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A study on intuitionistic fuzzy 2-absorbing primary ideals in Γ-ring

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Abstract: In this paper, we initiate the study of a generalization of intuitionistic fuzzy primary ideals in Γ-ring by introducing intuitionistic fuzzy 2-absorbing primary ideals. We investigate the structural characteristics of intuitionistic fuzzy 2-absorbing primary ideals and study their properties.

Keywords: 2-absorbing ideal, 2-absorbing primary ideal, Intuitionistic fuzzy 2-absorbing primary ideal.
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1 Introduction

The concept of a Γ-ring has a special place among generalizations of rings. One of the most interesting examples of a ring would be the endomorphism ring of an Abelian group, i.e., $\mathrm{End}\, M$ or $\mathrm{Hom}(M, M)$ where M is an Abelian group. Now if two Abelian groups, say A and B instead of one are taken, then $\mathrm{Hom}(A, B)$ is no longer a ring in the way as $\mathrm{End}\, M$ becomes a ring because the composition is no longer defined. However, if one takes an element of $\mathrm{Hom}(B, A)$ and put it in between two elements of $\mathrm{Hom}(A, B)$, then the composition can be defined. This served as a motivating factor for introducing and studying the notion of a Γ-ring. The notion of a Γ-ring, a generalization of the concept of associative rings, has been introduced and studied by Nobusawa in [11]. Barnes [4] slightly weakened the conditions in the definition of a Γ-ring in the sense of Nobusawa. The structure of Γ-rings was investigated by several authors such as Barnes in [4], Kyuno in [9,10]. Warsi in [21] studied the decomposition of primary ideal of Γ-rings. Paul in [17] studied various types of ideals of Γ-rings and the corresponding operator rings.

The concept of an intuitionistic fuzzy set as a generalization of Zadeh's fuzzy sets is introduced by Atanassov [1]. Moreover, the notion of intuitionistic fuzzy subring and ideal is presented by Hur, Kang and Song in [6, 7]. Kim, Jun and Ozturk in [8] tried the idea of intuitionistic fuzzy sets to the theory of Γ-rings and developed the notion of intuitionistic fuzzy ideal in Γ-ring. Ramachandran and Palaniappan in [13,15,16,18] studied in detail the properties of intuitionisti fuzzy ideals of Γ-rings. The concept of intuitionistic fuzzy prime ideal in Γ-ring was also innovated by Palaniappan and Ramachandran in [14]. Sharma and Lata in [19] innovate the study of intuitionistic fuzzy characteristic ideals of Γ-ring and its operator ring. The concepts of intuitionistic fuzzy prime radical and intuitionistic fuzzy primary ideal of a Γ-ring was introduced and studied by Sharma et al. in [20].

The concept of a 2-absorbing ideal, which is a generalization of prime ideal, was introduced by Badawi in [2] and which was also studied in [3]. At present, studies on the 2-absorbing ideal theory are progressing rapidly. Elkettani and Kasem [5] unify the notion of 2-absorbing ideal and 2-absorbing primary ideal to δ-2-absorbing primary ideal of Γ-ring and derived many interesting results. Yavuz, Onar, and Ersoy in [12, 22] studied the intuitionistic fuzzy 2-absorbing primary ideal (IF2API) and semi-primary ideal of a commutative ring. The purpose of present paper is to study the structural characteristics of the concept of IF2API of a commutative Γ-ring.

2 Preliminaries

Let us recall some definitions and results, which are necessary for the development of the paper,

Definition 2.1. ([4, 11]) *If $(R, +)$ and $(\Gamma, +)$ are additive Abelian groups. Then R is called a Γ-ring (in the sense of Barnes, [4]) if there exist mapping $R \times \Gamma \times R \to R$ [image of (x, α, y) is denoted by $x\alpha y$, $x, y \in R, \alpha \in \Gamma$] satisfying the following conditions:*

(1) $x\alpha y \in R$.

(2) $(x + y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha + \beta)y = x\alpha y + x\beta y$, $x\alpha(y + z) = x\alpha y + x\alpha z$.

(3) $(x\alpha y)\beta z = x\alpha(y\beta z)$. for all $x, y, z \in M$, and $\alpha, \beta \in \Gamma$.

The Γ-ring R is called commutative if $x\gamma y = y\gamma x, \forall x, y \in R, \gamma \in \Gamma$. An element $1 \in R$ is said to be the unity of R if for each $x \in R$ there exists $\gamma \in \Gamma$ such that $x\gamma 1 = 1\gamma x = x$. A subset I of a Γ-ring R is a left (right) ideal of R if I is an additive subgroup of R and $R\Gamma I = \{x\alpha y | x \in R, \alpha \in \Gamma, y \in I\}$, $(I\Gamma R)$ is contained in I. If I is both a left and a right ideal, then I is a two-sided ideal, or simply an ideal of R. A mapping $f : R \to R'$ of Γ-rings is called a Γ-homomorphism, [4] if $f(x+y) = f(x) + f(y)$ and $f(x\alpha y) = f(x)\alpha f(y)$ for all $x, y \in R, \alpha \in \Gamma$.

Definition 2.2. ([4]) *Let R be a Γ-ring. A proper ideal K of R is called prime if for all pairs of ideals I and J of R, $I\Gamma J \subseteq K$ implies that $I \subseteq K$ or $J \subseteq K$.*

Theorem 2.3. ([10, 17]) *If K is an ideal of a Γ-ring R, the following conditions are equivalent:*

(i) K is a prime ideal of R;

(ii) If $a, b \in R$ and $a\Gamma R\Gamma b \subseteq K$, then $a \in K$ or $b \in K$.

