Notes on Intuitionistic Fuzzy Sets
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On (*r*, *s*)-connectedness in intuitionistic fuzzy topological spaces

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Abstract: The aim of this paper is to establish the (r, s)-connectedness in intuitionistic fuzzy topological space. Here we give two new notions of (r, s)-connectedness and total (r, s)-connectedness in intuitionistic fuzzy topological space. Also, we find a relation about classical topology and intuitionistic fuzzy topology. Furthermore, using some provisos we will show that (r, s)-connectedness in intuitionistic fuzzy topological spaces are productive and some of its features.
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1 Introduction

The basic concept of a fuzzy set was given by Zadeh [42] in 1965, after then fuzzy topology by Chang [13] in 1968. The generalized concept of intuitionistic fuzzy set was introduced by Atanassov [9] which take into account both the degrees of membership and non-membership subject to the condition that their sum does not exceed 1. Çoker [11, 12, 14–17] and his colleagues introduced intuitionistic fuzzy topological spaces and connectedness in intuitionistic fuzzy topological spaces was introduced by Ozcag and Çoker [32]. M. S. Islam et al. [23, 24], Das [18], Lee et al. [25, 26], Minana et al. [31], R. Srivastava et al. [36, 37], Tiwari et al. [40], Estiaq Ahmed et al. [1–5], Ying-Ming et al. [41], Talukder et al. [38], Fang et al. [19], Hasan et al. [20], R. Islam et al. [22], Ahmad et al. [6], Ali et al. [7, 8], Ramadan et al. [33], Immaculate et al. [21], Mahbub et al. [27–30] and N. X. Tan et al. [39] subsequently developed the study of intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we define two new notions of (r, s)-connectedness and total (r, s)-connectedness in intuitionistic fuzzy topological space and some of their features.

2 Notations and preliminaries

Through this paper, X will be a nonempty set, T is a topology, t is a fuzzy topology, $\mathcal{T}$ is an intuitionistic topology and τ is an intuitionistic fuzzy topology. λ and μ are fuzzy sets, $A = (\mu_A, \nu_A)$ is intuitionistic fuzzy set. Particularly, we denote by $\underline{0}$ and $\underline{1}$ the constant fuzzy sets taking values 0 and 1, respectively.

Definition 2.1 ([13]). *Let X be a nonempty set. A family t of fuzzy sets in X is called a fuzzy topology on X if the following conditions hold.*

(1) $\underline{0}, \underline{1} \in t$,

(2) $\lambda \cap \mu \in t$ for all $\lambda, \mu \in t$,

(3) $\cup \lambda_j \in t$ for any arbitrary family $\{\lambda_j \in t, j \in J\}$.

Definition 2.2 ([14]). *Suppose X is a nonempty set. An intuitionistic fuzzy set A on X is an object having the form $A = (X, A_1, A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A.*

In this paper, we use the simpler notation $A = (A_1, A_2)$ instead of $A = (X, A_1, A_2)$ for an intuitionistic fuzzy set.

Definition 2.3 ([9]). *Let X be a nonempty set. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where μ_A and ν_A are fuzzy sets in X, denoting the degree of membership and the degree of non-membership, respectively, subject to the condition $\mu_A(x) + \nu_A(x) \leq 1$.*

Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ for IFSs.

Definition 2.4 ([9]). *Let X be a nonempty set and IFSs A, B in X be given by $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$, respectively, then:*

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$,

(c) $\bar{A} = (\nu_A, \mu_A)$,

(d) $A \cap B = (\mu_A \cap \mu_B, \nu_A \cup \nu_B)$,

(e) $A \cup B = (\mu_A \cup \mu_B, \nu_A \cap \nu_B)$.

Definition 2.5 ([9]). *Let* $\{A_j = \left(\mu_{A_j}, \nu_{A_j}\right), j \in J\}$ *be an arbitrary family of IFSs in* X. *Then*

(a) $\cap A_j = (\cap \mu_{A_j}, \cup \nu_{A_j})$,

(b) $\cup A_j = (\cup \mu_{A_j}, \cap \nu_{A_j})$,

(c) $0_\sim = (\underline{0}, \underline{1}), 1_\sim = (\underline{1}, \underline{0})$.

Definition 2.6 ([14]). *An intuitionistic fuzzy topology (IFT, in short) on a nonempty set* X *is a family* τ *of IFSs in* X *satisfying the following axioms:*

(1) $0_\sim, 1_\sim \in \tau$,

(2) $A \cap B \in \tau$, *for all* $A, B \in \tau$,

(3) $\cup A_j \in \tau$ *for any arbitrary family* $\{A_j \in \tau, j \in J\}$.

The pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS, in short), members of* τ *are called intuitionistic fuzzy open sets (IFOS, in short).*

Definition 2.7 ([35]). *Let* $A = (\mu_A, \nu_A)$ *be a IFS in* X *and* U *be a nonempty subset of* X. *The restriction of* A *to* U *is a IFS in* U, *denoted by* $A|U$ *and defined by* $A|U = (\mu_A|U, \nu_A|U)$.

Definition 2.8 ([23]). *Let* (X, τ) *be an intuitionistic fuzzy topological space and* U *is a nonempty subset of* X *then* $\tau_U = \{A|U: A \in \tau\}$ *is an intuitionistic fuzzy topology on* U *and* (U, τ_U) *is called subspace of* (X, τ).

