = k_1 - s_1, \mu_2(v_6 v_7) = s_1$ and so on, $\nu_2(v_2 v_3) = s_2, \nu_2(v_4 v_5) = k_2 - s_2, \nu_2(v_6 v_7) = s_2$ and so on.
Since $n - 1 \not\equiv 0(mod\ 4)$, $\mu_2(v_{n-1} v_n) = s_1 and\ \nu_2(v_{n-1} v_n) = s_2$
Now proceeding as above, $d_\mu(v_n v_1) = k_1 \Rightarrow \mu_2(v_{n-1} v_n) + \mu_2(v_1 v_2) = k_1 \Rightarrow s_1 + s_1 = k_1$ $\Rightarrow s_1 = \frac{k_1}{2}$, $\mu_2(v_i v_{i+1}) = \frac{k_1}{2}, \forall\, i = 1,2, \dots n$, where $v_{n+1} = v_1$. Similarly we get $s_2 = \frac{k_2}{2}$, $\nu_2(v_i v_{i+1}) = \frac{k_2}{2}, \forall\, i = 1,2, \dots n$, where $v_{n+1} = v_1$. Hence in the both cases, $\mu_2$ and $\nu_2$ are constant functions. Conversely, assume that $\mu_2$ and $\nu_2$ are constant functions with constant values $c_1$ and $c_2$ respectively. Then $d_\mu(v_i v_{i+1}) = 2c_1$ and $d_\nu(v_i v_{i+1}) = 2c_2$. Therefore $G$ is a $(2c_1, 2c_2) -$ edge regular. □

**Theorem 4.7:** Let $G : (V, E)$ be an intuitionistic fuzzy graph on an even cycle $G^*$ with $n$ vertices and let $n \not\equiv 0(mod\ 4)$. Then $G$ is an edge regular intuitionistic fuzzy graph if and only if $\mu_2$ and $\nu_2$ are constant functions.
*Proof:* Let $G$ be an intuitionistic fuzzy graph on an even cycle $v_1 v_2, \dots, v_n v_1$, where $n \not\equiv 0(mod\ 4)$. If $\mu_2$ & $\nu_2$ are constant functions with constant values $c_1$ and $c_2$ respectively.
$d_G(v_i v_{i+1}) = (\, d_\mu(v_i v_{i+1}), d_\nu(v_i v_{i+1})) = (2c_1, 2c_2)$.

Therefore $G$ is a $(2c_1, 2c_2) -$ edge regular intuitionistic fuzzy graph.

Conversely, let $G$ is a $(k_1, k_2) -$ edge regular intuitionistic fuzzy graph. Since $n$ is even and $n \not\equiv 0(mod\ 4)$, we have $n - 2 \equiv 0(mod\ 4)$. Therefore the number of edges that lie alternatively from $v_1 v_2$ is $\frac{n}{2} - 1$ (an even number).Let $\mu_2(v_1 v_2) = s_1$ and $\nu_2(v_1 v_2) = s_2$. Then proceeding as above $\mu_2(v_3 v_4) = k_1 - s_1$ and $\nu_2(v_3 v_4) = k_2 - s_2$, $\mu_2(v_5 v_6) = s_1$ and $\nu_2(v_5 v_6) = s_2$ ,…., $\mu_2(v_{n-3} v_{n-2}) = k_1 - s_1$ and $\nu_2(v_{n-3} v_{n-2}) = k_2 - s_2$, $\mu_2(v_{n-1} v_n) = s_1$ and $\nu_2(v_{n-1} v_n) = s_2$.

$$d_\mu(v_n v_1) = \mu_2(v_{n-1} v_n) + \mu_2(v_1 v_2) = k_1 \Rightarrow s_1 + s_1 = k_1 \Rightarrow s_1 = \frac{k_1}{2}.$$