Definition 2.4. ([21]) *Let R be a Γ-ring. Then the radical of an ideal K of R is denoted by $\sqrt{K}$ and is defined as the set*

$$\sqrt{K} = \{x \in R : (x\gamma)^{n-1}x \in K, \textit{ for some } n \in \mathbf{N} \textit{ and for all } \gamma \in \Gamma\ \},$$

where $(x\gamma)^{n-1}x = x$ for $n = 1$.

Definition 2.5. ([4]) *An ideal K of a commutative Γ-ring R is said to be primary if for any two ideals I and J of R, $I\Gamma J \subseteq K$ implies either $I \subseteq K$ or $J \subseteq \sqrt{K}$, where $\sqrt{K}$ is the prime radical of K.*

Definition 2.6. ([5]) *A proper ideal I of a Γ-ring R is called a 2-absorbing ideal (2AI) if whenever $x, y, z \in R, \gamma_1, \gamma_2 \in \Gamma$ and $x\gamma_1 y\gamma_2 z \in I$ imply that $x\gamma_1 y \in I$ or $x\gamma_2 z \in I$ or $y\gamma_2 z \in I$.*

Definition 2.7. ([5]) *A proper ideal I of Γ-ring R is called 2-absorbing primary ideal (2API) of R if whenever $x, y, z \in R, \gamma_1, \gamma_2 \in \Gamma$ and $x\gamma_1 y\gamma_2 z \in I$, then $x\gamma_1 y \in I$ or $x\gamma_2 z \in \sqrt{I}$ or $y\gamma_2 z \in \sqrt{I}$.*

Remark 2.8. *Every 2-absorbing ideal in R is a 2-absorbing primary ideal in R.*

However, the converse of the above remark does not hold.

For example: Consider $R = \mathbb{Z}, \Gamma = 5\mathbb{Z}$. Then R is a Γ-ring. Consider $I = 12\mathbb{Z}$. Take $\gamma_1, \gamma_2 \in \Gamma$ such that $2\gamma_1 2\gamma_2 3 \in I$ implies $2\gamma_1 2 \notin I$, but $2\gamma_2 3 \in \sqrt{I}$. Thus I is a 2API of R, however I is not a 2AI of R, for $2\gamma_1 2\gamma_2 3 \in I$, but $2\gamma_1 2 \notin I$ and $2\gamma_2 3 \notin I$.

We now review some intuitionistic fuzzy logic concepts. We refer the reader to follow [1, 6, 19, 20] for complete details.

Definition 2.9. ([6]) *An intuitionistic fuzzy set (IFS) A in X can be represented as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A, \nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.*

Remark 2.10. ([6])

(i) When $\mu_A(x) + \nu_A(x) = 1, \forall x \in X$. Then A is called a fuzzy set.

(ii) We denote by $IFS(X)$ the set of all IFSs of X.

If $A, B \in IFS(X)$, then $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x), \forall x \in X$ and $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$. For any subset Y of X, the intuitionistic fuzzy characteristic function χ_Y is an intuitionistic fuzzy set of X, defined as $\chi_Y(x) = (1, 0), \forall x \in Y$ and $\chi_Y(x) = (0, 1), \forall x \in X \backslash Y$. Let $p, q \in [0, 1]$ with $p + q \leq 1$. Then the crisp set $A_{(p,q)} = \{x \in X : \mu_A(x) \geq p \text{ and } \nu_A(x) \leq q\}$ is called the (p, q)-level cut subset of A. Also the IFS $x_{(p,q)}$ of X defined as $x_{(p,q)}(y) = (p, q)$, if $y = x$, otherwise $(0, 1)$ is called the intuitionistic fuzzy point (IFP) in X with support x. By $x_{(p,q)} \in A$ we mean $\mu_A(x) \geq p$ and $\nu_A(x) \leq q$. Thus $x_{(p,q)} \in A$ if and only if $x_{(p,q)} \subseteq A$. Further if $f : X \to Y$ is a mapping and A, B are respectively IFSs of X and Y, then the image $f(A)$ is an IFS of Y that is defined as $\mu_{f(A)}(y) = Sup\{\mu_A(x) : f(x) = y\}$, $\nu_{f(A)}(y) = Inf\{\nu_A(x) : f(x) = y\}$, for all $y \in Y$ and the inverse image $f^{-1}(B)$ is an IFS of X that is defined as $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$, $\nu_{f^{-1}(B)}(x) = \nu_B(f(x))$, for all $x \in X$, i.e., $f^{-1}(B)(x) = B(f(x))$, for all $x \in X$. Also the IFS A of X is said to be f-invariant if for any $x, y \in X$, whenever $f(x) = f(y)$ implies $A(x) = A(y)$.

Definition 2.11. ([15]) *Let A and B be two IFSs of a Γ-ring M and $\gamma \in \Gamma$. Then the product $A\Gamma B$ of A and B is defined by*

$$(\mu_{A\Gamma B}(x), \nu_{A\Gamma B}(x)) = \begin{cases} (\vee_{x=y\gamma z}(\mu_A(y) \wedge \mu_B(z)), \wedge_{x=y\gamma z}(\nu_A(y) \vee \nu_B(z)), & \text{if } x = y\gamma z \\ (0, 1), & \text{otherwise} \end{cases}$$

Remark 2.12. ([15]) *If A and B are two IFSs of a Γ-ring R, then $A\Gamma B \subseteq A \cap B$*

Definition 2.13. ([15]) *Let A be an IFS of a Γ-ring R. Then A is called an intuitionistic fuzzy ideal (IFI) of R if for all $x, y \in R, \gamma \in \Gamma$, the following are satisfied*

(i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$;

(ii) $\mu_A(x\gamma y) \geq \mu_A(x) \vee \mu_A(y)$;

(iii) $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$;

(iv) $\nu_A(x\gamma y) \leq \nu_A(x) \wedge \nu_A(y)$.

