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IDEAS FOR INTUITIONISTIC FUZZY EQUATIONS, INEQUALITIES AND OPTIMIZATION

Krassimir T. Atanassov
CLBME - Bulgarian Academy of Sciences and
MRL, P.O. Box 12, Sofia-1113, BULGARIA
e-mail: krat@bgcict.bitnet

The continuation of the results from [1,2] are introduced here. Both the results from [1,2], and the following ones are related to the Intuitionistic Fuzzy Sets (IFSs) [3], and to the Intuitionistic Fuzzy Propositional Calculus (IFPC) [4] and Intuitionistic Fuzzy Modal Logic (IFML) [5] as well. Here, for brevity we shall show the results related to the IFML, but their modification for the case of IFSs is trivial.

Let S be the set of propositional forms (c.f., [6]: each proposition is a propositional form; if A is a propositional form then $\neg A$ is a propositional form; if A and B are propositional forms, then $A \,\&\, B$, $A \vee B$, $A \supset B$ are propositional forms) and let $V: S \to [0, 1] \times [0, 1]$, be defined for every $p \in S$ as $V(p) = \langle \mu(p), \gamma(p) \rangle$, where $\mu(p)$ and $\gamma(p)$ are the degrees of validity and non-validity of p, respectively, and let $\mu(p) + \gamma(p) \leq 1$ (see [4]). The evaluation of the negation $\neg p$ of the proposition p will be defined through: $V(\neg p) = \langle \gamma(p), \mu(p) \rangle$. When the values $V(p)$ and $V(q)$ of the propositions p and q are known, the evaluation function V can be extended also for the operations "&", "$\vee$" and "$\supset$" through the definition:

$$V(p \,\&\, q) = \langle \min(\mu(p), \mu(q)), \max(\gamma(p), \gamma(q)) \rangle,$$
$$V(p \vee q) = \langle \max(\mu(p), \mu(q)), \min(\gamma(p), \gamma(q)) \rangle,$$
$$V(p \supset q) = \langle \max(\gamma(p), \mu(q)), \min(\mu(p), \gamma(q)) \rangle.$$

By analogy with the operations over IFSs it will be convenient to define for the propositions $p, q \in S$:

$$\neg V(p) = V(\neg p),$$
$$V(p) \wedge V(q) = V(p \,\&\, q),$$
$$V(p) \vee V(q) = V(p \vee q),$$
$$V(p) \to V(q) = V(p \supset q).$$

Let everywhere below $p = q$ <u>iff</u> $V(p) = V(q)$.

In [5] the basic definitions of the Intuitionistic Fuzzy Modal Logic (IFML) are introduced. There, the following seven intuitionistic fuzzy logic operators are defined for a fixed propositional form A and for fixed $\alpha, \beta \in [0, 1]$:

$$V(D_\alpha(A)) = \langle a + \alpha.(1 - a - b), b + (1 - \alpha).(1 - a - b) \rangle,$$

$$V(F_{\alpha,\beta}(A)) = \langle a + \alpha.(1 - a - b), b + \beta.(1 - a - b) \rangle, \text{ for } \alpha+\beta \leq 1,$$

$$V(G_{\alpha,\beta}(A)) = \langle \alpha.a, \beta.b \rangle,$$

$$V(H_{\alpha,\beta}(A)) = \langle \alpha.a, b + \beta.(1 - a - b) \rangle,$$

$$V(H^*_{\alpha,\beta}(A)) = \langle \alpha.a, b + \beta.(1 - \alpha.a - b) \rangle,$$

$$V(J_{\alpha,\beta}(A)) = \langle a + \alpha.(1 - a - b), \beta.b \rangle,$$

$V(J^{*}_{\alpha,\beta}(A)) = \langle a + \alpha.(1 - a - \beta.b),\ \beta.b\rangle,$

$V(P_{\alpha,\beta}(A)) = \langle \max(\alpha, a),\ \min(\beta, b)\rangle,$ for $\alpha + \beta \le 1,$