Definition 2.9 ([35]). *Let* $\alpha, \beta \in (0,1)$ *and* $\alpha + \beta \leq 1$. *An intuitionistic fuzzy point (IFP, for short)* $p^x_{(\alpha,\beta)}$ *of* X *defined by* $p^x_{(\alpha,\beta)} = \langle x, \mu_p, \nu_p \rangle$,
for $y \in X$

$$\mu_p(y) = \begin{cases} \alpha, & if \quad y = x \\ 0, & if \quad y \neq x \end{cases},$$

and

$$\nu_p(y) = \begin{cases} \beta, & if \quad y = x \\ 1, & if \quad y \neq x \end{cases}.$$

In this case, x *is called the support of* $p^x_{(\alpha,\beta)}$. *An IFP* $p^x_{(\alpha,\beta)}$ *is said to belong to an IFS* $A = \langle x, \mu_A, \nu_A \rangle$ *of* X, *denoted by* $p^x_{(\alpha,\beta)} \in A$, *if* $\alpha \leq \mu_A(x)$ *and* $\beta \geq \nu_A(x)$.

Proposition 2.1 ([3]). *An IFS* A *in* X *is the union of all IFP belonging to* A.

Definition 2.10 ([10]). *Let* $A = (x, \mu_A, \nu_A)$ *and* $B = (y, \mu_B, \nu_B)$ *be IFSs in* X *and* Y, *respectively. Then the product of IFSs* A *and* B *denoted by* $A \times B$ *is defined by* $A \times B = \left\{(x,y), \mu_A \overset{\times}{\underset{\cdot}{}} \mu_B, \nu_A \overset{\cdot}{\underset{\times}{}} \nu_B)\right\}$ *where* $\left(\mu_A \overset{\times}{\underset{\cdot}{}} \mu_B\right)(x,y) = \min\left(\mu_A(x), \mu_B(y)\right)$ *and* $\left(\nu_A \overset{\cdot}{\underset{\times}{}} \nu_B\right)(x,y) = \max\left(\nu_A(x), \nu_B(y)\right)$ *for all* $(x,y) \in X \times Y$. *Obviously,* $0 \leq \left(\mu_A \overset{\times}{\underset{\cdot}{}} \mu_B\right) + \left(\nu_A \overset{\cdot}{\underset{\times}{}} \nu_B\right) \leq 1$. *This definition can be extended to an arbitrary family of IFSs.*

Definition 2.11 ([37]). *Two disjoint non-empty intuitionistic fuzzy subsets* $A = (\mu_A, \nu_A)$ *and* $B = (\mu_B, \nu_B)$ *of an IFTS* (X, τ) *are said to be separated if there exist* $U_i \in \tau$ $(i = 1, 2)$ *such that* $U_1 \supseteq A, U_2 \supseteq B$ *and* $U_1 \cap A = U_2 \cap B = 0_\sim$.

Definition 2.12 ([37]). *Let (X,τ) be an IFTS and A be an IFS in X which is strictly positive, i.e., $A(x) \gg 0_{\sim}$ (i.e., $\mu_A(x) > 0, \nu_A(x) < 1, \forall x \in X$). A pair $U_1, U_2 \in \tau$ is called (C_1)-separation of A if $U_1 \neq A, U_2 \neq A, U_1 \cup U_2 = A$ and $U_1 \cap U_2 = 0_{\sim}$.*

Definition 2.13 ([34]). *A fuzzy topological space X is said to be disconnected if $X = A \cup B$, where A and B are non-empty open fuzzy sets in X such that $A \cap B = \emptyset$. Hence a fuzzy topological space X cannot be represented as the union of two non-empty, disjoint open fuzzy sets on X.*

Definition 2.14 ([27]). *Let (X,τ) be an intuitionistic fuzzy topological space. A family $\{(\mu_{G_i}, \nu_{G_i}): i \in J\}$ of IFOS in X is called open cover of X if $\cup\, \mu_{G_i} = 1$ and $\cap\, \nu_{G_i} = 0$. If every open cover of X has a finite subcover, then X is said to be intuitionistic fuzzy compact (IF-compact, in short).*

Definition 2.15 ([27]). *A family $\{(\mu_{G_i}, \nu_{G_i}): i \in J\}$ of IFOS in X is called (α,β)-level open cover of X if $\cup\, \mu_{G_i} \geq \alpha$ and $\cap\, \nu_{G_i} \leq \beta$ with $\alpha + \beta \leq 1$. If every (α,β)-level open cover of X has a finite subcover, then X is said to be (α,β)-level IF-compact.*

Definition 2.16 ([30]). *An intuitionistic fuzzy subsets $A = (\mu_A, \nu_A)$ of an IFTS X is disconnected if there exists an open intuitionistic fuzzy subsets $G = (\mu_G, \nu_G)$ and $H = (\mu_H, \nu_H)$ of X such that $(A \cap G) \cup (A \cap H) = (1,0)$ and $(A \cap G) \cap (A \cap H) = (0,1)$. In this case, $G \cup H$ is called a disconnection.*

Definition 2.17 ([30]). *An IFTS (X,τ) is T_1-space if $\forall$ IF-singleton $x_{\alpha,\beta}, y_{m,n} \in X$ with $x_{\alpha,\beta} \neq y_{m,n}$, then $\exists A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $x_{\alpha,\beta} \in A, y_{m,n} \notin A$ and $x_{\alpha,\beta} \notin B$, $y_{m,n} \in B$.*

3 (*r*, *s*)-connectedness in intuitionistic fuzzy topological space

In this section, we have introduced (*r*, *s*)-connectedness in intuitionistic fuzzy topological spaces. Furthermore, we have established some theorems and examples of (*r*, *s*)-connectedness in intuitionistic fuzzy topological spaces and discussed different characterizations of (*r*, *s*)-connectedness.