The set of all intuitionistic fuzzy ideals of a Γ-ring R is denoted by $IFI(R)$. Note that if $A \in IFI(R)$, then $\mu_A(0_R) \geq \mu_A(x)$ and $\nu_A(0_R) \leq \nu_A(x), \forall x \in R$ (see [15]).

Remark 2.14. ([13–15]) *If A, B and C are IFIs of a Γ-ring R, then $A\Gamma B$, $A \cap B$ are also IFIs of R.*

Definition 2.15. ([14]) *Let P be an intuitionistic fuzzy ideal (IFI) of a Γ-ring R. Then P is said to be an intuitionistic fuzzy prime ideal (IFPI) of R if P is non-constant and for any IFIs A, B of R, $A\Gamma B \subseteq P$ imply $A \subseteq P$ or $B \subseteq P$.*

Lemma 2.16. ([14]) *Let A be an IFI of a commutative Γ-ring R. Then the subsequent assertions are equivalent:*

(i) $x_{(p,q)}\Gamma y_{(t,s)} = (x\Gamma y)_{(p\wedge t, q\vee s)}$, *where* $x_{(p,q)}, y_{(t,s)} \in IFP(R)$;

(ii) $\langle x_{(p,q)}\rangle\Gamma\langle y_{(t,s)}\rangle = \langle (x\Gamma y)_{(p\wedge t, q\vee s)}\rangle$, *where* $\langle x_{(p,q)}\rangle$ *is the IFI of* R *generated by* $x_{(p,q)}$.

Theorem 2.17. ([14]) *Let A be an IFI of Γ-ring R. Then the subsequent assertions are equivalent:*

(i) $x_{(p,q)}\Gamma y_{(t,s)} \subseteq A \Rightarrow x_{(p,q)} \subseteq A$ or $y_{(t,s)} \subseteq A$, where $x_{(p,q)}, y_{(t,s)} \in IFP(R)$.

(ii) A is an intuitionistic fuzzy prime ideal of R.

Definition 2.18. ([20]) *Let A be an IFI of a Γ-ring R. Then the IFS $\sqrt{A}$ of R defined by*

$$\sqrt{A} = \cap\{B : B \in IFPI(R); A \subseteq B\}$$

is said to be the intuitionistic fuzzy prime radical of A. Note that $\sqrt{A}$ is also an IFI of R.

Proposition 2.19. ([20]) *Let A, B be two IFIs of a Γ-ring R. Then*

$$\sqrt{A\Gamma B} = \sqrt{A \cap B} = \sqrt{A} \cap \sqrt{B}.$$

Definition 2.20. ([20]) *Let Q be a non-constant IFI of a Γ-ring R. Then Q is termed as an intuitionistic fuzzy primary ideal of R if for any two IFIs A, B of R such that $A\Gamma B \subseteq Q$ implies that either $A \subseteq Q$ or $B \subseteq \sqrt{Q}$.*

Theorem 2.21. ([20]) *Let R be a commutative Γ-ring and Q be an IFI of R. Then for any two IFPs $x_{(p,q)}, y_{(t,s)} \in IFP(R)$ the subsequent assertions are equivalent:*

(i) Q is an intuitionistic fuzzy primary ideal of R;

(ii) $x_{(p,q)}\Gamma y_{(t,s)} \subseteq Q$ implies $x_{(p,q)} \subseteq Q$ or $y_{(t,s)} \subseteq \sqrt{Q}$.

Proposition 2.22. ([20]) *Let Q be an IFI of a Γ-ring R. If Q is an intuitionistic fuzzy primary ideal of R, then for all $x, y \in R, \gamma \in \Gamma$ such that $\mu_Q(x\gamma y) > \mu_Q(x), \nu_Q(x\gamma y) < \nu_Q(x)$ implies that $\mu_Q(x\gamma y) < \mu_{\sqrt{Q}}(y), \nu_Q(x\gamma y) > \nu_{\sqrt{Q}}(y)$.*

Example 2.23. ([20]) *Every IFPI of a Γ-ring R is an IF-primary ideal of R.*

Theorem 2.24. ([20]) *Let f be a homomorphism of a Γ-ring R onto a Γ-ring R'. If A is an IFI of R such that A is constant on $\operatorname{Ker} f$, then $\sqrt{f(A)} = f(\sqrt{A})$.*

Theorem 2.25. ([20]) *Let f be a homomorphism of a Γ-ring R into a Γ-ring R'. If B is an IFI of R , then $\sqrt{f^{-1}(B)} = f^{-1}(\sqrt{B})$.*

3 Intuitionistic fuzzy 2-absorbing primary ideals of a Γ-ring

In this section, we introduce and study intuitionistic fuzzy 2-absorbing primary ideal (IF2API) of a Γ-ring R. Throughout this paper we assume that R is a commutative Γ-ring with unity.

Definition 3.1. *Let Q be a non-constant IFI of a Γ-ring R. Then Q is called an intuitionistic fuzzy 2-absorbing primary ideal of R if for any $IFPs$ $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of R and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ implies that either $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq Q$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$.*

Proposition 3.2. *Every intuitionistic fuzzy primary ideal of a Γ-ring R is an intuitionistic fuzzy 2-absorbing primary ideal of R.*

Proof. The proof is straightforward. $\square$

Theorem 3.3. *Let Q be an IFI of a Γ-ring R. If Q is an IF2API of R, then $Q_{(\alpha,\beta)}$ is a 2API of Γ-ring R for all $\alpha \in [0, \mu_Q(0)]$ and $\beta \in [\nu_Q(0), 1]$ with $\alpha + \beta \leq 1$ and $Q_{(\alpha,\beta)} \neq R$.*