Therefore $k_1 - s_1 = \frac{k_1}{2}$, Similarly $s_2 = \frac{k_2}{2}$ and $k_2 - s_2 = \frac{k_2}{2}$.
$\mu_2(v_1 v_2) = \mu_2(v_3 v_4) = \cdots = \mu_2(v_{n-1} v_n) = \frac{k_1}{2}$, $\nu_2(v_1 v_2) = \nu_2(v_3 v_4) = \cdots = \nu_2(v_{n-1} v_n) = \frac{k_2}{2}$. Similarly if $\mu_2(v_2 v_3) = r_1$ and $\nu_2(v_2 v_3) = r_2$ then proceeding as above, $\mu_2(v_n v_1) = r_1$ and $\nu_2(v_n v_1) = r_2$. $d_\mu(v_1 v_2) = k_1$ and $d_\nu(v_1 v_2) = k_2, \Rightarrow r_1 = \frac{k_1}{2}$ and $r_2 = \frac{k_2}{2}$.
Therefore $\mu_2$ and $\nu_2$ are constant functions. □

**Theorem 4.8:** Let $G : (V, E)$ be an intuitionistic fuzzy graph on an even cycle $G^*$ with $n$ vertices and let $n \equiv 0(mod\ 4)$. Then $G$ is a $(k_1, k_2) -$ edge regular intuitionistic fuzzy graph if and only if $\mu_2$ and $\nu_2$ assumes exactly eight values $r_i, s_i, t_i$ and $l_i$, $i = 1,2$ such that consecutive adjacent edges receives these values in cyclic order with $r_i + t_i = k_i$ and $s_i + l_i = k_i$, $i = 1,2$.
*Proof:* Let $G$ be a $(k_1, k_2) -$ edge regular intuitionistic fuzzy graph on an even cycle $v_1 v_2, \dots, v_n v_1$, where $n \equiv 0(mod\ 4)$. Let $\mu_2(v_1 v_2) = r_1$ and $\nu_2(v_1 v_2) = r_2$.

Since $n \equiv 0(mod\ 4)$, $n - 2$ is an even and $n - 2 \not\equiv 0(mod\ 4)$.

$\Rightarrow \frac{n}{2} - 1$ is an odd number. Therefore the number of edges that lie alternatively from $v_1v_2$ is $\frac{n}{2} - 1$ (an odd number). Now $\mu_2(v_1v_2) = r_1$ and $\mu_2(v_2v_3) = k_1 \Rightarrow \mu_2(v_3v_4) = k_1 - r_1$. Similarly, $\mu_2(v_5v_6) = r_1, \dots, \mu_2(v_{n-1}v_n) = k_1 - r_1$. If $\mu_2(v_2v_3) = s_1$, then proceeding as above $\mu_2(v_4v_5) = k_1 - s_1$, $\mu_2(v_6v_7) = s_1, \dots,$ $\mu_2(v_nv_1) = k_1 - s_1$.

Thus the consecutive adjacent edges of the cycle receives the four membership values $r_1, s_1, k_1 - r_1 and\ k_1 - s_1$in cyclic order. Similarly the consecutive adjacent edges of the cycle receives the four non membership values $r_2, s_2, k_2 - r_2 and\ k_2 - s_2$ in cyclic order. Conversely, assume that $\mu_2$ and $\nu_2$ exactly takes eight values respectively as $r_i, s_i, t_i\ and\ l_i, i = 1,2$ such that consecutive adjacent edges receives these values in cyclic order with $r_i + t_i = k_i$ and $s_i + l_i = k_i, i = 1,2$. $d(e) = (r_1 + t_1, r_2 + t_2)$ (or)$(s_1 + l_1, s_2 + l_2) = (k_1, k_2)$(or)$(k_1, k_2) \Rightarrow d(e) = (k_1, k_2)\ \forall e \in E$.

Therefore $G$ is a $(k_1, k_2) -$ edge regular intuitionistic fuzzy graph. □

**Theorem 4.9:** Let $G : (V, E)$ be an intuitionistic fuzzy graph on a cycle $G^*$ with $n \not\equiv 0 (mod\ 4)$, where $|V| = n$. Then $G : (\mu, \nu)$ is an edge regular intuitionistic fuzzy graph if and only if $G^{(t)}$ is an edge regular intuitionistic fuzzy graph, where $0 < t \leq 1$.