Definition 3.1. *An IFTS (X,τ) is said to be (r,s)-disconnected for $r \in I_0, s \in I_1$ if there exist non-empty open IFSs $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ in X such that $(A \cup B)(x) > (s,r)$ and $(A \cap B)(x) < (r,s), \forall x \in X$.*

Theorem 3.2. *Let (X,τ) is an IFTS. If (X,τ) is IF-connected, then (X,τ) is IF-*(*r*, *s*)-connected. *But converse of the above theorem is not true in general.*

Proof. Let (X,τ) is not IF-connected then there exist non-empty open IFSs $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ in X such that $A \cup B = (1,0)$ and $A \cap B = (0,1)$.

Now, $A \cup B = (1,0)$, i.e., $(\mu_A, \nu_A) \cup (\mu_B, \nu_B) = (1,0)$

$$\Rightarrow \mu_A \cup \mu_B = 1,\ \ \nu_A \cap \nu_B = 0$$
$$\Rightarrow \mu_A \cup \mu_B > s,\ \ \nu_A \cap \nu_B < r,$$

as $r \in I_0 = (0,1], s \in I_1 = [0,1)$

$\Rightarrow A \cup B > (s,r)$

Again, $A \cap B = (0,1)$, i.e.,

$(\mu_A, \nu_A) \cap (\mu_B, \nu_B) = (0,1)$

$\Rightarrow \mu_A \cap \mu_B = 0,\ \nu_A \cup \nu_B = 1$

$\Rightarrow \mu_A \cap \mu_B < r,\ \nu_A \cup \nu_B > s,$

as $r \in I_0 = (0,1], s \in I_1 = [0,1)$

$\Rightarrow A \cap B < (r,s)$

So, (X,τ) is *IF*-(*r*, *s*)-disconnected. Hence if (X,τ) is IF-connected, then (X,τ) is *IF*-(*r*, *s*)-connected.

The second part of the theorem can be prove by an example.

Let τ be an IFT and *A* and *B* are two IFS on *X*, where $A = \{\langle x, (0.8, 0.5), (0.5, 0.3\rangle : x \in X\}$ and $B = \{\langle x, (0.4, 0.2), (0.3, 0.5)\rangle : x \in X\}$, then $A \cup B = \{\langle x, (0.8, 0.2), (0.5, 0.3)\rangle : x \in X\} > (s,r)$ and $A \cap B = \{\langle x, (0.4, 0.5), (0.3, 0.5)\rangle ; x \in X\} < (r,s)$ where $r = 0.8, s = 0.3$. So, (X,τ) is (*r*, *s*)-disconnected. But $A \cup B = \{\langle x, (0.8, 0.2), (0.5, 0.3)\rangle : x \in X\} \neq (1,0)$ and $A \cap B = \{\langle x, (0.4, 0.5), (0.3, 0.5)\rangle : x \in X\} \neq (0,1)$, so (X,τ) is not IF-disconnected. □

Theorem 3.3. *An IFTS* (X,τ) *is IF-*(*r*, *s*)-connected *if and only if there exists no non-empty IFOS A and B in X such that* $A = B^C$.

The proof of the above theorem is obvious.

Theorem 3.4. *The continuous image of an IF-*(r,s)*-connected space X is IF-(r, s)-connected*.

Proof. Let $f: (X,\tau) \to (Y,\delta)$ be a continuous function from an IFTS (X,τ) to (Y,δ). Consider (X,τ) is IF-(*r*, *s*)-connected, we shall prove that (Y,δ) is also *IF*-(*r*, *s*)-connected. Suppose (Y,δ) is not IF-(*r*, *s*)-connected, i.e., (Y,δ) has an (*r*, *s*)-disconnection. Let this be $G = (\mu_G, \nu_G)$ and $H = (\mu_H, \nu_H)$ be two IFS on X, then $G \cup H > (s,r)$, i.e., $\mu_G \cup \mu_H > s$ and $\nu_G \cap \nu_H < r$. Again, $G \cap H < (r,s)$, i.e., $\mu_G \cap \mu_H < r$ and $\nu_G \cup \nu_H > s$.

Now, $f^{-1}(G) = (f^{-1}(\mu_G), f^{-1}(\nu_G))$ and $f^{-1}(H) = (f^{-1}(\mu_H), f^{-1}(\nu_H))$.

So, $f^{-1}(G) \cup f^{-1}(H) = (\max\big(f^{-1}(\mu_G), f^{-1}(\mu_H)\big)(x), \min(f^{-1}(\nu_G), f^{-1}(\nu_H))(x))$

$= \big(\max(f^{-1}(\mu_G)(x), f^{-1}(\mu_H)(x)\big), \min(f^{-1}(\nu_G)(x), f^{-1}(\nu_H)(x)))$

$= (\max(\mu_G(f(x), \mu_H\big(f(x)\big)), \min(\nu_G(f(x), \nu_H(f(x)))$

$= ((\mu_G \cup \mu_H)(f(x)), (\nu_G \cap \nu_H)(f(x)))$

$= (f^{-1}(\mu_G \cup \mu_H)(x), f^{-1}(\nu_G \cap \nu_H)(x))$

$> (s,r)$

Again, $f^{-1}(G) \cap f^{-1}(H) = (\min\big(f^{-1}(\mu_G), f^{-1}(\mu_H)\big)(x), \max(f^{-1}(\nu_G), f^{-1}(\nu_H))(x))$

$= \big(\min(f^{-1}(\mu_G)(x), f^{-1}(\mu_H)(x)\big), \max(f^{-1}(\nu_G)(x), f^{-1}(\nu_H)(x)))$

$= (\min(\mu_G(f(x), \mu_H\big(f(x)\big)), \max(\nu_G(f(x), \nu_H(f(x)))$

$= ((\mu_G \cap \mu_H)(f(x)), (\nu_G \cup \nu_H)(f(x)))$

$= (f^{-1}(\mu_G \cap \mu_H)(x), f^{-1}(\nu_G \cup \nu_H)(x))$

$< (r,s)$

Hence , $f^{-1}(G)$ and $f^{-1}(H)$ give a (r,s)-disconnection for X, which gives the proof. □

Theorem 3.5. *Let* $\{(X_i, \tau_{X_i}), i \in J\}$ *be a family of subspaces of an IFTS* (X, τ) *such that* $\cap X_i \neq \emptyset$, *if* (X_i, τ_{X_i}) *is IF-(r, s)-connected, then* $(\cup X_i, \tau_{\cup X_i})$ *is also IF-(r, s)-connected.*

Proof. Suppose that $(\cup X_i, \tau_{\cup X_i})$ is not IF-(r, s)-connected, so there exist $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau_{\cup X_i}$ such that $A \cup B > (s, r)$ and $A \cap B < (r, s)$.