Proof. Let Q be an intuitionistic fuzzy 2-absorbing primary ideal of R and suppose that $x, y, z \in R, \gamma_1, \gamma_2 \in \Gamma$ are such that $x\gamma_1 y\gamma_2 z \in Q_{(\alpha,\beta)}$ for all $\alpha \in [0, \mu_Q(0)]$ and $\beta \in [\nu_Q(0), 1]$ with $\alpha + \beta \leq 1$ and $Q_{(\alpha,\beta)} \neq R$. Then $\mu_Q(x\gamma_1 y\gamma_2 z) \geq \alpha$, $\nu_Q(x\gamma_1 y\gamma_2 z) \leq \beta$ implies $\mu_{(x\gamma_1 y\gamma_2 z)_{(\alpha,\beta)}}(x\gamma_1 y\gamma_2 z) = \alpha \leq \mu_Q(x\gamma_1 y\gamma_2 z)$ and $\nu_{(x\gamma_1 y\gamma_2 z)_{(\alpha,\beta)}}(x\gamma_1 y\gamma_2 z) = \beta \geq \nu_Q(x\gamma_1 y\gamma_2 z)$ and so we have $(x\gamma_1 y\gamma_2 z)_{(\alpha,\beta)} \subseteq Q$, i.e., $x_{(\alpha,\beta)}\gamma_1 y_{(\alpha,\beta)}\gamma_2 z_{(\alpha,\beta)} \subseteq Q$. Since Q is an intuitionistic fuzzy 2-absorbing primary ideal of Γ-ring R, we have $x_{(\alpha,\beta)}\gamma_1 y_{(\alpha,\beta)} \subseteq Q$ or $x_{(\alpha,\beta)}\gamma_2 z_{(\alpha,\beta)} \subseteq \sqrt{Q}$ or $y_{(\alpha,\beta)}\gamma_2 z_{(\alpha,\beta)} \subseteq \sqrt{Q}$, i.e., $(x\gamma_1 y)_{(\alpha,\beta)} \subseteq Q$ or $(x\gamma_2 z)_{(\alpha,\beta)} \subseteq \sqrt{Q}$ or $(y\gamma_2 z)_{(\alpha,\beta)} \subseteq \sqrt{Q}$.

Thus $x\gamma_1 y \in Q_{(\alpha,\beta)}$ or $x\gamma_2 z \in (\sqrt{Q})_{(\alpha,\beta)} = \sqrt{Q_{(\alpha,\beta)}}$ or $y\gamma_2 z \in \sqrt{Q_{(\alpha,\beta)}}$.

Therefore $Q_{(\alpha,\beta)}$ is a 2-absorbing primary ideal of Γ-ring R. $\square$

The next example reveal that the opposite of the theorem is not generally true.

Example 3.4. *Let $R = \mathbb{Z}$ and $\Gamma = 2\mathbb{Z}$, so that R is a Γ-ring. Define the IFI Q of R by*

$$\mu_Q(x) = \begin{cases} 1, & \text{if } x = 0 \\ 1/3, & \text{if } x \in 15\mathbb{Z} - \{0\} \\ 0, & \text{if } x \in \mathbb{Z} - 15\mathbb{Z} \end{cases}; \quad \nu_Q(x) = \begin{cases} 0, & \text{if } x = 0 \\ 1/2, & \text{if } x \in 15\mathbb{Z} - \{0\} \\ 1, & \text{if } x \in \mathbb{Z} - 15\mathbb{Z}. \end{cases}.$$

Since $Q_{(0,1)} = \mathbb{Z}, Q_{(1/3,1/2)} = 15\mathbb{Z}, Q_{(1,0)} = \{0\}$, then we get $Q_{(\alpha,\beta)}$ is a 2-absorbing primary ideal of Γ-ring R. But for $\gamma_1, \gamma_2 \in 2\mathbb{Z}$, we get

$$3_{(1/2,1/3)}\gamma_1 5_{(1/2,1/3)}\gamma_2 1_{(1/3,1/2)} = (3\gamma_1 5\gamma_2 1)_{(1/2\wedge 1/2\wedge 1/3, 1/3\vee 1/3\vee 1/2)} = (3\gamma_1 5\gamma_2 1)_{(1/3,1/2)} \subseteq Q$$

and

$$\mu_{3_{(1/2,1/3)}\gamma_1 5_{(1/2,1/3)}}(3\gamma_1 5) = \mu_{(3\gamma_1 5)_{(1/2,1/3)}}(3\gamma_1 5) = 1/2 > 1/3 = \mu_Q(3\gamma_1 5).$$

Similarly, we get $\nu_{3_{(1/2,1/3)}\gamma_1 5_{(1/2,1/3)}}(3\gamma_1 5) < \nu_Q(3\gamma_1 5)$.
This implies that $3_{(1/2,1/3)}\gamma_1 5_{(1/2,1/3)} \nsubseteq Q$.

$$\mu_{3_{(1/2,1/3)}\gamma_2 1_{(1/3,1/2)}}(3\gamma_2 1) = \mu_{(3\gamma_2 1)_{(1/3,1/2)}}(3\gamma_2 1) = 1/3 > 0 = \mu_{\sqrt{Q}}(3\gamma_2 1).$$

Similarly, $\nu_{3_{(1/2,1/3)}\gamma_2 1_{(1/3,1/2)}}(3\gamma_2 1) < \nu_{\sqrt{Q}}(3\gamma_2 1)$. This implies $3_{(1/2,1/3)}\gamma_2 1_{(1/3,1/2)} \nsubseteq \sqrt{Q}$. In the same way, we can show that $5_{(1/2,1/3)}\gamma_2 1_{(1/3,1/2)} \nsubseteq \sqrt{Q}$. Thus Q is not an intuitionistic fuzzy 2-absorbing primary ideal of Γ-ring R.