$V(G_{\alpha,\beta}(A)) = \langle \min(\alpha, a),\ \max(\beta, b)\rangle,$ for $\alpha + \beta \le 1,$

$V(\forall x\ A(x)) = \langle \min_{x\in S} \mu_A(x),\ \max_{x\in S} \gamma_A(x)\rangle,$

$V(\exists x\ A(x)) = \langle \max_{x\in S} \mu_A(x),\ \min_{x\in S} \gamma_A(x)\rangle.$

Obvuiously, $D_\alpha(A) = F_{\alpha, 1-\alpha}(A)$ and thus we will not use the operator D_α. On the other hand, the modal operators "necessity" and "possibility" satisfy the equalities $\Box A = D_0(A)$ and $\Diamond A = D_1(A)$.

Following [1], we shall call the propositional form A a pseudo-fixed point if $V(A) = V(Y(A))$, where Y is a some IF-operator. A theorem related to IFS case is proved in [1], which in the case of IFML can be formulated as follows, where $V(A) = \langle a, b\rangle$ and $0 \le a, b \le 1$.

THEOREM 1 [1]: For all $\alpha, \beta \in [0, 1]$ the pseudo-fixed point(s) of

(a) $\Box$, $\Diamond$, D_α, $F_{\alpha,\beta}$ are all elements $A \in S$ for which $a + b = 1$, and $\alpha + \beta \le 1$;

(b) $G_{\alpha,\beta}$ are all elements $A \in S$ for which $a = b = 0$,

(c) $H_{\alpha,\beta}$, $H^{*}_{\alpha,\beta}$ are the elements $A \in S$ for which $a = 0$ and $b = 1$,

(d) $J_{\alpha,\beta}$, $J^{*}_{\alpha,\beta}$ are the elements $A \in S$ for which $a = 1$ and $b = 0$,

(e) $P_{\alpha,\beta}$ are all elements $A \in S$ for which $\alpha \le a \le 1$ and $0 \le b \le \beta$,

(f) $G_{\alpha,\beta}$ are all elements $A \in S$ for which $0 \le a \le \alpha$ and $\beta \le b \le 1$,

(g) $\exists$ are all elements $A \in S$ for which $a = 1$ and $b = 0$,

(h) $\forall$ are all elements $A \in S$ for which $a = 0$ and $b = 1$.

Therefore, the theorem shows all solutions of the expression

$$V(Y_{\alpha,\beta}(A)) = V(A), \qquad (1)$$

where α and β have the above sense or of the expression

$$Y_{\alpha,\beta}(A) = A.$$

An expression with the form of (1) will be called an Intuitionistic Fuzzy Equation (IFE). The following theorem gives other similar examples.

THEOREM 2 [2]: For every two propositional forms $A, B \in S$ there exists an operator Y from the above type and there exist real numbers $\alpha, \beta \in [0, 1]$ such that

$$V(A) = V(Y_{\alpha,\beta}(B)). \qquad (2)$$

Obviously, Theorem 2 (resp. (2)) is a generalization of Theorem 1 (resp. (1)). In both cases in one of the sizes there is a constant (right in (1) and left in (2)). The next four simple examples are interesting as illustrations of other possible constructions for IFEs. Let $a, b, \alpha, \beta, \gamma, \delta \in [0, 1]$, $a + b \le 1$, $\alpha + \beta \le 1$,

$\Gamma + \partial \le 1$. Then the solutions of

$$\Box A = P_{\alpha,\beta}(A) \qquad (3)$$

are obtained as follows. (1) is equivalent to the expression

$$\langle u, 1 - u\rangle = \langle \max(\alpha, u), \min(\beta, v)\rangle$$

where $u, v \in [0, 1]$, $V(A) = \langle u, v\rangle$ and u and v are unknown quantities. Therefore the following equations must hold:

$$u = \max(\alpha, u)$$
$$1 - u = \min(\beta, v)$$

Their solutions are the following: $\langle u, v\rangle = \langle p, 1 - p\rangle$, for every $p \in [\alpha, 1]$, if $\beta \ge 1 - p$. If $\beta < 1 - p$, then (3) does not have any solutions.