Now, $A \cup B > (s, r)$

$$\Rightarrow (A \cup B)|X_i > (s, r), \forall X_i \subseteq \cup X_i$$
$$\Rightarrow ((\mu_A, \nu_A) \cup (\mu_B, \nu_B))|X_i > (s, r)$$
$$\Rightarrow (\mu_A \cup \mu_B, \nu_A \cap \nu_B)|X_i > (s, r),$$

which gives, $(\mu_A \cup \mu_B)|X_i > s$ and $(\nu_A \cap \nu_B)|X_i < r$

From $(\mu_A \cup \mu_B)|X_i < s$, we get $(\mu_{A_i}|X_i) \cup (\mu_{B_i}|X_i) > s$ and from $(\nu_A \cap \nu_B)|X_i < r$ we get $(\nu_{A_i}|X_i) \cap (\nu_{B_i}|X_i) < r$, where $(\mu_{A_i}|X_i, \nu_{A_i}|X_i), (\mu_{B_i}|X_i, \nu_{B_i}|X_i) \in \tau_{X_i}$.

Again from $A \cap B < (r, s)$

$$\Rightarrow (A \cap B)|X_i < (r, s), \forall X_i \subseteq \cup X_i$$
$$\Rightarrow ((\mu_A, \nu_A) \cap (\mu_B, \nu_B))|X_i < (r, s)$$
$$\Rightarrow (\mu_A \cap \mu_B, \nu_A \cup \nu_B)|X_i < (r, s),$$

which gives, $(\mu_A \cap \mu_B)|X_i < r$ and $(\nu_A \cup \nu_B)|X_i > s$. Therefore, $(\mu_{A_i}|X_i) \cap (\mu_{B_i}|X_i) < r$ and $(\nu_{A_i}|X_i) \cup (\nu_{B_i}|X_i) > s$.

Hence, (X_i, τ_{X_i}) is not IF-(r, s)-connected. □

Theorem 3.6. *Let* (X, T) *be a topological space and* (X, τ) *be its corresponding IFTS, where* $\tau = \{(1_A, 1_{A^c}) : A \in T\}$. *If* (X, T) *is connected, then* (X, τ) *is IF-(r, s)-connected.*

Proof. Suppose that (X, T) is disconnected, so there exist two nonempty subsets A, B of X such that $A \cup B = X, A \cap B = \emptyset$. Since $A, B \in T$, then $1_A = (1_A, 1_{A^c}) \in \tau$ and $1_B = (1_B, 1_{B^c}) \in \tau$. Now,

$$\begin{aligned} 1_A \cup 1_B &= (1_A, 1_{A^c}) \cup (1_B, 1_{B^c}) \\ &= (1_A \cup 1_B, 1_{A^c} \cap 1_{B^c}) \\ &= (1_{A\cup B}, 1_{A^c \cap B^c}) = (1_{A\cup B}, 1_{(A\cup B)^c}) \\ &= (1_X, 1_\emptyset) = (1, 0) \\ &> (s, r), \end{aligned}$$

as $r \in I_0 = (0, 1], s \in I_1 = [0, 1)$.

Again,

$$\begin{aligned} 1_A \cap 1_B &= (1_A, 1_{A^c}) \cap (1_B, 1_{B^c}) \\ &= (1_A \cap 1_B, 1_{A^c} \cup 1_{B^c}) \\ &= (1_{A\cap B}, 1_{A^c \cup B^c}) = (1_{A\cap B}, 1_{(A\cap B)^c}) \\ &= (1_\emptyset, 1_X) = (0, 1) \\ &< (r, s), \end{aligned}$$

as $r \in I_0 = (0, 1], s \in I_1 = [0, 1)$.

So, (X, τ) is IF-(r, s)-disconnected.

Hence if (X, T) is connected, then (X, τ) is IF-(r, s)-connected. □

Theorem 3.7. *Let $(X, \mathcal{T})$ be an intuitionistic topological space and (X, τ) be its corresponding IFTS, where $\tau = \{(1_{A_1}, 1_{A_2}) : A = (A_1, A_2) \in \mathcal{T}\}$. If $(X, \mathcal{T})$ is connected then (X, τ) is IF-(r, s)-*connected.

Proof. Suppose $(X, \mathcal{T})$ is disconnected, so there exist two nonempty subsets A, B of X such that $A \cup B = (X, \emptyset), A \cap B = (\emptyset, X)$. Since $A, B \in \mathcal{T}$, then $1_A = (1_{A_1}, 1_{A_2}) \in \tau$ and $1_B = (1_{B_1}, 1_{B_2}) \in \tau$.

