15th Int. Workshop on Generalized Nets
Burgas, 16 October 2014, 8–16

Intuitionistic fuzzy evaluation of the work of places in generalized nets and generalized nets with characteristics of the places

Velin Andonov

Institute of Biophysics and Biomedical Engineering
Bulgarian Academy of Sciences
105 "Acad. G. Bonchev" Str., 1113 Sofia, Bulgaria,
e-mail: velin_andonov@yahoo.com

Abstract: In two previous papers we have proposed methods for evaluation of tokens in generalized nets. In the ordinary generalized nets these evaluations are based on the characteristics obtained by the tokens during their stay in the net. In generalized nets with characteristics of the places we have proposed evaluation of tokens based on the characteristics of the places. In the present paper we propose two methods for evaluation of the places. In the ordinary generalized nets the evaluation is based on the characteristics obtained by the tokens in the place. In generalized nets with characteristics of the places the evaluation can be based on the characteristics obtained by the places.

Keywords: Evaluation of places, Generalized nets, Intuitionistic fuzzy pairs.
AMS Classification: 68Q85, 03E72.

1 Introduction

In [2, 3] two ways of evaluation of tokens in Generalized Nets (GNs) are proposed. Here we shall briefly mention them along with the notation used there. For arbitrary token α by $\overline{x}^\alpha = \langle x_0^\alpha, x_1^\alpha, ..., x_{fin}^\alpha \rangle$ we denote the vector of all characteristics obtained by the token during its transfer in the net. A way to evaluate the tokens based on their characteristics with respect to a given criterion is proposed in [2]. In the simplest case when all characteristics of the tokens belong to one type, for instance they are all real numbers, let the criterion be such that all characteristics less than a given threshold T are "bad" (i.e. they do not meet the criterion) and all characteristics greater or equal to T are "good" (i.e. they meet the criterion). Using the indicator function

$$I^\alpha(x_i^\alpha) = \begin{cases} 0, & \text{if } x_i^\alpha < T \\ 1, & \text{if } x_i^\alpha \geq T \end{cases} \tag{1}$$

an evaluation of the token α with respect to the criterion can be obtained through the function

$$\mu_\alpha = \frac{\sum\limits_{i=0}^{fin} I^\alpha(x_i^\alpha)}{fin+1}\,. \tag{2}$$

In this special case, μ_α is a fuzzy membership function (see [8]).

In the definition of GN there is no restriction over the types of characteristics that a token can receive. We may have tokens the characteristics of which belong to one type as well as tokens the characteristics of which belong to different types. For instance take the vector with characteristics $\langle 5, 6, \text{“}blue\text{”}, \text{“}green\text{”}, 0, 12\rangle$. For such vector we cannot determine whether the characteristic *“green”* satisfy a cretrion of evaluation that regards numerical characteristics. Therefore, the criterion of evaluation may be such that we have two disjoint sets Δ^α and Ξ^α the first of which is the set of all characteristics that satisfy the criterion while the second is the set of all characteristics that do not satisfy the criterion. In such case, through the indicator functions of these two sets:

$$I_\Delta^\alpha(x_i^\alpha) = \begin{cases} 0, & \text{if } \; x_i^\alpha \notin \Delta^\alpha \\ 1, & \text{if } \; x_i^\alpha \in \Delta^\alpha \end{cases}\,, \tag{3}$$

$$I_\Xi^\alpha(x_i^\alpha) = \begin{cases} 0, & \text{if}\, x_i^\alpha \notin \Xi^\alpha \\ 1, & \text{if}\, x_i^\alpha \in \Xi^\alpha \end{cases}\,, \tag{4}$$

token α can be evaluated with the pair $\langle \mu_\alpha, \nu_\alpha\rangle$ where

$$\mu_\alpha = \frac{\sum\limits_{i=0}^{fin} I_\Delta^\alpha(x_i^\alpha)}{fin+1}\,, \tag{5}$$

$$\nu_\alpha = \frac{\sum\limits_{i=0}^{fin} I_\Xi^\alpha(x_i^\alpha)}{fin+1}. \tag{6}$$

It is easy to see that $\mu_\alpha, \nu_\alpha \in [0, 1]$ and $\mu_\alpha + \nu_\alpha \leq 1$. The pair $\langle \mu_\alpha, \nu_\alpha\rangle$ is an intuitionistic fuzzy pair (see [7]). The number $\pi_\alpha = 1 - \mu_\alpha - \nu_\alpha$ is the degree of indeterminacy of the evaluation. In [3] two sources of indeterminacy are pointed out: indeterminacy due to the criterion; and indeterminacy due to the GN model.