Corollary 3.5. *If Q is an intuitionistic fuzzy 2-absorbing primary ideal of Γ-ring R, then*

$$Q_* = \{x \in R : \mu_Q(x) = \mu_Q(0) \text{ and } \nu_Q(x) = \nu_Q(0)\}$$

is a 2-absorbing primary ideal of Γ-ring R.

Proof. Since Q is a non-constant intuitionistic fuzzy ideal of Γ-ring R, then $Q_* \neq R$. Now the result follows from the above theorem. $\square$

In the sequel of the paper, for the sake of simplicity, we denote $x^m = x\gamma_1 x\gamma_2 x \cdots \gamma_{m-1} x$ for some $\gamma_1, \gamma_2, \dots, \gamma_{m-1} \in \Gamma$ and for some $m \in \mathbb{Z}^+$.

Theorem 3.6. *Let J be a 2-absorbing primary ideal of Γ-ring R. Then the intuitionistic fuzzy characteristic function χ_J w.r.t. J defined by*

$$\mu_{\chi_J}(x) = \begin{cases} 1, & \text{if } x \in J \\ 0, & \text{otherwise} \end{cases}, \quad \nu_{\chi_J}(x) = \begin{cases} 0, & \text{if } x \in J \\ 1, & \text{otherwise} \end{cases}$$

is an IF2API of Γ-ring R.

Proof. We have $J \neq R$ and so $Q = \chi_J$ is non-constant because J is a 2-absorbing primary ideal of R. Assume that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$, but $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq Q$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \nsubseteq \sqrt{Q}$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \nsubseteq \sqrt{Q}$, where $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ are $IFPs$ of R and $\gamma_1, \gamma_2 \in \Gamma$.

Then $\mu_Q(x\gamma_1 y) < p \wedge t$, $\nu_Q(x\gamma_1 y) > q \vee s$, and $\mu_Q\{(x\gamma_2 z)^m\} < \mu_{\sqrt{Q}}(x\gamma_2 z) = p \wedge u$, $\nu_Q\{(x\gamma_2 z)^m\} > \nu_{\sqrt{Q}}(x\gamma_2 z) = q \vee v$, and $\mu_Q\{(y\gamma_2 z)^m\} < \mu_{\sqrt{Q}}(y\gamma_2 z) = t \wedge u$, $\nu_Q\{(y\gamma_2 z)^m\} > \mu_{\sqrt{Q}}(y\gamma_2 z) = s \vee v$ for all $m \in \mathbb{Z}$. Hence $\mu_Q(x\gamma_1 y) = 0, \nu_Q(x\gamma_1 y) = 1$ and so $x\gamma_1 y \notin J$; $\mu_Q\{(x\gamma_2 z)^m\} = 0$, $\nu_Q\{(x\gamma_2 z)^m\} = 1$ and so $(x\gamma_2 z)^m \notin Q$ implies that $x\gamma_2 z \notin \sqrt{Q}$; $\mu_Q\{(y\gamma_2 z)^m\} = 0, \nu_Q\{(y\gamma_2 z)^m\} = 1$ and so $(y\gamma_2 z)^m \notin Q$ implies that $y\gamma_2 z \notin \sqrt{Q}$.

Since J is a 2-absorbing ideal of R, we have $x\gamma_1 y\gamma_2 z \notin J$ and so $\mu_Q(x\gamma_1 y\gamma_2 z) = 0$, $\nu_Q(x\gamma_1 y\gamma_2 z) = 1$ for all $x, y, z \in R$ and $\gamma_1, \gamma_2 \in \Gamma$.

By our hypothesis, we have $(x\gamma_1 y\gamma_2 z)_{(p\wedge t\wedge u, q\vee s\vee v)} = x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ and $p \wedge t \wedge u < \mu_Q(x\gamma_1 y\gamma_2 z) = 0$, $q \vee s \vee v > \nu_Q(x\gamma_1 y\gamma_2 z) = 1$. Hence $p \vee t = 0, q \vee s = 1$ or $p \vee u = 0, q \vee v = 1$ or $t \vee u = 0, s \vee v = 1$, which is a contradiction. Hence $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq Q$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$ and Q is an intuitionistic fuzzy 2-absorbing primary ideal of Γ-ring R. □

Definition 3.7. *Let Q be a non-constant IFI of a Γ-ring R. Then Q is called an intuitionistic fuzzy 2-absorbing ideal (IF2AI) of R if for any $IFPs$ $x_{(p,q)}, y_{(t,s)}, z_{(u,v)}$ of R and $\gamma_1, \gamma_2 \in \Gamma$ such that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ implies that either $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq Q$ or $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq Q$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$.*

Theorem 3.8. *Every IF2AI of Γ-ring R is an IF2API of R.*

Proof. The proof is straightforward. □

The next example reveal that the opposite of the above theorem is not true.

Example 3.9. *Let $R = \mathbb{Z}$ and $\Gamma = 5\mathbb{Z}$, so R is a Γ-ring. Let $Q = \chi_{12\mathbb{Z}}$. Then Q is an IFI of Γ-ring R. It can be easily verified that Q is an IF2API of R, but it is not an IF2AI of R for $\gamma_1, \gamma_2 \in \Gamma$ such that $2_{(p,q)}\gamma_1 2_{(t,s)}\gamma_2 3_{(u,v)} = (2\gamma_1 2\gamma_2 3)_{(p\wedge t\wedge u, q\vee s\vee v)} \subseteq Q$ implies $2_{(p,q)}\gamma_1 2_{(t,s)} = (2\gamma_1 2)_{(p\wedge t, q\vee s)} \nsubseteq Q$, $2_{(p,q)}\gamma_2 3_{(u,v)} = (2\gamma_2 3)_{(p\wedge u, q\vee v)} \nsubseteq Q$, $2_{(t,s)}\gamma_2 3_{(u,v)} = (2\gamma_2 3)_{(t\wedge u, s\vee v)} \nsubseteq Q$.*

Proposition 3.10. *If Q is an IF2API of Γ-ring R, then $\sqrt{Q}$ is an IF2AI of R.*

Proof. Suppose that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$ and $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq \sqrt{Q}$, where $x_{(p,q)}$, $y_{(t,s)}$, $z_{(u,v)} \in IFPs(R)$ and $\gamma_1, \gamma_2 \in \Gamma$.