Here, $A \cup B = (X, \emptyset)$

$$\Rightarrow (A_1, A_2) \cup (B_1, B_2) = (X, \emptyset)$$
$$\Rightarrow (A_1 \cup B_1, A_2 \cap B_2) = (X, \emptyset)$$
$$\Rightarrow A_1 \cup B_1 = X, A_2 \cap B_2 = \emptyset.$$

Again, $A \cap B = (\emptyset, X)$

$$\Rightarrow (A_1, A_2) \cap (B_1, B_2) = (\emptyset, X)$$
$$\Rightarrow (A_1 \cap B_1, A_2 \cup B_2) = (\emptyset, X)$$
$$\Rightarrow A_1 \cap B_1 = \emptyset, A_2 \cup B_2 = X$$

Now,

$$\begin{aligned} 1_A \cup 1_B &= (1_{A_1}, 1_{A_2}) \cup (1_{B_1}, 1_{B_2}) \\ &= (1_{A_1} \cup 1_{B_1}, 1_{A_2} \cap 1_{B_2}) \\ &= (1_{A_1 \cup B_1}, 1_{A_2 \cap B_2}) = (1_X, 1_\emptyset) \\ &= (1, 0) \\ &> (s, r), \end{aligned}$$

as $r \in I_0 = (0, 1], s \in I_1 = [0, 1)$.

Again,

$$\begin{aligned} 1_A \cap 1_B &= (1_{A_1}, 1_{A_2}) \cap (1_{B_1}, 1_{B_2}) \\ &= (1_{A_1} \cap 1_{B_1}, 1_{A_2} \cup 1_{B_2}) \\ &= (1_{A_1 \cap B_1}, 1_{A_2 \cup B_2}) = (1_\emptyset, 1_X) \\ &= (0,1) \\ &> (r, s), \end{aligned}$$

as $r \in I_0 = (0, 1], s \in I_1 = [0, 1)$.

So, (X, τ) is IF-(r, s)-disconnected.

Hence if $(X, \mathcal{T})$ is connected then (X, τ) is IF-(r, s)-connected. □

Theorem 3.9. *If (X, τ) and (Y, δ) are IF-(r, s)-connected space, then $(X \times Y, \tau \times \delta)$ is also IF-(r, s)-connected.*

Proof. Consider $(X \times Y, \tau \times \delta)$ is not IF-(r, s)-connected, then $\exists A, B \in \tau \times \delta$ such that $A \cup B > (s, r)$ and $A \cap B < (r, s)$. Since $A, B \in \tau \times \delta$ then $A = C \times D$ and $B = E \times F$ where $C = (\mu_C, \nu_C), E = (\mu_E, \nu_E) \in \tau$, and $D = (\mu_D, \nu_D), F = (\mu_F, \nu_F) \in \delta$.

Now $C \times D = (\mu_C \overset{\times}{\underset{\cdot}{}} \mu_D, \nu_C \underset{\times}{\overset{\cdot}{}} \nu_D)$, where $\left(\mu_C \overset{\times}{\underset{\cdot}{}} \mu_D\right)(x, y) = \min(\mu_C(x), \mu_D(y))$ and $(\nu_C \underset{\times}{\overset{\cdot}{}} \nu_D)(x, y) = \max(\nu_C(x), \nu_D(y)), \forall (x, y) \in \tau \times \delta$.

Similarly, $E \times F = (\mu_E \overset{\times}{\underset{\cdot}{}} \mu_F, \nu_E \underset{\times}{\overset{\cdot}{}} \nu_F)$.

Now, $A \cup B > (s, r)$

$$\Rightarrow (C \times D) \cup (E \times F) > (s, r)$$

$\Rightarrow \left(\mu_C \overset{\times}{\underset{\cdot}{}} \mu_D, \nu_C \overset{\cdot}{\underset{\times}{}} \nu_D\right) \cup \left(\mu_E \overset{\times}{\underset{\cdot}{}} \mu_F, \nu_E \overset{\cdot}{\underset{\times}{}} \nu_F\right) > (s, r)$

$\Rightarrow$ (min$\big(\mu_C(x), \mu_D(y)\big) \cup$ min$\big(\mu_E(x), \mu_F(y)\big)$, max$\big(\nu_C(x), \nu_D(y)\big) \cap$ max$\big(\nu_E(x), \nu_F(y)\big)$) $> (s, r)$

i.e., min$\big(\mu_C(x), \mu_D(y)\big) \cup$ min$\big(\mu_E(x), \mu_F(y)\big) > s$

$\Rightarrow$ Either, min$\big(\mu_C(x), \mu_D(y)\big) > s$ or, min$\big(\mu_E(x), \mu_F(y)\big) > s$

$\Rightarrow$ Either $\mu_C(x) > s, \mu_D(y) > s$ or, $\mu_E(x) > s, \mu_F(y) > s$

For, max$\big(\nu_C(x), \nu_D(y)\big) \cap$ max$\big(\nu_E(x), \nu_F(y)\big) < r$

$\Rightarrow$ max$\big(\nu_C(x), \nu_D(y)\big) < r$ and max$\big(\nu_E(x), \nu_F(y)\big) < r$

$\Rightarrow \nu_C(x) < r, \nu_D(y) < r, \nu_E(x) < r, \nu_F(y) < r.$

<u>Case I:</u> Suppose $\mu_C(x) > s, \mu_D(y) > s.$

Then $C \cup E = (\mu_C, \nu_C) \cup (\mu_E, \nu_E) = (\mu_C \cup \mu_E, \nu_C \cap \nu_E) > (s, r)$ as $\mu_C(x) > s.$

<u>Case II:</u> Suppose $\mu_E(x) > s, \mu_F(y) > s.$

Then $D \cup F = (\mu_D, \nu_D) \cup (\mu_F, \nu_F) = (\mu_D \cup \mu_F, \nu_D \cap \nu_F) > (s, r)$ as $\mu_F(y) > s.$

Again, $A \cap B < (r, s)$

$\Rightarrow (C \times D) \cap (E \times F) < (r, s)$

$\Rightarrow \left(\mu_C \overset{\times}{\underset{\cdot}{}} \mu_D, \nu_C \overset{\cdot}{\underset{\times}{}} \nu_D\right) \cap \left(\mu_E \overset{\times}{\underset{\cdot}{}} \mu_F, \nu_E \overset{\cdot}{\underset{\times}{}} \nu_F\right) < (r, s)$