Further, in the same paper, weights for the characteristics are considered. The use of weights allows for the time moments when the characteristics are obtained to be taken into account. One justification for this is that it is natural to think that the newly obtained characteristics are more important for the evaluation than those obtained in the past. In another scenario, the weights can be connected with the places so as to give more importance to particular places.

The tokens in GNs do not preserve all of their characteristics during their stay in the net. Therefore, in order to evaluate the tokens on the basis of all characteristics during the functioning of the net we need to store the characteristics of those tokens which are object of evaluation. In [2] a modification of a given GN model is proposed which allows for the evaluations to be obtained after each transfer of token during the functioning of the net.

Evaluation of tokens in Generalized Nets with Characteristics of the Places (GNCP) based on the characteristics obtained by the places is proposed in [3]. GNCP as defined in [1] is the ordered four-tuple

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle\,, \tag{7}$$

where all components with the exception of the characteristic functions Y and Ψ have the same meaning as in the standard GNs. Here Y assigns initial characteristics to some of the the places and Ψ assigns characteristics to some places when tokens enter them. In the graphical representation of GNCP in Fig. 1 the two concentric circles denote that the places can obtain characteristics.

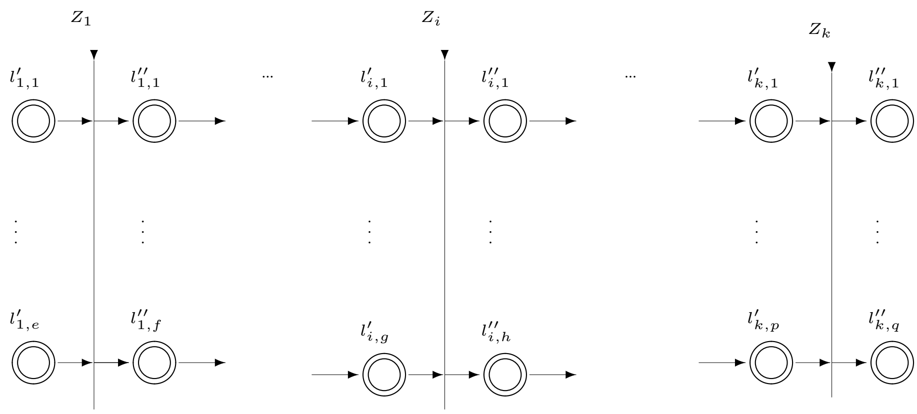

Figure. 1

Let $\langle l_0, l_1, ..., l_k\rangle$ be the places through which token α has consecutively passed and as a result characteristic has been assigned to them. Let $\langle \psi^{\alpha}_{l_0}, \psi^{\alpha}_{l_1}, ..., \psi^{\alpha}_{l_k}\rangle$ be the characteristics obtained by the places upon the entering of the token. Let Δ^{l_j} be the set of all possible "good" characteristics, i.e. that satisfy the criterion for evaluation and Ξ^{l_j} be the set of all "bad" characteristics, i.e. those that do not satisfy the criterion for evaluation of place l_j. Using the indicator functions of these two sets:

$$I^{l_j}_{\Delta}(\psi^{\alpha}_{l_j}) = \begin{cases} 0, & \text{if } \psi^{\alpha}_{l_j} \notin \Delta^{l_j} \\ 1, & \text{if } \psi^{\alpha}_{l_j} \in \Delta^{l_j} \end{cases}\,, \tag{8}$$

$$I^{l_j}_{\Xi}(\psi^{\alpha}_{l_j}) = \begin{cases} 0, & \text{if}\, \psi^{\alpha}_{l_j} \notin \Xi^{l_j} \\ 1, & \text{if}\, \psi^{\alpha}_{l_j} \in \Xi^{l_j} \end{cases}\,, \tag{9}$$

we evaluate the token with the pair $\langle \mu^l_{\alpha}, \nu^l_{\alpha}\rangle$ where

$$\mu^l_{\alpha} = \frac{\sum\limits_{j=0}^{k} I^{l_j}_{\Delta}(\psi^{\alpha}_{l_j})}{k+1}\,, \tag{10}$$

$$\nu^l_{\alpha} = \frac{\sum\limits_{j=0}^{k} I^{l_j}_{\Xi}(\psi^{\alpha}_{l_j})}{k+1}. \tag{11}$$