The second example is the following:

$$V(G_{\alpha,\beta}(F_{\Gamma,\partial}(A))) = \langle a, b\rangle \qquad (4)$$

It is solved similarly. The basic difference between (2) and (4) is in the number of the operators applied over A. (4) illustrates the possibility to apply more than one operator. We can complicate (4), e.g., to the form

$$(G_{\alpha,\beta}(F_{\Gamma,\partial}(A)) \ \& \ H^{*}_{\alpha,\partial}(A \supset P_{\beta,\Gamma}(A))) \vee D_{\partial}(A) = J^{*}_{\beta,\alpha}(\neg A).$$

Obviously, the solution of this expression will be more complex than the above solutions. All these examples are related to expressions having degrees of validity and non-validity as unknown quantities. The second possibility is the one, when the unknown quantities are operator parameters. The next IFE illustrates this:

$$V(H_{\alpha,\beta}(A)) = \langle c, d\rangle, \qquad (5)$$

where $V(A) = \langle a, b\rangle$ and $a, b, c, d \in [0, 1]$ and $a + b \le 1$, $c + d \le 1$. Here the unknown quantities are the operator parameters α and β. In this case we obtain

$$\langle \alpha.a, b + \beta.(1 - a - b)\rangle = \langle c, d\rangle.$$

If $a < c$ (particularly, if $a = 0$ and $c \ne 0$), then (5) does not have any solutions. If $c \le a \le 1$, then $\alpha = \frac{c}{a}$. If $b = 1 - a$ and $b \ne d$, or if $b > d$, then (5) does not have any solutions; if $b = d = 1 - a$, then all real numbers $\beta \in [0, 1]$ are solutions of (5); if $b < 1 - a$, then $\beta = \dfrac{d - a}{1 - a - b}$.

If in some of the above IFEs we change the sign "=" with one of the signs "≥" or "≤", we shall obtain an Intuitionistic Fuzzy Inequality (IFI). The solutions of the IFIs are obtained similarly to the solutions of the IFEs.

Every system with the form

$$Y_1(x_1, x_2, \ldots, x_s) *_1 Z_1(x_1, x_2, \ldots, x_s)$$
$$Y_2(x_1, x_2, \ldots, x_s) *_2 Z_2(x_1, x_2, \ldots, x_s)$$
$$\vdots$$
$$Y_s(x_1, x_2, \ldots, x_s) *_s Z_s(x_1, x_2, \ldots, x_s)$$

where $*_1, *_2, \ldots, *_s \in \{=, \le, \ge\}$ is called a system of IFEs (if all $*_i$ are "="; $1 \le i \le s$); or a system of IFIs, otherwise. We shall call these systems "systems of an argument type", if the unknow quantities are in the degrees of the arguments, and "systems of a parameter type", if the unknow quantities are operator parameters. We can construct a third type of systems "heterogeneous systems", if the unknown quantities are of both types simultaneously. In this case the system must have the form

$$\begin{array}{c} Y_1(x_1, x_2, \ldots, x_s) *_1 Z_1(x_1, x_2, \ldots, x_s) \\ Y_2(x_1, x_2, \ldots, x_s) *_2 Z_2(x_1, x_2, \ldots, x_s) \\ . \quad . \quad . \\ Y_r(x_1, x_2, \ldots, x_s) *_r Z_r(x_1, x_2, \ldots, x_s) \end{array}$$

where $r = s + t$ and s and t are numbers of the IFIs (in particular - IFEs) of the first and of the second types.

