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ON SOME PROPERTIES OF INTUITIONISTIC FUZZY NEGATION $\neg_{@}$

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Abstract

Some properties of the intuitionistic fuzzy negation $\neg_{@}$ related to De Morgan's Laws and Law of Excluding Middle are discussed.

In a series of papers a lot of new implications and negations were defined in the frames of the intuitionistic fuzzy logic (see, e.g., [2]). Some of them were defined by the author in [5]. Here we shall study some strange properties of the new negation.

In intuitionistic fuzzy propositional calculus, if x is a variable then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b\rangle,$$

so that $a, b, a + b \in [0, 1]$, where a and b are degrees of validity and of non-validity of x.

Below we shall assume that for the two variables x and y the equalities: $V(x) = \langle a, b\rangle$, $V(y) = \langle c, d\rangle$ $(a, b, c, d, a + b, c + d \in [0, 1])$ hold.

For the needs of the discussion below, following the definition from [1, 2], we shall define the notion of Intuitionistic Fuzzy Tautology (IFT) by:

$$x \text{ is an IFT, if and only if for } V(x) = \langle a, b\rangle \text{ holds: } a \geq b,$$

while x will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, x is a tautology, if $V(x) = \langle 1, 0\rangle$.

For two variables x and y the operations "conjunction" (&) and "disjunction" ($\vee$) are defined (see [1, 2]) by:

$$V(x\&y) = \langle \min(a, c), \max(b, d)\rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d)\rangle.$$

In the intuitionistic fuzzy sets theory operation @ is defined over two IFSs

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in E\}$$

by:

$$A@B = \{\langle x, (\frac{\mu_A(x) + \mu_B(x))}{2}, \frac{(\nu_A(x) + \nu_B(x))}{2}\rangle | x \in E\}.$$

In [5], we introduced a modification of this operation for the case of intuitionistic fuzzy logic in the form:

$$V(x \rightarrow_{@} y) = \langle \frac{b+c}{2}, \frac{a+d}{2} \rangle.$$

The new implication generates the following negation:

$$V(\neg_{@} x) = \langle \frac{b}{2}, \frac{a+1}{2} \rangle$$

that does not have analogues among the other intuitionistic fuzzy negations.

In [5] there were proved that implication $\rightarrow_{@}$

(a) does not satisfy Modus Ponens in the case of tautology,

(b) satisfies Modus Ponens in the IFT-case.

For the new intuitionistic fuzzy implication and negation none of the following three properties:

Property P1: $A \rightarrow_{@} \neg_{@}\neg_{@} A$,

Property P2: $\neg_{@}\neg_{@} A \rightarrow_{@} A$,

Property P3: $\neg_{@}\neg_{@}\neg_{@} A = \neg_{@} A$

is valid.

Now, the question about the form of expression $\neg_{@}\neg_{@} ... \neg_{@} A$ is interesting.

Let us define:

$$\neg_{@}^{1} A = \neg_{@} A$$

$$\neg_{@}^{n+1} A = \neg_{@} \neg_{@}^{n} A.$$

Let $n \geq 0$ be a natural number. In [5] it is proved that

$$\neg_{@}^{2n+1} \langle a, b \rangle = \langle \frac{b}{2^{2n+1}} + \frac{2}{3} . \frac{4^n - 1}{2^{2n+1}}, \frac{a}{2^{2n+1}} + \frac{1}{3} . \frac{4^{n+1} - 1}{2^{2n+1}} \rangle,$$

$$\neg_{@}^{2n+2} \langle a, b \rangle = \langle \frac{a}{2^{2n+2}} + \frac{1}{3} . \frac{4^{n+1} - 1}{2^{2n+2}}, \frac{b}{2^{2n+2}} + \frac{2}{3} . \frac{4^{n+1} - 1}{2^{2n+2}} \rangle,$$

and

$$\lim_{n \to \infty} \neg_{@}^{n} \langle a, b \rangle = \langle \frac{1}{3}, \frac{2}{3} \rangle.$$

These assertions show that the new implication and negation are non-standard ones. Now we shall give new examples proving their specific nature.

In [4] it is mentioned that De Morgan's Laws have the forms:

$$\neg x \wedge \neg y = \neg (x \vee y), \tag{1}$$

$$\neg x \vee \neg y = \neg (x \wedge y), \tag{2}$$

and

$$\neg(\neg x \vee \neg y) = x \wedge y, \tag{3}$$

$$\neg(\neg x \wedge \neg y) = x \vee y, \tag{4}$$

and it is shown that some negation do not satisfy these equalities.

For the new negation we can prove

Theorem 1: Negation $\neg_{@}$ satisfies equalities (1)-(2), but they do not satisfy equalities (3)-(4).

Proof: Let x and y be given. Then, for (1) we obtain

$$\neg_{@}x \wedge \neg_{@}y = \neg_{@}\langle a, b\rangle \wedge \neg_{@}\langle c, d\rangle$$

$$= \langle \frac{b}{2}, \frac{a+1}{2}\rangle \wedge \langle \frac{d}{2}, \frac{c+1}{2}\rangle$$

$$= \langle \min(\frac{b}{2}, \frac{d}{2}), \max(\frac{a+1}{2}, \frac{c+1}{2})\rangle$$

$$= \langle \frac{\min(b, d)}{2}, \frac{\max(a, c)+1}{2}\rangle$$

$$= \neg_{@}\langle \max(a, c), \min(b, d)\rangle = x \vee y.$$

In [4] the following De Morgan's Laws modification are also discussed

$$\neg(\neg x \vee \neg y) = \neg\neg x \wedge \neg\neg y, \tag{5}$$

$$\neg(\neg x \wedge \neg y) = \neg\neg x \vee \neg\neg y, \tag{6}$$

and it is shown that some negation do not satisfy these equalities.

Theorem 2: Negation $\neg_{@}$ satisfies equalities (5)-(6).

The proof is analogous to the above one.

In [3] the validity of the Law of Excluded Middle is studied in the forms:

$$\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle 1, 0\rangle \tag{7}$$

(tautology-form) and

$$\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle p, q\rangle, \tag{8}$$

and a Modified Law for Excluded Middle in the forms:

$$\neg\neg\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle 1, 0\rangle \tag{9}$$

(tautology-form) and

$$\neg\neg\langle a, b\rangle \vee \neg\langle a, b\rangle = \langle p, q\rangle, \tag{10}$$

(IFT-form), where $1 \geq p \geq q \geq 0$ and it is proved that no one (from the defined in [4]) negation satisfies the Law of Excluded Middle (LEM) in the tautological form (7), some negations satisfy it in the IFT-form (8), some of them satisfy the Modified Law of Excluded Middle in the tautological form (9) and all negations (discussed in [4]) satisfy the Modified Law of Excluded Middle in the IFT-form (10).

Now, we can prove

Theorem 3: Negation $\neg_{@}$ does not satisfy any of the equalities (7)-(10).

Really, for $a = 0.0$ and $b = 0.8$ we obtain that the values of the left sides of (7)-(10) are $\langle 0.4, 0.5\rangle$ that is neither a tautology, nor an IFT.

Finally, we will formulate another assertion, related to the well known equality from the first order logic:

$$x \rightarrow y = \neg x \vee (x \wedge y). \tag{11}$$

Now, for it is valid the following
Theorem 4: Implication $\rightarrow_{@}$ does not satisfy equality (11), neither as a tautology nor as an IFT.

These assertions show that the new implication and negation are non-standard ones. They do not have analogues among the other already defined implications and negations.

References

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