Further in [3] evaluations with weights are considered. A modification of a given GNCP which allows for the evaluation to be obtained during the functioning of the net is also proposed.

In the present paper, we look at the problem for evaluation in GNs from different angle. We propose evaluation of the places which in ordinary GNs is based on the characteristics of the tokens obtained in the place and it is, in fact, evaluation of the characteristic function Φ which is connected to the place. However, we shall say "evaluation of the place" as usually it is the place that has some physical meaning in the models. One justification for such evaluation to be considered is that the characteristics of the tokens carry information about the place because a given characteristic is obtained not elsewhere but in a particular place. In general the place where a characteristic has been obtained is not included in the characteristic of the token. Thus we need to modify a given GN model so that it can also give evaluation of some of the places.

Operations and relations over transitions and GNs are defined in [4, 5] and many assertions about them are proved. Of these, related to the present paper is the relation $\subset_*$. We shall define it for GNCP. Let

$$E_i = \langle\langle A_i, \pi_A^i, \pi_L^i, c^i, f^i, \theta_1^i, \theta_2^i\rangle, \langle K_i, \pi_K^i, \theta_K^i\rangle, \langle T_i, t_i^0, t_i^*\rangle, \langle X_i, Y_i, \Phi_i, \Psi_i, b_i\rangle\rangle \ .$$

Definition 1. *For every two GNCP E_1 and E_2:*
$E_1 \subset_* E_2$ *iff* $(\forall Z_1 \in A_1)(\exists Z_2 \in A_2)(Z_1 \subset Z_2)\&(\pi_A^1 = \pi_A^2|E_1)\&(\pi_L^1 = \pi_L^2|E_1)\&(c^1 = c^2|E_1)\&(f^1 = f^2|E_1)\&(\theta_1^1 = \theta_1^2|E_1)\&(\theta_2^1 = \theta_2^2|E_1)\&(K_1 \subset K_2)\&(\pi_K^1 = \pi_K^2|E_1)\&(\theta_K^2 = \theta_K^2|E_1)\&(T_2 \leq T_1 \leq T_1 + t_1^* \leq T_2 + t_2^*)\&(t_1^0 = t_2^0)\&(X_1 = X_2|E_1)\&(Y_1 = Y_2|E_1)\&(\Phi_1 = \Phi_2|E_1)\&(\Psi_1 = \Psi_2|E_1)\&(b_1 \leq b_2|E_1)$.

2 Evaluation of places on the basis of the characteristics of the tokens

In analogy to the evaluation of the tokens, we shall discuss two ways to evaluate the behavior of a place — first, on the basis of the characteristics obtained by tokens in this place, and second, on the basis of the characteristics of the place in the sense of GNCP.

Let l be a place of an ordinary GN. We shall consider that it is not input place of the net. Let $\langle \alpha_1, \alpha_2, ..., \alpha_k\rangle$ be the tokens that have passed through l up to the current time moment and $\langle x_l^{\alpha_1}, x_l^{\alpha_2}, ..., x_l^{\alpha_k}\rangle$ be the vector with characteristics assigned to the tokens. We assume that for every token a criterion for evaluation is set such that we have two sets of possible characteristics — Δ^{α_i} and Ξ^{α_i} — which are respectively the set of "good" (those that satisfy the criterion) and "bad" (those that do not satisfy the criterion). Using the indicator functions of these two sets we obtain evaluation of place l in the form of IFP $\langle \mu_l^\alpha, \nu_l^\alpha\rangle$, where

$$\mu_l^\alpha = \frac{\sum\limits_{j=1}^{k} I_\Delta^{\alpha_j}(x_l^{\alpha_j})}{k} \ , \tag{12}$$

$$\nu_l^\alpha = \frac{\sum\limits_{j=1}^{k} I_\Xi^{\alpha_j}(x_l^{\alpha_j})}{k} \ . \tag{13}$$