When some of the relations $*_i$ $(1 \le i \le r)$ are "$\le$" (or "$\ge$"), the solutions can be elements of some sets (e.g., parts of the triangle, which determines the geometrical representation of S; see [7]). It is suitable to introduce the following definitions for every $x_1, x_2, \ldots, x_s \in S$ and for $\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_t, \beta_t$ (see above): $\langle x_1, x_2, \ldots, x_s\rangle$ is a maximal (minimal) s-tuple of arguments <u>iff</u> for every $y_1, y_2, \ldots, y_s \in S$ it is valid: $V(x_i) \ge V(y_i)$ $(V(x_i) \le V(y_i))$ for every i $(1 \le i \le s)$; $\langle\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_t, \beta_t\rangle$ is a maximal (minimal) 2.t-tuple of operators parameters if the differences about the norn σ_3 of their left and right parts for every one of the members of the system of IFIs

$$\begin{array}{c} V(Y_1(x_1, x_2, \ldots, x_s)) *_1 V(Z_1(x_1, x_2, \ldots, x_s)) \\ V(Y_2(x_1, x_2, \ldots, x_s)) *_2 V(Z_2(x_1, x_2, \ldots, x_s)) \\ . \quad . \quad . \\ V(Y_r(x_1, x_2, \ldots, x_s)) *_r V(Z_r(x_1, x_2, \ldots, x_s)) \end{array}$$

obtain their maximal (minimal) values. Therefore, $\langle\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_t, \beta_t\rangle$ is a maximal (minimal) 2.t-tuple of operators parameters if the numbers

$$\begin{array}{c} \sigma_3(Y_1(x_1, x_2, \ldots, x_s)) - \sigma_3(Z_1(x_1, x_2, \ldots, x_s)), \\ \sigma_3(Y_2(x_1, x_2, \ldots, x_s)) - \sigma_3(Z_2(x_1, x_2, \ldots, x_s)), \\ . \quad . \quad . \\ \sigma_3(Y_r(x_1, x_2, \ldots, x_s)) - \sigma_3(Z_r(x_1, x_2, \ldots, x_s)), \end{array}$$

obtain their maximal (minimal) values.

By σ_3 we denote the norm which juxtaposes to $A \in S$ the number

$$\sigma_3(A) = \frac{1 + \mu(A) - \gamma(A)}{2}.$$

(see [8]).

If in the result of solving of a given system of IFIs we obtain that for every i $(1 \le i \le r)$: $V(x_i) \in T_i$, where T_i are the parts of the above mentioned triangle, then we can formulate the following problem: to determine the maximal (minimal) s-tuple $\langle x_1, x_2, \ldots, x_s\rangle$. The procedure of determining this s-tuple $\langle x_1, x_2, \ldots, x_s\rangle$ we shall call an Intuitionistic Fuzzy Optimization (IFO) about the arguments. Analogically, we can define the concept IFO about the operators parameters.

The ways for solving of the IFEs and IFIs and the ways for implementing of the two types of IFO are open problems, and will be promising a direction for further research.

An alternative IFO problem can be based on the extension of Bellman-Zadeh's principle (see, e.g. [9]) in the sense of P. Angelov's research [10, 11].

Another possible continuation of the above ideas can be the following. It is related to the concept of an Intuitionistic Fuzzy relation (IFR) [12-16] and its Index Matrix (IM) [17] representation. For example, if we have the observations (alternatives, etc.) X and the criteria Y, where $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ are fixed finite sets (universes), we can construct the following IM

	y_1	y_2	$\ldots$	y_n
x_1	$\langle\mu(x_1, y_1), \nu(x_1, y_1)\rangle$	$\langle\mu(x_1, y_2), \nu(x_1, y_2)\rangle$	$\ldots$	$\langle\mu(x_1, y_n), \nu(x_1, y_n)\rangle$
x_2	$\langle\mu(x_2, y_1), \nu(x_2, y_1)\rangle$	$\langle\mu(x_2, y_2), \nu(x_2, y_2)\rangle$	$\ldots$	$\langle\mu(x_2, y_n), \nu(x_2, y_n)\rangle$
$\vdots$	.	.	.	
x_m	$\langle\mu(x_m, y_1), \nu(x_m, y_1)\rangle$	$\langle\mu(x_m, y_2), \nu(x_m, y_2)\rangle$	$\ldots$	$\langle\mu(x_m, y_n), \nu(x_m, y_n)\rangle$

where $\langle\mu_i, \nu_j\rangle$ is the estimation of the observation x_i on the criterion y_j.