$\Rightarrow$ (min$\big(\mu_C(x), \mu_D(y)\big) \cap$ min$\big(\mu_E(x), \mu_F(y)\big)$, max$\big(\nu_C(x), \nu_D(y)\big) \cup$ max$\big(\nu_E(x), \nu_F(y)\big)$) $< (r, s)$,

i.e., min$\big(\mu_C(x), \mu_D(y)\big) \cap$ min$\big(\mu_E(x), \mu_F(y)\big) < r$

$\Rightarrow$ min$\big(\mu_C(x), \mu_D(y)\big) < r$ and min$\big(\mu_E(x), \mu_F(y)\big) < r$

$\Rightarrow$ Either $\mu_C(x) < r$, or $\mu_D(y) < r$ and either $\mu_E(x) < r$ or $\mu_F(y) < r.$

Again, for, max$\big(\nu_C(x), \nu_D(y)\big) \cup$ max$\big(\nu_E(x), \nu_F(y)\big) > s$

$\Rightarrow$ Either max$\big(\nu_C(x), \nu_D(y)\big) > s$ or, max$\big(\nu_E(x), \nu_F(y)\big) > s$

$\Rightarrow$ Either $\nu_C(x) > s$ or $\nu_D(y) > s$, or, either $\nu_E(x) > s$ or $\nu_F(y) > s.$

<u>Case III:</u> Suppose $\mu_C(x) < r$, or $\mu_D(y) < r$ and $\nu_C(x) > s.$

Then $C \cap E = (\mu_C, \nu_C) \cap (\mu_E, \nu_E) = (\mu_C \cap \mu_E, \nu_C \cup \nu_E) < (r, s).$

<u>Case IV:</u> Suppose $\mu_E(x) < r$ or $\mu_F(y) < r$ and $\nu_F(y) > s.$

Then $D \cap F = (\mu_D, \nu_D) \cap (\mu_F, \nu_F) = (\mu_D \cap \mu_F, \nu_D \cup \nu_F) < (r, s).$

So, (X, τ) and (Y, δ) are not (r, s)-connected, hence if (X, τ) and (Y, δ) are IF-(r, s)-connected, then $(X \times Y, \tau \times \delta)$ is IF-(r, s)-connected. □

Theorem 3.10. The product of IF-(r, s)-connected spaces is IF-(r, s)-connected.

Proof. Let (X_i, τ_i) be a collection of IF-(r, s)-connected spaces. Also let $(X, \tau) = (\Pi_i X_i, \Pi_i \tau_i)$ be the product space. Consider $(\Pi_i X_i, \Pi_i, \tau_i)$ are not IF-(r, s)-connected, then there exist $A, B \in \tau_1 \times \tau_2 \times \tau_3 \times \dots$ such that $A \cup B > (s, r)$ and $A \cap B < (r, s)$. Since $A, B \in \tau_1 \times \tau_2 \times \tau_3 \times \dots$, then $A = A_1 \times A_2 \times A_3 \times \dots$ and $B = B_1 \times B_2 \times B_3 \times \dots$, where $A_i = (\mu_{A_i}, \nu_{A_i}) \in \tau$ and $B_i = (\mu_{B_i}, \nu_{B_i}) \in \tau$.

Now, $A \cup B > (s, r)$

$\Rightarrow (A_1 \times A_2 \times A_3 \times \dots) \cup (B_1 \times B_2 \times B_3 \times \dots) > (s, r)$

$\Rightarrow \left((\mu_{A_1}, \nu_{A_1}) \times (\mu_{A_2}, \nu_{A_2}) \times (\mu_{A_3}, \nu_{A_3}) \times \dots\right) \cup$

$\left((\mu_{B_1}, \nu_{B_1}) \times (\mu_{B_2}, \nu_{B_2}) \times (\mu_{B_3}, \nu_{B_3}) \times \dots\right) > (s, r)$

$\Rightarrow (\inf(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \mu_{A_3}(x_3), \dots) \cup \inf(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots),$

$\sup(\nu_{A_1}(x_1), \nu_{A_2}(x_2), \nu_{A_3}(x_3), \dots) \cap \sup(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots)) > (s, r)$, where $(x_1, x_2, x_3, \dots) \in \Pi_i X_i$, i.e., $\inf(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \mu_{A_3}(x_3), \dots) \cup$ $\inf(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots) > s$.

$\Rightarrow$ Either, $\inf(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \mu_{A_3}(x_3), \dots) > s$

or, $\inf(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots) > s$

$\Rightarrow$ Either $\mu_{A_1}(x_1) > s, \mu_{A_2}(x_2) > s, \mu_{A_3}(x_3) > s, \dots$

or, $\mu_{B_1}(x_1) > s, \mu_{B_2}(x_2) > s, \mu_{B_3}(x_3) > s, \dots$

Again, $\sup(\nu_{A_1}(x_1), \nu_{A_2}(x_2), \nu_{A_3}(x_3), \dots) \cap \sup(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots) < r$

$\Rightarrow \sup(\nu_{A_1}(x_1), \nu_{A_2}(x_2), \nu_{A_3}(x_3), \dots) < r$ and $\sup(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots) < r$

$\Rightarrow \nu_{A_1}(x_1) < r, \nu_{A_2}(x_2) < r, \nu_{A_3}(x_3) < r, \dots, \nu_{B_1}(x_1) < r, \nu_{B_2}(x_2) < r, \nu_{B_3}(x_3) < r, \dots$

<u>Case I:</u> Suppose $\mu_{A_1}(x_1) > s, \mu_{B_i}(x_i) > s, \nu_{A_1}(x_1) < r, \nu_{B_i}(x_i) < r$.