Similarly to the evaluation of tokens proposed in the previous two sections weights can be used if some of the tokens are more significant than others with respect to the place. The IFP with weights we denote by $\langle \mu_l^{\alpha,w}, \nu_l^{\alpha,w} \rangle$:

$$\mu_l^{\alpha,w} = \frac{\sum_{j=1}^{k} w_j I_{\Delta}^{\alpha_j}(x_l^{\alpha_j})}{k}, \tag{14}$$

$$\nu_l^{\alpha,w} = \frac{\sum_{j=1}^{k} w_j I_{\Xi}^{\alpha_j}(x_l^{\alpha_j})}{k}, \tag{15}$$

where the weights should satisfy the condition $w_j \in [0,1]$, $\forall j \in \{1,2,...,k\}$. If the tokens are equally significant to the place, but we want the newly obtained characteristics to have greater impact on the evaluations then a reasonable choice for the weights will be $w_j = \frac{j}{k}$ for $j = 1,2,...,k$.

As in the case of evaluation of tokens through their characteristics we need to preserve the characteristics of the tokens obtained in the place which is being evaluated. This is required because each token α can keep only the last $b(\alpha)$ characteristics. We shall discuss two possible modifications of a given GN model that would allow for the characteristics to be preserved and the evaluation of the places to be obtained during the functioning of the net.

2.1 Evaluation of the places using additional place

We consider that the place which is to be evaluated is output for a transition Z_i. Without loss of generality we assume that the place is $l''_{i,h}$. Let $Z_i = \langle L'_i, L''_i, t_1^i, t_2^i, r_i, M_i, \square_i \rangle$. We add a new place l_i^* to this transition which is both input and output. The new modified transition is denoted by $Z_i^* = \langle L_i'^*, L_i''^*, t_i^1, t_i^2, r_i^*, M_i^*, \square_i^* \rangle$ (see Fig. 2).

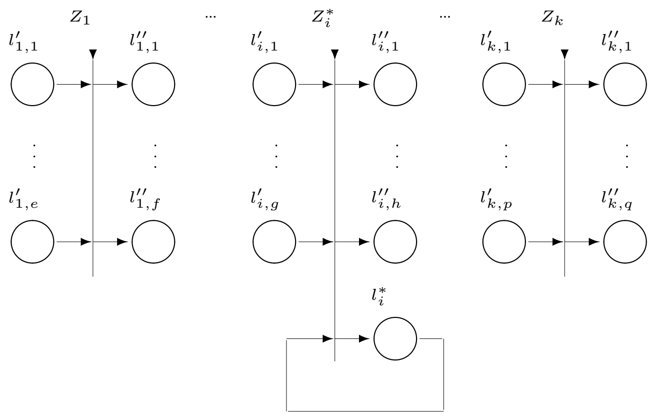

Figure 2. Modified GN model aimed at evaluation of output place of transition Z_i^*, based on the characteristics the tokens.

The components of the modified transition Z_i^* are obtained from the components of Z_i in the following way:

$$L_i'^* = L'_i \cup \{l_i^*\},$$

$$L''^*_i = L''_i \cup \{l^*_i\}.$$

If $r_i = [L'_i, L''_i, r_{l'_{i,s}, l''_{i,t}}]$ is the index matrix of transition's conditions, then

$$r^*_i = [L'^*_i, L''^*_i, \{r^*_{l'_{i,s}, l''_{i,t}}\}] \,,$$

where

$$(\forall l'_{i,s} \in L'_i)(\forall l''_{i,t} \in L''_i)(r^*_{l'_{i,s}, l''_{i,t}} = r_{l'_{i,s}, l''_{i,t}}) \,,$$
$$(\forall l'_{i,s} \in L'_i)(r^*_{l'_{i,s}, l^*_i} = \text{“}false\text{”}) \,,$$
$$(\forall l''_{i,t} \in L''_i)(r^*_{l^*_i, l''_{i,t}} = \text{“}false\text{”}) \,,$$
$$r^*_{l^*_i, l^*_i} = \text{“at least one token has been transferred to place } l''_{i,h}\text{”} \,.$$

