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The $\ominus$ operation over intuitionistic fuzzy pairs

Peter Vassilev[1] and Simeon Ribagin[2]

[1] Department of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
e-mail: peter.vassilev@gmail.com

[2] Department of Bioinformatics and Mathematical Modelling,
Institute of Biophysics and Biomedical Engineering,
Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
e-mail: simribagin@gmail.com

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Abstract: In this paper we introduce an inverse operation to the operation "$+$" defined over intuitionistic fuzzy pairs. We investigate the main properties of the introduced operation and compared it to other "$-$" operations previously introduced over intuitionistic fuzzy pairs.
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1 Introduction

Our purpose in the present research is to introduce an operation defined over intuitionistic fuzzy pairs which is in some sense inverse to the operation "$+$". We note in passing that there are two other operations "$-$" introduced by Traneva et al. in [3,4], which however have no such property. We base our construction on some properties established regarding the operation "$+$" in [5]. First we remind the important notions that will be used further in the exposition.

Definition 1 (cf. [2])**.** *An* intuitionistic fuzzy pair *(IFP) is an object of the form* $\langle a, b\rangle$, *where*

$$0 \leq \min(a, b) \leq \max(a, b) \leq \min(a, b) + \max(a, b) \leq 1. \tag{1.1}$$

Definition 2 (cf. [2]). *We say that the IFP $\langle a, b\rangle$ is greater than or equal to the IFP $\langle c, d\rangle$, and we write $\langle a, b\rangle \geq \langle c, d\rangle$ if and only if (iff)*

$$\begin{cases} a \geq c \\ b \leq d. \end{cases} \tag{1.2}$$

Alternatively, we say that $\langle a, b\rangle \leq \langle c, d\rangle$ iff $\langle c, d\rangle \geq \langle a, b\rangle$.

Definition 3 (cf. [2]). *The operation "$+$" between the IFPs $\langle a, b\rangle$, and $\langle c, d\rangle$, is defined as*

$$\langle a, b\rangle + \langle c, d\rangle = \langle a + c - ac, bd\rangle. \tag{1.3}$$

An alternative form of (1.3) *is*

$$\langle a, b\rangle + \langle c, d\rangle = \langle 1 - (1 - a)(1 - c), bd\rangle. \tag{1.4}$$

From Definition 3 and (1.4) it can be easily seen that our desired operation $\ominus$ should have the property (for $\langle a, b\rangle \geq \langle c, d\rangle$):

$$\langle a, b\rangle \ominus \langle c, d\rangle = \left\langle 1 - \frac{1-a}{1-c}, \frac{b}{d} \right\rangle,$$

whenever the last is an IFP.

Since we only want to apply the said operation over IFPs that will produce an IFP, we will define for every IFP $u = \langle u_1, u_2\rangle$ the set of feasible IFPs $\mathcal{F}_\ominus(u)$, such that for every IFP $v \in \mathcal{F}_\ominus(u)$, we have $z = u \ominus v$ is an IFP.

2 The new operation $\ominus$

Let the IFP $u = \langle u_1, u_2\rangle$ be given. Let us denote by $T(u)$ the set of all pairs $\langle w_1, w_2\rangle$, such that

$$\begin{cases} 1 - w_1 \geq \max(w_2, 1 - u_1) \\ w_2 - u_2 > 0 \\ \begin{vmatrix} w_2 & 1 - w_1 \\ u_2 & 1 - u_1 \end{vmatrix} \geq 0 \end{cases} \tag{2.1}$$

If $u_2 = 0$, we will also use the set

$$S_0(u) = \{\langle v, 0\rangle | 0 \leq v \leq u_1\} \tag{2.2}$$

We are now ready to define the feasibility set for u:

$$\mathcal{F}_\ominus(u) = \begin{cases} \{u\} \cup T(u) \cup S_0(u), & \text{if } u_2 = 0. \\ \{u\} \cup T(u), & \text{otherwise} \end{cases} \tag{2.3}$$

We note that every $w \in \mathcal{F}_\ominus(u)$ is by necessity an IFP. Indeed, if $w = u$, this is true, the same is evident when $w \in S_0(u)$. If $w \in T(u)$ by the first two inequalities of (2.1), we have that $1 - w_1 \geq w_2 > u_2 \geq 0$. Hence, $0 < w_1 + w_2 \leq 1$, i.e. w is an IFP in all cases.