*Proof:* Given $G$ is an intuitionistic fuzzy graph on a cycle $G^*$ with $n \not\equiv 0 (mod\ 4)$. Then by Theorem 4.6 and 4.7, we have $\mu_2$ and $\nu_2$ are constant functions then $G$ is an edge regular intuitionistic fuzzy graph if and only if $G^{(t)}$ is an edge regular intuitionistic fuzzy graph, where $0 < t \leq 1$. □

**Definition 4.10:** The adjacency sequence of a vertex $v$ in an intuitionistic fuzzy graph $G$ is defined as a sequence of both membership and non membership values of edges incident at $v$ arranged in increasing order. It is denoted by $as(v)$.

**Definition 4.11:** The adjacency sequence of an edge $e$ in an intuitionistic fuzzy graph $G$ is defined as a sequence of both membership and non membership values of edges adjacent to $e$ arranged in increasing order. It is denoted by $as(e)$.

**Theorem 4.12:** If all the edges of $G$ have the same adjacency sequence, then all the edges of $G_{(t)}$ have the same adjacency sequence.

*Proof:* Suppose that all the edges of $G$ have the same adjacency sequence, say $(k_1, k_2, \dots, k_n)$ in membership and $(s_1, s_2, \dots, s_n)$in non membership. If $t > k_n$ and $t < s_n$, then there is no edge in $G_{(t)}$. If $t \leq k_1$ and $t \geq s_1$, then adjacency sequence of $e$ is $(k_1, k_2, \dots, k_n)$ in membership and $(s_1, s_2, \dots, s_n)$in non membership for each $e \in E_{(t)}$.

If $k_{i-1} < t \leq k_i$ and $s_{i-1} > t \geq s_i$, then the adjacency sequence of $e$ is $(k_i, k_{i+1}, \dots, k_n)$ in membership and $(s_i, s_{i+1}, \dots, s_n)$ in non-membership for each $e \in E_{(t)}$. □

**Remark 4.13:** Converse of Theorem 4.13 need not be true. For example all the edges in $G_{(t)}, t = (.5, .3)$ have the same adjacency sequence ((.6, .2), (.7, .3)). But in $G$,

$$as(p) = \big((.4, .1), (.6, .2), (.7, .3)\big) \neq \big((.6, .2), (.6, .2), (.7, .3), (.7, .3)\big) = as(t)$$

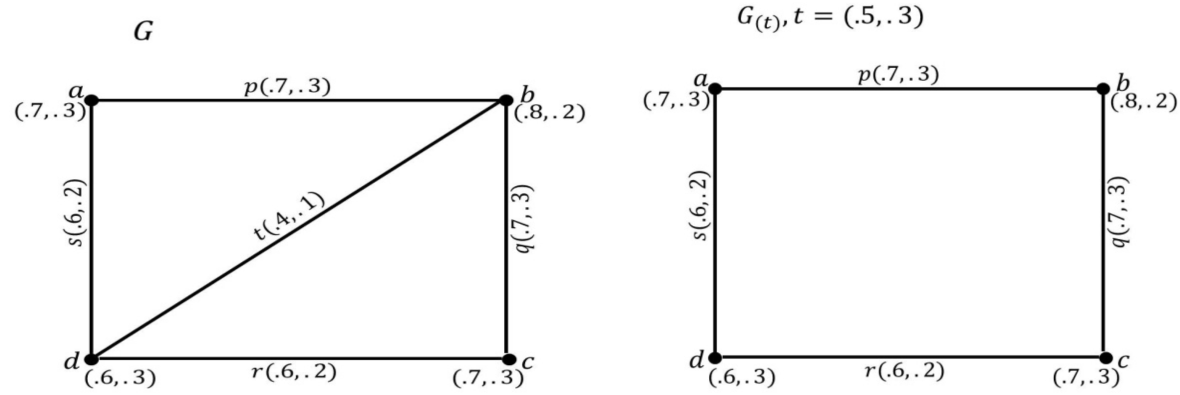


Figure 4.3

**Theorem 4.14:** If all the edges of $G$ have the same adjacency sequence, then all the edges of $G^{(t)}$ have the same adjacency sequence.