Since $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} = (x\gamma_1 y\gamma_2 z)_{(p\wedge t\wedge u, q\vee s\vee v)} \subseteq \sqrt{Q} \Rightarrow \mu_{\sqrt{Q}}(x\gamma_1 y\gamma_2 z) \geq p \wedge t \wedge u$ and $\nu_{\sqrt{Q}}(x\gamma_1 y\gamma_2 z) \leq q \vee s \vee v$.

From the definition of $\sqrt{Q}$, we have $\mu_{\sqrt{Q}}(x\gamma_1 y\gamma_2 z) = \mathrm{Inf}\{\mu_Q((x\gamma_1 y\gamma_2 z)^m) : m \in \mathbb{N}\} \geq \mathrm{Inf}\{\mu_Q(x^m\gamma_3 y^m\gamma_4 z^m) : m \in \mathbb{N}\} \geq p \wedge t \wedge u$, for some $\gamma_3, \gamma_4 \in \Gamma$. Similarly, we can show that $\nu_{\sqrt{Q}}(x\gamma_1 y\gamma_2 z) \leq q \vee s \vee v$.

Then there exists $k \in \mathbb{Z}^+$ such that for some $\gamma_1', \gamma_2' \in \Gamma$, $\mu_Q((x\gamma_1 y\gamma_2 z)^k) \geq \mu_Q(x\gamma_1 y\gamma_2 z) \geq p \wedge t \wedge u$ and $\nu_Q((x\gamma_1 y\gamma_2 z)^k) \leq \nu_Q(x\gamma_1 y\gamma_2 z) \leq q \vee s \vee v$. This implies that $(x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)})^k \in Q$. If $x_{(p,q)}\gamma_1 y_{(t,s)} \notin \sqrt{Q}$, then for all $k \in \mathbb{Z}^+$ and for some $\gamma \in \Gamma$, we have $\mu_Q(x_{(p,q)}\gamma_1 y_{(t,s)})^k \geq \mu_Q(x^k_{(p,q)}\gamma y^k_{(t,s)})$ and $\nu_Q(x_{(p,q)}\gamma_1 y_{(t,s)})^k \leq \nu_Q(x^k_{(p,q)}\gamma y^k_{(t,s)})$ implies that $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq \sqrt{Q}$. Since Q is an IF2API of R, then $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$. Hence $\sqrt{Q}$ is an IF2AI of R. □

Definition 3.11. *Let Q be an IF2API of Γ-ring R and $P = \sqrt{Q}$ which is an IF2AI of R. Then Q is called an intuitionistic fuzzy P-2-absorbing primary ideal (IFP2API)of R.*

Theorem 3.12. *Let $Q_1, Q_2, \ldots, Q_n$ be IFP2APIs of Γ-ring R for some IF2AI P of R. Then $Q = \bigcap_{i=1}^n Q_i$ is an IFP2API of R.*

Proof. Assume that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ and $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq Q$, for any $x_{(p,q)}$, $y_{(t,s)}$, $z_{(u,v)} \in IFP(R)$ and $\gamma_1, \gamma_2 \in \Gamma$. Then $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq Q_j$, for some $j \in \{1, 2, \ldots, n\}$ and $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q_j$, for all $j \in \{1, 2, \ldots, n\}$. Since Q_j is an IFP2API of R, we have

$$y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q_j} = P = \bigcap_{i=1}^{n} \sqrt{Q_i} = \sqrt{\bigcap_{i=1}^{n} Q_i} = \sqrt{Q}$$

or

$$x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q_j} = P = \bigcap_{i=1}^{n} \sqrt{Q_i} = \sqrt{\bigcap_{i=1}^{n} Q_i} = \sqrt{Q}.$$

Thus Q is an IFP2API of R. □

In the next example, we convey that if Q_1, Q_2 are two IF2APIs of a Γ-ring R, then $Q_1 \cap Q_2$ need not to be an IF2API of R.

Example 3.13. *Let $R = \mathbb{Z}$ and $\Gamma = p\mathbb{Z}$, where $p > 5$ is a prime integer. So that R is a Γ-ring. Take $Q_1 = \chi_{50\mathbb{Z}}, Q_2 = \chi_{75\mathbb{Z}}$. Clearly Q_1, Q_2 are IF2APIs of R. But $Q_1 \cap Q_2 = \chi_{150\mathbb{Z}}$ and as such $\sqrt{Q_1 \cap Q_2} = \chi_{30\mathbb{Z}}$, then for $\gamma_1, \gamma_2 \in \Gamma$ such that $25_{(p,q)}\gamma_1 3_{(t,s)}\gamma_2 2_{(u,v)} \subseteq Q_1 \cap Q_2$, but $25_{(p,q)}\gamma_1 3_{(t,s)} \nsubseteq Q_1 \cap Q_2$, $25_{(p,q)}\gamma_2 2_{(u,v)} \nsubseteq \sqrt{Q_1 \cap Q_2}$ and $3_{(t,s)}\gamma_2 2_{(u,v)} \nsubseteq \sqrt{Q_1 \cap Q_2}$. Therefore, $Q_1 \cap Q_2$ is not an IF2API of R.*

Theorem 3.14. *Let Q be an IFI of a Γ-ring R. If $\sqrt{Q}$ is an IFPI of R, then Q is an IF2API of R.*

Proof. Suppose that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq Q$ and $x_{(p,q)}\gamma_1 y_{(t,s)} \nsubseteq Q$, for any $x_{(p,q)}, y_{(t,s)}$, $z_{(u,v)} \in IFP(R)$ and $\gamma_1, \gamma_2 \in \Gamma$.