Definition 4. *For a fixed IFP* $u = \langle u_1, u_2\rangle$ *and* $v = \langle v_1, v_2\rangle \in \mathcal{F}_\ominus(u)$, *we define the operation* $\ominus$ *as follows*

$$\begin{cases} u \ominus v = \langle 0, 1\rangle, & \text{if } u = v \\ u \ominus v = \left\langle \dfrac{u_1 - v_1}{1 - v_1}, 0 \right\rangle, & \text{if } u_2 = 0 \\ u \ominus v = \left\langle \dfrac{u_1 - v_1}{1 - v_1}, \dfrac{u_2}{v_2} \right\rangle, & \text{otherwise} \end{cases} \tag{2.4}$$

In order to verify that this definition is correct, we need to show that if $z = u \ominus v$, for $v \in \mathcal{F}_\ominus(u)$, then $z \in \mathcal{F}_\ominus(u)$. If $v = u$, we have $z = \langle 0, 1\rangle$, which either satisfies (2.4) or coincides with u, when it does not, i.e., it is always in $\mathcal{F}_\ominus(u)$. The case when $u_2 = 0$, is a matter of direct check as we have:

$$z = \left\langle \frac{u_1 - v_1}{1 - v_1}, 0 \right\rangle \in S_0(u)$$

(see (2.2)), and, therefore, by (2.3) $z \in \mathcal{F}_\ominus(u)$. Thus, without loss of generality we may assume $z \neq u$ and $z_2 \neq 0$, i.e., $z \notin S_0(u) \cup \{u\}$. We have to show that $z \in T(u)$. We have

$$\frac{1 - u_1}{1 - v_1} \geq 1 - u_1$$

(since $v_1 \geq 0$) and due to (2.1)

$$\frac{1 - u_1}{1 - v_1} \geq \frac{u_2}{v_2}.$$

Thus the first inequality is satisfied. We also have $z_2 > u_2$ (since $1 > v_2 \geq u_2$.) We only have to show that:

$$\frac{u_2}{v_2}(1 - u_1) - \frac{1 - u_1}{1 - v_1} u_2 \geq 0.$$

After simplification,

$$\frac{1}{v_2} - \frac{1}{1 - v_1} \geq 0$$

But the last is equivalent to the fact that v is an IFP. Thus, our definition is correct.

Remark 1. It is important to note that $u \ominus v = w$ does not mean that $u \ominus w = v$, in general. For instance,

$$\langle 0.6, 0.0\rangle \ominus \langle 0.5, 0.4\rangle = \langle 0.2, 0.0\rangle,$$

while

$$\langle 0.6, 0.0\rangle \ominus \langle 0.2, 0.0\rangle = \langle 0.5, 0.0\rangle,$$

In both cases, however, we have that:

$$w = u \ominus v \Leftrightarrow v + w = u.$$

Also, we have $v \leq u$ and $w \leq u$.

We shall now compare our operation $\ominus$ with the two operations "$-$" introduced by Traneva et al. in [3, 4], respectively.

Definition 5 ([3]). *Let the IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$ be given. Then, the operation "$-$" between these IFPs is defined as follows:*

$$x - y = \langle \min(a, d), \max(b, c)\rangle. \tag{2.5}$$

Remark 2. Again it can easily be seen that in the general case for this operation $x - y = z$, does not mean $x - z = y$. For this particular operation, we also have the equality:

$$\langle a, b\rangle - \langle c, d\rangle = \langle d, c\rangle - \langle b, a\rangle.$$

Definition 6 ([4]). *Let the IFPs $x = \langle a, b\rangle$ and $y = \langle c, d\rangle$ be given. Then, the operation "$-$" between these IFPs is defined as follows:*

$$x - y = \langle \max(a - c, 0), \min(1, b + d, 1 - a + c)\rangle. \tag{2.6}$$

The three operations defined by (2.4), (2.5) and (2.6) do not usually coincide. We will consider some instances below. We have (for (2.5)):

$$x - x = \langle \min(a, b), \max(a, b)\rangle$$

but (for (2.6)):

$$x - x = \langle 0, \min(b + d, 1)\rangle$$

and from (2.4)

$$x \ominus x = \langle 0, 1\rangle.$$

Let us now consider the equation:

$$x - y = y.$$

For (2.5) we obtain that the equation only has solution when $a = b = c = d$. For (2.6) this only has solution when $a = c = 0$ and either $b = 0$, or $d = 1$.

For (2.4) we obtain

$$x \ominus y = y \Leftrightarrow y_1 = 1 - \sqrt{1 - x_1}, y_2 = \sqrt{x_2}.$$

We will now consider the equation:

$$x - y = x.$$

For (2.5) we obtain that the solution to the equation is $y = \langle b, a\rangle$. For (2.6) we obtain that the solution to the equation is $y = \langle 0, 0\rangle$. For (2.4) we obtain that:

$$x \ominus y = x \Leftrightarrow y = \langle 0, 1\rangle$$

This shows that all the three operations are significantly different and cover various use scenarios.

3 Conclusion

We have introduced a new operation $\ominus$ between IFPs, and we have compared it to two other operations denoted by "$-$" that have been defined over IFPs. Our operation requires the definition of a feasibility set depending on a fixed IFP to limit the possible IFPs admissible in the operation, while the other two operations have no restriction over the potentially participating IFPs. We have outlined some of the most important properties of the new operation—namely, that it is somewhat inverse to the operation "$+$" between IFPs. We have highlighted the main differences by considering the solutions of three equations for each operation.

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