If $M_i = [L'_i, L''_i, \{m_{l'_{i,s}, l''_{i,t}}\}]$ is the index matrix with the capacities of the arcs, then

$$r^*_i = [L'^*_i, L''^*_i, \{r^*_{l'_{i,s}, l''_{i,t}}\}] \,,$$

$$M^*_i = [L'^*_i, L''^*_i, \{m^*_{l'_{i,s}, l''_{i,t}}\}] \,,$$

where

$$(\forall l'_{i,s} \in L'_i)(\forall l''_{i,t} \in L''_i)(m^*_{l'_{i,s}, l''_{i,t}} = m_{l'_{i,s}, l''_{i,t}}) \,,$$
$$(\forall l'_{i,s} \in L'_i)(m^*_{l'_{i,s}, l^*_i} = 0) \,,$$
$$(\forall l''_{i,t} \in L''_i)(m^*_{l^*_i, l''_{i,t}} = 0) \,,$$
$$m^*_{l^*_i, l^*_i} = 1 \,.$$
$$\Box^*_i = \Box_i \,.$$

The modified GN we denote by

$$E^* = \langle\langle A^*, \pi^*_A, \pi^*_L, c^*, f^*, \theta^*_1, \theta^*_2\rangle, \langle K^*, \pi^*_K, \theta^*_K\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b^*\rangle\rangle \,,$$

where $A^* = A \setminus \{Z_i\} \cup \{Z^*_i\}$. The priorities of the corresponding transitions in E and E^* are equal, i.e.

$$(\forall Z_j \in A \setminus \{Z_i\})(\pi^*_A(Z_j) = \pi_A(Z_j)) \,,$$
$$\pi^*_A(Z^*_i) = \pi_A(Z_i) \,.$$

All other functions of E^* are defined analogously — they coincide with their corresponding functions of E over the common components for both nets and we shall only describe the differences related to the new place l^*_i. The priority of l^*_i should be the lowest among all input places of Z^*_i, i.e. $\pi^*_L(l^*_i) < \min_{l'_{i,j} \in pr_1 Z_i} \pi_L(l'_{i,j})$. For the capacity we have $c^*(l^*_i) = 1$. In the initial time moment a token α^*_i stays in place l^*_i with initial characteristic the place which is to be evaluated and a list with the tokens and the criterion for evaluation of each of them:

$$\text{“}l''_{i,h}, \langle\alpha_1, \text{criterion for } \alpha_1\rangle, \langle\alpha_2, \text{criterion for } \alpha_2\rangle, ..., \langle\alpha_j, \text{criterion for } \alpha_j\rangle\text{”} \,.$$

The priority of the token α^*_i has no effect on the functioning of the net and it can be chosedn to be the lowest among all other tokens. The time coponents are the same for both nets. If

$\alpha_{i_1}, \alpha_{i_2}, ..., \alpha_{i_k}$ are the tokens which have entered place $l''_{i,h}$ at the current time step, then α_i^* obtains the characteristic

$$\Phi^*_{l_i^*}(\alpha_i^*) = \text{“}\langle \alpha_{i_1}, \Phi_{l''_{i,h}}(\alpha_{i_1})\rangle, \langle \alpha_{i_2}, \Phi_{l''_{i,h}}(\alpha_{i_2})\rangle, ..., \langle \alpha_{i_j}, \Phi_{l''_{i,h}}(\alpha_{i_j})\rangle, \langle \mu^{\alpha}_{l''_{i,h}}, \nu^{\alpha}_{l''_{i,h}}\rangle\text{”}\ ,$$

where the IFP $\langle \mu^{\alpha}_{l''_{i,h}}, \nu^{\alpha}_{l''_{i,h}}\rangle$ is obtained from formulae (12) and (13). If we want to use fixed weights for the evaluation, then these weights should be included in the initial characteristic of α_i^*:

$$\text{“}l''_{i,h}, \langle \alpha_1, \text{criterion for } \alpha_1, w_1\rangle, \langle \alpha_2, \text{criterion for } \alpha_2, w_2\rangle, ..., \langle \alpha_j, \text{criterion for } \alpha_j, w_j\rangle\text{”}\ .$$

In this case the IFP $\langle \mu^{\alpha,w}_{l''_{i,h}}, \nu^{\alpha,w}_{l''_{i,h}}\rangle$ is obtained from formulae (14) and (15).