Since $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \in Q$ and R is commutative Γ-ring R, we have

$$x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)}\gamma_2 z_{(u,v)} = (x_{(p,q)}\gamma_1 z_{(u,v)})\gamma_2 (y_{(t,s)}\gamma_2 z_{(u,v)}) \subseteq Q \subseteq \sqrt{Q}.$$

Thus $x_{(p,q)}\gamma_1 z_{(u,v)} \subseteq \sqrt{Q}$ or $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq \sqrt{Q}$. Since $\sqrt{Q}$ is an IFPI of R. Therefore we conclude that Q is an IF2API of R. □

4 Homomorphic behaviour of intuitionistic fuzzy 2-absorbing primary ideals

In this section we shall discuss the behaviour of IF2APIs of Γ-ring under Γ-ring homomorphism.

Theorem 4.1. *Let $f : R \to R^{'}$ be a surjective Γ-homomorphism. If Q is an IF2API of R which is constant on* $\operatorname{Ker} f$*, then $f(Q)$ is an IF2API of $R^{'}$.*

Proof. Suppose that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} = (x\gamma_1 y\gamma_2 z)_{(p\wedge t\wedge u, q\vee s\vee v)} \subseteq f(Q)$, where $x_{(p,q)}$, $y_{(t,s)}$, $z_{(u,v)} \in IFP(R^{'})$ and $\gamma_1, \gamma_2 \in \Gamma$. Since f is a surjective Γ-homomorphism, then there exist $a, b, c \in R$ such that $f(a) = x, f(b) = y, f(c) = z$. Thus

$$\begin{aligned}
\mu_{a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)}}(a\gamma_1 b\gamma_2 c) &= \mu_{(a\gamma_1 b\gamma_2 c)_{(p\wedge t\wedge u, q\vee s\vee v)}}(a\gamma_1 b\gamma_2 c) \\
&= p \wedge t \wedge u \\
&\leq \mu_{f(Q)}(x\gamma_1 y\gamma_2 z) \\
&= \mu_{f(Q)}(f(a)\gamma_1 f(b)\gamma_2 f(c)) \\
&= \mu_{f(Q)}(f(a\gamma_1 b\gamma_2 c)) \\
&= \mu_{f^{-1}(f(Q))}(a\gamma_1 b\gamma_2 c) \text{ [As } Q \text{ is constant on Ker} f\text{, so } f^{-1}(f(Q)) = Q] \\
&= \mu_Q(a\gamma_1 b\gamma_2 c).
\end{aligned}$$

Thus $\mu_{a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)}}(a\gamma_1 b\gamma_2 c) \leq \mu_Q(a\gamma_1 b\gamma_2 c)$.

Similarly, we can show that $\nu_{a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)}}(a\gamma_1 b\gamma_2 c) \geq \nu_Q(a\gamma_1 b\gamma_2 c)$. Then we get $a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq Q$. Since Q is an IF2API of R, then $a_{(p,q)}\gamma_1 b_{(t,s)} \subseteq Q$ or $a_{(p,q)}\gamma_2 c_{(u,v)} \subseteq \sqrt{Q}$ or $b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq \sqrt{Q}$. Thus

$$\begin{aligned}
p \wedge t &\leq \mu_Q(a\gamma_1 b) = \mu_{f(Q)}(f(a\gamma_1 b)) \\
&= \mu_{f(Q)}(f(a)\gamma_1 f(b)) \\
&= \mu_{f(Q)}(x\gamma_1 y).
\end{aligned}$$

Similarly, we can show that $q \vee s \geq \mu_{f(Q)}(x\gamma_1 y)$ and so $(x\gamma_1 y)_{(p\wedge t, q\vee s)} \subseteq f(Q)$.
Thus $x_{(p,q)}\gamma_1 y_{(t,s)} \subseteq f(Q)$ or

$$\begin{aligned}
p \wedge u &\leq \mu_{\sqrt{Q}}(a\gamma_2 c) = \mu_{f(\sqrt{Q})}(f(a\gamma_2 c)) \\
&= \mu_{f(\sqrt{Q})}(f(a)\gamma_2 f(c)) \\
&= \mu_{f(\sqrt{Q})}(x\gamma_2 z).
\end{aligned}$$

Similarly, we can show that $q \vee v \geq \nu_{f(\sqrt{Q})}(x\gamma_2 z)$ and so $(x\gamma_2 z)_{(p\wedge u, q\vee v)} \subseteq f(\sqrt{Q})$. Thus $x_{(p,q)}\gamma_2 z_{(u,v)} \subseteq f(\sqrt{Q})$ or

$$\begin{aligned} t \wedge u &\leq \mu_{\sqrt{Q}}(b\gamma_2 c) = \mu_{f(\sqrt{Q})}(f(b\gamma_2 c)) \\ &= \mu_{f(\sqrt{Q})}(f(b)\gamma_2 f(c)) \\ &= \mu_{f(\sqrt{Q})}(y\gamma_2 z). \end{aligned}$$

Similarly, we can show that $s \vee v \geq \nu_{f(\sqrt{Q})}(y\gamma_2 z)$ and so $(y\gamma_2 z)_{(t\wedge u, s\vee v)} \subseteq f(\sqrt{Q})$.
Thus $y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f(\sqrt{Q})$. Hence $f(Q)$ is an IF2API of R'. □

Corollary 4.2. *Let $f : R \to R'$ be a surjective Γ-homomorphism. If Q is an IF2API of R which is constant on* $\mathrm{Ker}\, f$*, then $f(\sqrt{Q})$ is an IF2AI of R'*