On the basis of this IM (which is an IFR on the universe X x Y) we can determine different interesting interior connections. For example, we cancalculate the couples:

$$\langle\mu_i^{opt}, \nu_i^{opt}\rangle = \langle\max_{1\le j\le n} \mu_{i,j}, \min_{1\le j\le n} \nu_{i,j}\rangle$$

(an optimistic evaluation),

$$\langle\mu_i^{pes}, \nu_i^{pes}\rangle = \langle\min_{1\le j\le n} \mu_{i,j}, \max_{1\le j\le n} \nu_{i,j}\rangle$$

(a pessimistic evaluation),

$$\langle\mu_i^{ave}, \nu_i^{ave}\rangle = \langle(\sum_{j=1}^{n} \mu_{i,j})/n, (\sum_{j=1}^{n} \nu_{i,j})/n\rangle$$

(an average evaluation), or generally (cf. [13-16]),

$$\langle\mu_i^S, \nu_i^T\rangle = \langle S(\mu_1, \mu_2, \ldots, \mu_n), T(\nu_1, \nu_2, \ldots, \nu_n)\rangle, \quad (6)$$

where S and T are two mappings, S, T: [0, 1] x [0, 1] -> [0, 1], which satisfy the properties:

i) Boundary conditions, $T(x, 1) = x$, $T(x, 0) = 0$, $S(x, 1) = 1$ and $S(x, 0) = x$ for every $x \in [0, 1]$;

ii) Monotonicity, $T(x, y) \le T(z, t)$ and $S(x, y) \le S(z, t)$ iff $x \le z$ and $y \le t$, where $x, y, z, t \in [0, 1]$;

iii) Commutativeness, $T(x, y) = T(y, x)$ and $S(x, y) = S(y, x)$ for every $x, y \in [0, 1]$;

iv) Associativeness, $T(T(x, y), z) = T(x, T(y, z))$ and $S(S(x, y), z) = S(x, S(y, z))$ for every $x, y, z \in [0, 1]$;

v) Connection, $S(x, y) + T(z, t) \le 1$ for every $x, y, z, t \in [0, 1]$, such that $x + z \le 1$ and $y + t \le 1$.

Analogically, we can determine similar values for the columns. For example, we can calculate the couples:

$$<\mu_j, \nu_j> = \max_{1 \le i \le m} <\mu_{i,j}, \nu_{i,j}>,$$

where for $a, b, c, d \in [0, 1]$ and $a + b \le 1$, $c + d \le 1$:

$$<a, b> \ge <c, d> \text{ iff } a \ge c \text{ and } b \le d.$$

If there are incomparable couples, they can be compared on the basis of their σ_3-norm values (see [8]), if their μ-components are $\ge 1/2$, or on the basis of their μ-components, if they are $\le 1/2$ (these questions are discussed in [18] more detail]).

Another couple can be juxtaposed to the elements of the i-th IM row:

$$<\mu_i, \nu_i> = F_{\mu_{i,n}, \nu_{i,n}}(\ldots(F_{\mu_{i,k-1}, \nu_{i,k-1}}(F_{\mu_{i,k+1}, \nu_{i,k+1}}(\ldots(F_{\mu_{i,1}, \nu_{i,1}}(<\mu_{i,k}, \nu_{i,k}>) \quad (7)$$

where k is the number of the best couple in the above sense.