Finally, token α_i^* keeps all of its characteristics, i.e. $b^*(\alpha_i^*) = \text{“}\infty\text{”}$.

Evaluations of other output places of the same transition can be obtained just by changing the characteristic function $\Phi^*_{l_i^*}$ and the predicate $r^*_{l_i^*, l_i^*}$. Evaluations of output places of other transitions can be obtained by applying the same modification to them. For the original and the modified net we have:

Theorem 1. $E \subset_* E^*$.

2.2 Evaluation of places using the characteristic function of the places Ψ of GNCP

A more convenient way to evaluate a place on the basis of the characteristics assigned to the tokens in it is through the characteristic function Ψ. Let E be a GN and

$$E^* = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, Y, \Phi, \Psi, b\rangle\rangle\ ,$$

be a GNCP obtained from E. All components except the characteristic functions Y and Ψ are the same as in E. The characteristic function Y assigns initial characteristic only to the place which is to be evaluated — $l''_{i,h}$ — in the form

$$Y_{l''_{i,h}} = \text{“}l''_{i,h}, \langle \alpha_1, \text{criterion for } \alpha_1\rangle, \langle \alpha_2, \text{criterion for } \alpha_2\rangle, ..., \langle \alpha_j, \text{criterion for } \alpha_j\rangle\text{”}\ .$$

If $\alpha_{i_1}, \alpha_{i_2}, ..., \alpha_{i_k}$ are the tokens which have entered place $l''_{i,h}$ during the current time step, then $l''_{i,h}$ obtains the characteristic

$$\Psi^*_{l''_{i,h}} = \text{“}\langle \alpha_{i_1}, \Phi_{l''_{i,h}}(\alpha_{i_1})\rangle, \langle \alpha_{i_2}, \Phi_{l''_{i,h}}(\alpha_{i_2})\rangle, ..., \langle \alpha_{i_j}, \Phi_{l''_{i,h}}(\alpha_{i_j})\rangle, \langle \mu^{\alpha}_{l''_{i,h}}, \nu^{\alpha}_{l''_{i,h}}\rangle\ .\text{”}$$

The use of GNCP allows us to evaluate the places without changing the graphical structure of the net. Evaluations of other places can be obtained by extending the characteristic functions Y and Ψ over them in the same way. Again it is clear that the modified net preserves the functioning and results of work of the given net, i.e.

Theorem 2. $E \subset_* E^*$.

3 Evaluation of places in GNCP based on their characteristics

Another approach to the evaluation of places, in the case of GNCP, is based on the characteristics assigned to them through the function Ψ. Let l be the place which we want to evaluate with respect to some criterion and $\langle \psi_0^l, \psi_1^l, ..., \psi_k^l \rangle$ be the vector of characteristics obtained by the place up to the current time moment. Let Δ^l and Ξ^l be respectively the set of all possible "good" characteristics, i.e. those that satisfy the criterion, and all possible "bad" characteristics, i.e. those that do not satisfy the criterion. Using the indicator functions of the two sets:

$$I_\Delta^l(\psi_i^l) = \begin{cases} 0, & \text{if } \psi_i^l \notin \Delta^l \\ 1, & \text{if } \psi_i^l \in \Delta^l \end{cases}, \tag{16}$$

$$I_\Xi^l(\psi_i^l) = \begin{cases} 0, & \text{if } \psi_i^l \notin \Xi^l \\ 1, & \text{if } \psi_i^l \in \Xi^l \end{cases}, \tag{17}$$

we obtain the evaluation of place l with the IFP $\langle \mu^l, \nu^l \rangle$ where

$$\mu^l = \frac{\sum_{j=0}^{k} I_\Delta^l(\psi_j^l)}{k+1}, \tag{18}$$

$$\nu^l = \frac{\sum_{j=0}^{k} I_\Xi^l(\psi_j^l)}{k+1}. \tag{19}$$

As discussed in the previous sections, weights can be used so that greater significance is given to some of the characteristics in comparison to the others:

$$\mu^{l,w} = \frac{\sum_{j=0}^{k} w_j I_\Delta^l(\psi_j^l)}{k+1}, \tag{20}$$

$$\nu^{l,w} = \frac{\sum_{j=0}^{k} w_j I_\Xi^l(\psi_j^l)}{k+1}, \tag{21}$$

where $w_j \in [0,1]$ for $j = 1, 2, ..., k$. A simple way to catch the tendencies in the behavior of the place is to use as weights $w_j = \frac{j+1}{k+1}$ for $j = 0, 1, ..., k$.

The possibility to assign characteristics to the places allows us to obtain evaluations of the places just by modifying the characteristic functions Y and Ψ while all other components of the net and the graphical representation remain the same. Let E be a GNCP (see Figure 1) and the place which we want to evaluate be $l''_{i,h}$. The modified GNCP we denote by

$$E^* = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, Y^*, \Phi, \Psi^*, b \rangle \rangle,$$

where all components except Y^* and Ψ^* remain the same as in the given GNCP E. The difference between the two nets comes from the values of Y^* and Ψ^* for place $l''_{i,h}$. The initial characteristics of the place is given by

$$Y^*_{l''_{i,h}} = \text{“}Y_{l''_{i,h}}, \text{criterion of evaluation”}.$$

When tokens enter place $l''_{i,h}$ the place obtains its new characteristic in the form

$$\Psi^*_{l''_{i,h}} = \text{``}\Psi_{l''_{i,h}}, \langle \mu^l, \nu^l \rangle\,,$$

where $\Psi_{l''_{i,h}}$ is the characteristic of the place in the original GNCP and $\langle \mu^l, \nu^l \rangle$ is the IFP given by (18) and (19) or (20) and (21). Again, it is clear that the modified net preserves the functioning and the results of the work of the given, i.e.

Theorem 3. $E \subset_* E^*$.

4 Conclusion and future work

The evaluation of places in GNs and GNCP proposed here is a logical continuation of the methods for evaluation of tokens suggested in [2, 3]. Now, we have two different approcaches to the problem of the evaluation of tokens and two to the problem of evaluation of places in GNs. This can be very useful if the net is used for control of some process. Simultaneously performed evaluations based on the charatceristics of the tokens and on the characteristics of the places would help for easy detection of problems related to the functioning of the net.

The next step of research in the direction of evaluating the work of GNs is to study possible approaches to the evaluation of transitions based on the characteristics of the tokens in the ordinary GNs and on the characteristics of the places in GNCP. The final step of this research would be to look at the possibilities for aggregation of the evaluations of places and transitions in order to obtain evaluation of the work of the whole net.

References

[1] Andonov, V., K. Atanassov, Generalized nets with characteristics of the places. *Compt. rend. Acad. bulg. Sci.*, Vol. 66, 2013, No. 12, 1673–1680.

[2] Andonov, V., Intuitionistic fuzzy evaluation of tokens in generalized nets based on their characteristics. *Notes on Intuitionistic Fuzzy Sets,* Vol. 20, No. 2, 2014, 109-118.

[3] Andonov, V., A. Shannon, Intuitionistic fuzzy evaluation of the behavior of tokens in generalized nets. *Intelligent Systems'2014. Advances in Intelligent Systems and Computing,* Vol. 322, 2015, 633-644.

[4] Atanassov, K., *Generalized Nets*. World Scientific, Singapore, 1991.

[5] Atanassov, K., *On Generalized Nets Theory*. Prof. M. Drinov Academic Publ. House, Sofia, 2007.

[6] Atanassov, K., *On Intuitionistic Fuzzy Sets Theory*. Springer, Physica-Verlag, Berlin, 2012.

[7] Atanassov, K., E. Szmidt, J. Kacprzyk, On intuitionistic fuzzy pairs. *Notes on Intuitionistic Fuzzy Sets,* Vol. 19, 2013, No. 3, 1–13.

[8] Zadeh, L. A., Fuzzy sets. *Information and Control*, Vol. 8, 1965, 338–353.