Then $A_1 \cup B_i = (\mu_{A_1}, \nu_{A_1}) \cup (\mu_{B_i}, \nu_{B_i}) = (\mu_{A_1} \cup \mu_{B_i}, \nu_{A_1} \cap \nu_{B_i}) > (s, r)$, for any $(\mu_{B_i}, \nu_{B_i}) \in \tau_i$.

Again, $A \cap B < (r, s)$

$\Rightarrow (A_1 \times A_2 \times A_3 \times \dots) \cap (B_1 \times B_2 \times B_3 \times \dots) < (r, s)$

$\Rightarrow \left((\mu_{A_1}, \nu_{A_1}) \times (\mu_{A_2}, \nu_{A_2}) \times (\mu_{A_3}, \nu_{A_3}) \times \dots\right) \cap$

$\left((\mu_{B_1}, \nu_{B_1}) \times (\mu_{B_2}, \nu_{B_2}) \times (\mu_{B_3}, \nu_{B_3}) \times \dots\right) < (r, s)$

$\Rightarrow (\inf(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \mu_{A_3}(x_3), \dots) \cap \inf(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots),$

$\sup(\nu_{A_1}(x_1), \nu_{A_2}(x_2), \nu_{A_3}(x_3), \dots) \cup \sup(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots)) < (r, s)$,

i.e., $\inf(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \mu_{A_3}(x_3), \dots) \cap \inf(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots) < r$.

$\Rightarrow \inf(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \mu_{A_3}(x_3), \dots) < r$ and $\inf(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots) < r$.

For, $\sup\left(\nu_{A_1}(x_1), \nu_{A_2}(x_2), \nu_{A_3}(x_3), \dots\right) \cup \sup\left(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots\right) > s$

$\Rightarrow$ Either $\sup\left(\nu_{A_1}(x_1), \nu_{A_2}(x_2), \nu_{A_3}(x_3), \dots\right) > s$

or, $\sup\left(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots\right) > s$.

Case II: Suppose $\inf\left(\mu_{B_1}(x_1), \mu_{B_2}(x_2), \mu_{B_3}(x_3), \dots\right) < r$, $\sup\left(\nu_{B_1}(x_1), \nu_{B_2}(x_2), \nu_{B_3}(x_3), \dots\right) > s$.

Then, $A_1 \cap B_i = \left(\mu_{A_1}, \nu_{A_1}\right) \cap \left(\mu_{B_i}, \nu_{B_i}\right) = \left(\mu_{A_1} \cap \mu_{B_i}, \nu_{A_1} \cup \nu_{B_i}\right) < (r, s)$.

Since $A_1 \in \tau_1$ and $B_i \in \tau_i$ gives $A_1 \cup B_i > (s, r)$ and $A_1 \cap B_i < (r, s)$, then $A_1 \cup B_1$ is an (r, s)-disconnection of τ_1. Thus, every coordinate space of τ_i is IF-(r, s)-disconnected. Hence, (X_i, τ_i) is a collection of IF-(r, s)-disconnected spaces, which is a contradiction. So, the product of IF-(r, s)-connected space is IF-(r, s)-connected. □

Definition 3.11. *An IFTS (X, τ) is said to be totally IF-(r, s)-disconnected for $r \in I_0, s \in I_1$ if for each pair of IFP $p_{\alpha,\beta}, q_{\rho,\theta} \in X$, there exists a (r, s)-disconnection $G \cup H$ of X with $p_{\alpha,\beta} \in G$ and $q_{\rho,\theta} \in H$, i.e., $G \cup H > (s, r)$ and $G \cap H < (r, s)$.*

Theorem 3.12. The continuous image of a totally IF-(r, s)-disconnected space is totally IF-(r, s)-disconnected.

Proof. Let $f: (X, \tau) \to (Y, \delta)$ be a continuous function from an IFTS (X, τ) to (Y, δ). Consider $x_{\alpha,\beta}, y_{r,s}$ be two IFP in $Y = f(X)$. Since f is continuous $f^{-1}(x_{\alpha,\beta})$ and $f^{-1}(y_{r,s})$ are IFP in X. If (X, τ) is totally IF-(r, s)-disconnected, then there exists an (r, s)-disconnection $G \cup H$ of X where $f^{-1}\left(x_{\alpha,\beta}\right) \in G = (\mu_G, \nu_G)$ and $f^{-1}\left(y_{r,s}\right) \in H = (\mu_H, \nu_H)$. Since $f^{-1}(x_{\alpha,\beta}) \in G \Rightarrow x_{\alpha,\beta} \in f(G)$ and $f^{-1}(y_{r,s}) \in H \Rightarrow y_{r,s} \in f(H)$. Again $G \cup H$ is an (r, s)-disconnection of X such that $G \cup H > (s, r)$ and $G \cap H < (r, s)$.

$$
\begin{aligned}
\text{Here, } G \cup H > (s, r) &\Rightarrow (\mu_G, \nu_G) \cup (\mu_H, \nu_H) > (s, r) \\
&\Rightarrow (\mu_G \cup \mu_H, \nu_G \cap \nu_H) > (s, r) \\
\text{and } \quad G \cap H < (r, s) &\Rightarrow (\mu_G, \nu_G) \cap (\mu_H, \nu_H) < (r, s) \\
&\Rightarrow (\mu_G \cap \mu_H, \nu_G \cup \nu_H) < (r, s).
\end{aligned}
$$