Proof. The result follows from Theorem (4.1), Proposition (3.10) and Theorem (2.24). □

Theorem 4.3. *Let $f : R \to R'$ be a Γ-homomorphism. If Q' is an IF2API of R', then $f^{-1}(Q')$ is an IF2API of R.*

Proof. Suppose that $x_{(p,q)}\gamma_1 y_{(t,s)}\gamma_2 z_{(u,v)} \subseteq f^{-1}(Q')$, where $x_{(p,q)}, y_{(t,s)}, z_{(u,v)} \in IFP(R)$ and $\gamma_1, \gamma_2 \in \Gamma$.

$$\begin{aligned} p \wedge t \wedge u &\leq \mu_{f^{-1}(Q')}(x\gamma_1 y\gamma_2 z) \\ &= \mu_{Q'}(f(x\gamma_1 y\gamma_2 z)) \\ &= \mu_{Q'}(f(x)\gamma_1 f(y)\gamma_2 f(z)), \end{aligned}$$

i.e., $p \wedge t \wedge u \leq \mu_{Q'}(f(x)\gamma_1 f(y)\gamma_2 f(z))$.

Similarly, we can show that $q \vee s \vee v \geq \nu_{Q'}(f(x)\gamma_1 f(y)\gamma_2 f(z))$. Let $f(x) = a, f(y) = b$, $f(z) = c$. Hence we have that $p\wedge t\wedge u \leq \mu_{Q'}(a\gamma_1 b\gamma_2 c)$ and $q\vee s\vee v \geq \nu_{Q'}(a\gamma_1 b\gamma_2 c)$ and as such $a_{(p,q)}\gamma_1 b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq Q'$. Since Q' is an intuitionistic fuzzy 2-absorbing primary ideal of R, then $a_{(p,q)}\gamma_1 b_{(t,s)} \subseteq Q'$ or $a_{(p,q)}\gamma_2 c_{(u,v)} \subseteq \sqrt{Q'}$ or $b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq \sqrt{Q'}$. If $a_{(p,q)}\gamma_1 b_{(t,s)} \subseteq Q'$, then

$$\begin{aligned} p \wedge t &\leq \mu_{Q'}(a\gamma_1 b) = \mu_{Q'}(f(x)\gamma_1 f(y)) \\ &= \mu_{Q'}(f(x\gamma_1 y)) \\ &= \mu_{f^{-1}(Q')}(x\gamma_1 y). \end{aligned}$$

i.e., $p \wedge t \leq \mu_{f^{-1}(Q')}(x\gamma_1 y)$.

Similarly, we can show that $q \vee s \geq \nu_{f^{-1}(Q')}(x\gamma_1 y)$. Thus we get $x_{(p,q)}\gamma_1 y_{(t,s)} = (x\gamma_1 y)_{(p\wedge t, q\vee s)} \subseteq f^{-1}(Q')$. If $a_{(p,q)}\gamma_2 c_{(u,v)} \subseteq \sqrt{Q'}$, then

$$\begin{aligned} p \wedge u &\leq \mu_{\sqrt{Q'}}(a\gamma_2 c) = \mu_{\sqrt{Q'}}(f(x)\gamma_2 f(z)) \\ &= \mu_{\sqrt{Q'}}(f(x\gamma_2 z)) \\ &= \mu_{f^{-1}(\sqrt{Q'})}(x\gamma_2 z). \end{aligned}$$

i.e., $p \wedge u \leq \mu_{f^{-1}(\sqrt{Q'})}(x\gamma_2 z)$.

Similarly, we can show that $q \vee v \geq \nu_{f^{-1}(\sqrt{Q'})}(x\gamma_2 z)$. Thus we get $x_{(p,q)}\gamma_2 z_{(u,v)} = (x\gamma_2 z)_{(p\wedge u, q\vee v)} \subseteq f^{-1}(\sqrt{Q'})$. If $b_{(t,s)}\gamma_2 c_{(u,v)} \subseteq \sqrt{Q'}$, then

$$\begin{aligned} t \wedge u &\leq \mu_{\sqrt{Q'}}(b\gamma_2 c) = \mu_{\sqrt{Q'}}(f(y)\gamma_2 f(z)) \\ &= \mu_{\sqrt{Q'}}(f(y\gamma_2 z)) \\ &= \mu_{f^{-1}(\sqrt{Q'})}(y\gamma_2 z). \end{aligned}$$

i.e., $t \wedge u \leq \mu_{f^{-1}(\sqrt{Q'})}(y\gamma_2 z)$.

Similarly, we can show that $s \vee v \geq \nu_{f^{-1}(\sqrt{Q'})}(y\gamma_2 z)$. Thus we get $y_{(t,s)}\gamma_2 z_{(u,v)} = (y\gamma_2 z)_{(t\wedge u, s\vee v)} \subseteq f^{-1}(\sqrt{Q'})$. Therefore, we see that $f^{-1}(Q')$ is an IF2API of R. $\square$

Corollary 4.4. *Let $f : R \to R'$ be a Γ-homomorphism. If Q' is an IF2API of R', then $f^{-1}(\sqrt{Q'})$ is an IF2AI of R*

Proof. The result follows from Theorem (4.3), Proposition (3.10) and Theorem (2.25). $\square$

5 Conclusion

In this paper, we contemplated IF2APIs of a Γ-ring R from a theoretical point of view. We proved that every IF2AI of Γ-ring R is an IF2API, but converse may not be true. With the help of an example we have shown that intersection of two IF2APIs of Γ-ring R need not an IF2API. However, we proved that intersection of a finite number of IFP2APIs of Γ-ring R be an IFP2API. The behaviour of IF2API under Γ-ring homomorphism has been investigated. It is shown that the notion of IF2APIs in Γ-ring inherits most of the essential properties of IF2APIs of commutative ring.

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