*Proof:* Suppose that all the edges of $G$ have the same adjacency sequence, say $(k_1, k_2, \dots, k_n)$ and $(s_1, s_2, \dots, s_n)$in membership and non membership respectively in $G$. If $t > k_n$ and $t < s_n$, then $(k_1, k_2, \dots, k_n)$ and $(s_1, s_2, \dots, s_n)$ are adjacency sequence in membership and non membership for each $e \in E^{(t)}$. If $t \leq k_1$ and $t \geq s_1$, then $(t, t, \dots, t)$ and $(t, t, \dots, t)$ are adjacency sequence in membership and non membership for each $e \in E^{(t)}$.

If $k_{i-1} < t \leq k_i$ and $s_{i-1} > t \geq s_i$, then the adjacency sequence in membership and non-membership is $(k_1, k_2, \dots, k_{i-1}, t, t, \dots, t)$ and $(s_1, s_2, \dots, s_{i-1}, t, t, \dots, t)$ respectively, $\forall e \in E^{(t)}$. □

**Remark 4.15:** Converse of Theorem 4.15 need not be true. For example all the edges in $G^{(t)}, t = (.4, .2)$ have the same adjacency sequence ((.2, .1), (.4, .2), (.4, .2)). But in $G$,

$as(p) = \big((.2, .1), (.5, .2), (.7, .3)\big) \neq \big((.2, .1), (.6, .25), (.7, .3)\big) = as(q)$

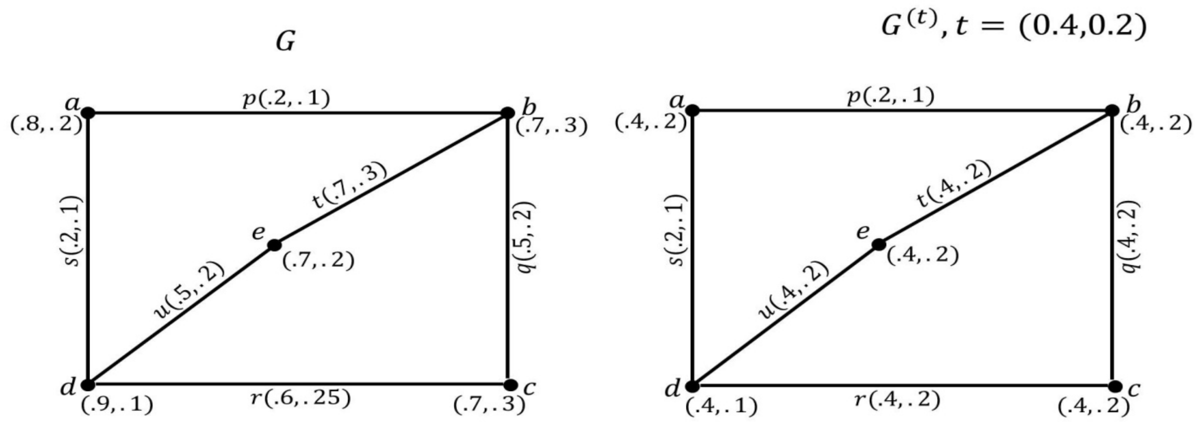


Figure 4.4

**Theorem 4.16:** If all the edges of $G$ have the same adjacency sequence, then $G_{(t)}$ and $G^{(t)}$ are edge regular intuitionistic fuzzy graphs.

*Proof:* When all the edges of $G$ have the same adjacency sequence, the same holds for $G_{(t)}$ and $G^{(t)}$ also. Since the sum of all the elements of the adjacency sequence of an edge is its degree $d_G(e) = ( d_\mu(e),\ d_\nu(e))$ , $G_{(t)}$ and $G^{(t)}$ are edge regular intuitionistic fuzzy graphs. □

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