We can modify the latter one and other similar constructions, using the above ideas, into optimization problems. For example, we can search these $(\mu_{i,j}, \nu_{i,j})$-values of an unclear (uncorrect), observation which can give the best final $<\mu_i, \nu_i>$-value for (7). We can serach these forms of S and T for which (6) obtains the optimal form.

Below, we shall unite some of the above ideas with the ideas from [19, 20].

Let the order of the criteria be not linear order (see [20]). Therefore, we can construct a standard graph or an Intuitionistic Fuzzy Graph (IFG) (see[19, 21]) which corresponds to this order. Following [19] we shall introduce some definitions.

Let us note the fact that the directed arc (the condition that the arc is an directed is not essential) has a beginning A and an end B by $A \vdash B$. If for the vertices A and B there are vertices $v_1, v_2, \ldots, v_s$, such that $A \vdash v_1 \vdash v_2 \vdash \ldots \vdash v_s \vdash B$, we shall write also $A \vdash_* B$.

Now, we can introduce the following definitions, too.

Let for a fixed vertex $v \in V$, where V is a set of the graph vertices:

$$Pred_0(v) = \{v\},$$

$$Pred_1(v) = \{w \,/\, w \vdash v\}$$

$$Pred_{i+1}(v) = \{w \,/\, (\exists u \in V)(w \in Pred_1(u) \,\&\, u \in Pred_i(v))\},$$

for every $i \ge 1$.

Let for every $i \geq 1$

$$c(v, i) = \mathrm{card}(\mathrm{Pred}_i(v))$$

(therefore $c(V, 0) = 1$), where card(X) is the cardinality of the set X.

If the graph has two fixed vertices - the maximal vertex T and the minimal vertex F, for which for every vertex v holds:

$$F \vdash v \vdash T$$

Therefore c(T, i) is the number of all vertices in the i-th level, numberedfrom T to F and

$$\mathrm{card}(v) = \sum_{i=0}^{\infty} c(T, i) = \sum_{i=0}^{l(F)} c(T, i),$$

where l(v) is the number of levels, numbered from T to F (therefore, l(F) is the total number of the levels).

For a given set of observations $\{O_1, O_2, \ldots, O_m\}$ and a given set of criteria $\{C_1, C_2, \ldots, C_n\}$ we can construct an IM A with elements $a_{i,j}$ $(1 \leq i \leq m, 1 \leq j \leq n)$ for which

$$a_{i,j} = \begin{cases} 1, & \text{if the object } O_i \text{ satisfies the criterion } C_j \\ 0, & \text{otherwise} \end{cases}$$

Therefore, we can construct the IM B with the same index-sets and with elements $\langle b_{i,j}, c_{i,j} \rangle$ for which

$$b_{i,j} = \sum_{k=0}^{l(C_j)} \frac{\sum_{v_s \in \mathrm{Pred}_k(C_j)} a_{i,s}}{c(T, k)}$$

$$c_{i,j} = \sum_{k=0}^{l(C_j)} \frac{c(T, k) - c(C_j, k)}{c(T, k)}$$

From the fact that $0 \leq a_{i,s} \leq 1$ it follows that

$$\sum_{v_s \in \mathrm{Pred}_k(C_j)} a_{i,s} \leq c(C_j, k)$$

and therefore, $0 \leq b_{i,j} + c_{i,j} \leq 1$, i.e., the so constructed graph is an IFG.

In this way we can determine the degrees of validity and of non-validity with which the different objects (observations) satisfy the criteria. This is another approach correcponding to the O. Asparoukhov's approach described in [18].

For this approach it is possible to formulate some optimization problems. They are related:

- to the graph structure (to found optimal paths in the graph (the IFG));
- to the relations between the criteria (are the criteria suitable; can they change with other ones which will generate another graph (an IFG)) with better structure);
- to the objects (observations) (to found optimal $a_{i,j}$-parameters) etc.

Moreover, we can search other ways for constructing the $a_{i,j}$-parameters, e.g., as a function of the level, of the weights of the IFG-arcs and other.

All these questions will be objects of further researches.

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