So, $f(G) = (f(\mu_G), f(\nu_G))$ and $f(H) = (f(\mu_H), f(\nu_H))$ gives

$$
\begin{aligned}
f(G) \cup f(H) &= (f(\mu_G), f(\nu_G)) \cup (f(\mu_H), f(\nu_H)) \\
&= (f(\mu_G) \cup f(\mu_H), f(\nu_G) \cap f(\nu_H)) \\
&= ((\mu_G \cup \mu_H)\left(f^{-1}(x)\right), (\nu_G \cap \nu_H)\left(f^{-1}(x)\right)) \\
&> (s, r)
\end{aligned}
$$

$$
\begin{aligned}
\text{and } \; f(G) \cap f(H) &= (f(\mu_G), f(\nu_G)) \cap (f(\mu_H), f(\nu_H)) \\
&= (f(\mu_G) \cap f(\mu_H), f(\nu_G) \cup f(\nu_H)) \\
&= ((\mu_G \cap \mu_H)\left(f^{-1}(x)\right), (\nu_G \cup \nu_H)\left(f^{-1}(x)\right)) \\
&< (r, s).
\end{aligned}
$$

So, $Y = f(X)$ is totally IF-(r, s)-disconnected. □

Theorem 3.13. *Every IF-* T_1 *space is a totally IF-*(r, s)*-disconnected space*.

Proof. Let (X, τ) be an IFTS and also an IF- T_1 space. Consider $x_{\alpha,\beta}, y_{m,n} \in X$ with $x_{\alpha,\beta} \neq y_{m,n}$, then $\exists A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ such that $x_{\alpha,\beta} \in A, y_{m,n} \notin A$ and $x_{\alpha,\beta} \notin B, y_{m,n} \in B$.
Now,

$$x_{\alpha,\beta} \in A = (\mu_A, \nu_A) \Rightarrow \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta$$

$$x_{\alpha,\beta} \notin B = (\mu_B, \nu_B) \Rightarrow \mu_B(x) < \alpha, \nu_B(x) > \beta$$

$$y_{m,n} \notin A = (\mu_A, \nu_A) \Rightarrow \mu_A(y) < m, \nu_A(y) > n$$

$$y_{m,n} \in B = (\mu_B, \nu_B) \Rightarrow \mu_B(y) \geq m, \nu_B(y) \leq n.$$

So,

$$(A \cup B)(x) = \big((\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x)\big) > (\alpha, \beta),$$

$$(A \cap B)(x) = ((\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x)) < (\alpha, \beta)$$

and

$$(A \cup B)(y) = \big((\mu_A \cup \mu_B)(y), (\nu_A \cap \nu_B)(y)\big) > (m, n),$$

$$(A \cap B)(y) = ((\mu_A \cap \mu_B)(y), (\nu_A \cup \nu_B)(y)) < (m, n).$$

This result is true for any $x_{\alpha,\beta}, y_{m,n} \in X$ with $x_{\alpha,\beta} \neq y_{m,n}$. Hence, it is clear that $A \cup B$ is an IF-(r, s)-disconnection of X, so (X, τ) is totally IF-(r, s)-disconnected. □

Theorem 3.14. *Every IF-* T_2 *space is a totally IF-*(r, s)*-disconnected space*.

Proof. Let (X, τ) be an IFTS and also an IF- T_2 space. Consider $x_{\alpha,\beta}, y_{m,n} \in X$ with $x_{\alpha,\beta} \neq y_{m,n}$, then $\exists A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \tau$ with $\mu_A(x_{\alpha,\beta}) = 1, \nu_A(x_{\alpha,\beta}) = 0, \mu_B(y_{m,n}) = 1$, $\nu_B(y_{m,n}) = 0$ and $A \cap B = (0, 1)$.
Now,

$$x_{\alpha,\beta} \in A = (\mu_A, \nu_A) \Rightarrow \mu_A(x_{\alpha,\beta}) = 1, \nu_A(x_{\alpha,\beta}) = 0,$$

$$x_{\alpha,\beta} \notin B = (\mu_B, \nu_B) \Rightarrow \mu_B(x_{\alpha,\beta}) = 0, \nu_B(x_{\alpha,\beta}) = 1,$$

$$y_{m,n} \notin A = (\mu_A, \nu_A) \Rightarrow \mu_A(y_{m,n}) = 0, \nu_A(y_{m,n}) = 1.$$

$$y_{m,n} \in B = (\mu_B, \nu_B) \Rightarrow \mu_B(y_{m,n}) = 1, \nu_B(y_{m,n}) = 0.$$

So,

$$(A \cup B)(x_{\alpha,\beta}) = \big((\mu_A \cup \mu_B)(x_{\alpha,\beta}), (\nu_A \cap \nu_B)(x_{\alpha,\beta})\big) = (1, 0) > (s, r),$$

$$(A \cap B)(x_{\alpha,\beta}) = \big((\mu_A \cap \mu_B)(x_{\alpha,\beta}), (\nu_A \cup \nu_B)(x_{\alpha,\beta})\big) = (0, 1) < (r, s)$$

as $r \in I_0 = (0, 1], s \in I_1 = [0, 1)$, and

$$(A \cup B)(y_{m,n}) = \big((\mu_A \cup \mu_B)(y_{m,n}), (\nu_A \cap \nu_B)(y_{m,n})\big) = (1, 0) > (s, r),$$

$$(A \cap B)(y_{m,n}) = \big((\mu_A \cap \mu_B)(y_{m,n}), (\nu_A \cup \nu_B)(y_{m,n})\big) = (0, 1) < (r, s)$$

as $r \in I_0 = (0, 1], s \in I_1 = [0, 1)$. □

4 Conclusion

The results presented in this paper indicate that many of the basic concepts in general topology can readily to extend to intuitionistic fuzzy topological spaces. Although the theory of intuitionistic fuzzy set is still in embryonic stage, it shows promise of having wide applications.

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