Issues in IFSs and GNs, Vol. 12, 2015, 61–67

Remark on Dworniczak's intuitionistic fuzzy implications. Part 2

Lilija Atanassova

Institute of Information and Communication Technologies
Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Bl. 2, Sofia-1113, Bulgaria
e-mail: l.c.atanassova@gmail.com

Abstract: The paper is a continuation of previous authors research. In it, on the basis of second implication, introduced by Piotr Dworniczak, four new intuitionistic fuzzy implications are defined. Some of their basic properties are studied.
Keywords and phrases: Intuitionistic fuzzy implication, Intuitionistic fuzzy set.
2000 Mathematics Subject Classification: 03E72.

1 Introduction

In [7, 8, 9], Piotr Dworniczak introduce three intuitionistic fuzzy implications, that generalized the defined by the author intuitionistic fuzzy implications in [3, 4, 5].

Here, continuating idea from [6], we introfuce four new intuitionistic fuzzy implications, modifying Dworniczak's intuitionistic fuzzy implication

$$\langle a, b\rangle \rightarrow_{151,\gamma} \langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1}\rangle,$$

where $\gamma \geq 1$.

2 Main Results

Let everywhere below variables x and y have truth values $\langle a, b\rangle$ and $\langle c, d\rangle$, where $a, b, c, d \in [0, 1], a + b \leq 1, c + d \leq 1$.

In [1, 2] the following operations and operators are defined:

$$\neg\langle a, b\rangle \equiv \neg_1\langle a, b\rangle = \langle b, a\rangle,$$

$$\langle a, b\rangle @\langle c, d\rangle = \langle \frac{a+c}{2}, \frac{b+d}{2}\rangle,$$

$$\Box\langle a, b\rangle = \langle a, 1-a\rangle,$$

$$\Diamond\langle a, b\rangle = \langle 1-b, b\rangle,$$

$$\boxplus_{\alpha,\beta,\gamma}\langle a, b\rangle = \langle \alpha a, \beta b + \gamma\rangle,$$

$$\boxtimes_{\alpha,\beta,\gamma}\langle a, b\rangle = \langle \alpha a + \gamma, \beta b\rangle,$$

where $\alpha, \beta, \gamma \in [0, 1]$ and $\max(\alpha, \beta) + \gamma \leq 1$,

$$\boxdot_{\alpha,\beta,\gamma,\delta}\langle a, b\rangle = \langle \alpha a + \gamma, \beta b + \delta\rangle,$$

where $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\max(\alpha, \beta) + \gamma + \delta \leq 1$.

We use the following formulas:

$$\langle a, b\rangle \to^1_{151,\gamma} \langle c, d\rangle = \Box\langle a, b\rangle \to_{151,\gamma} \Box\langle c, d\rangle,$$

$$\langle a, b\rangle \to^2_{151,\gamma} \langle c, d\rangle = \Box\langle a, b\rangle \to_{151,\gamma} \Diamond\langle c, d\rangle,$$

$$\langle a, b\rangle \to^3_{151,\gamma} \langle c, d\rangle = \Diamond\langle a, b\rangle \to_{151,\gamma} \Diamond\langle c, d\rangle,$$

$$\langle a, b\rangle \to^4_{151,\gamma} \langle c, d\rangle = \Diamond\langle a, b\rangle \to_{151,\gamma} \Box\langle c, d\rangle,$$

where $\gamma \geq 1$.

So, we obtain the explicit forms of the new four implications as follows:

$$\langle a, b\rangle \to^1_{151,\gamma} \langle c, d\rangle = \langle \frac{1-a+c+\gamma}{2\gamma+1}, \frac{a-c+\gamma}{2\gamma+1}\rangle,$$

$$\langle a, b\rangle \to^2_{151,\gamma} \langle c, d\rangle = \langle \frac{2-a-d+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1}\rangle,$$

$$\langle a, b\rangle \to^3_{151,\gamma} \langle c, d\rangle = \langle \frac{b-d+\gamma+1}{2\gamma+1}, \frac{-b+d+\gamma}{2\gamma+1}\rangle,$$

$$\langle a,b\rangle \to^4_{151,\gamma} \langle c,d\rangle = \langle \frac{b+c+\gamma}{2\gamma+1}, \frac{1-b-c+\gamma}{2\gamma+1}\rangle.$$

First, we check that for every $i = 1,2,3,4$ and for every $\lambda \geq 1$:

$$\langle 0,1\rangle \to^i_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma+2}{2\gamma-1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,1\rangle \to^i_{151,\gamma} \langle 1,0\rangle = \langle \frac{\gamma+2}{2\gamma-1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 1,0\rangle \to^i_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 1,0\rangle \to^i_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle.$$

Second, we check that for every $\gamma \geq 1$:

$$\langle 0,0\rangle \to^1_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^2_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^3_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^4_{151,\gamma} \langle 0,1\rangle = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^1_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^2_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^3_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^4_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^1_{151,\gamma} \langle 1,0\rangle = \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^2_{151,\gamma} \langle 1,0\rangle = \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^3_{151,\gamma} \langle 1,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,0\rangle \to^4_{151,\gamma} \langle 1,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,1\rangle \to^1_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 0,1\rangle \to^2_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle,$$

$$\langle 0,1\rangle \to^3_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+2}{2\gamma+1}, \frac{\gamma-1}{2\gamma+1}\rangle,$$

$$\langle 0,1\rangle \to^4_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 1,0\rangle \to^1_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle,$$

$$\langle 1,0\rangle \to^2_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 1,0\rangle \to^3_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma+1}{2\gamma+1}, \frac{\gamma}{2\gamma+1}\rangle,$$

$$\langle 1,0\rangle \to^4_{151,\gamma} \langle 0,0\rangle = \langle \frac{\gamma}{2\gamma+1}, \frac{\gamma+1}{2\gamma+1}\rangle.$$

Using definition from [1, 2]

$$\langle a,b\rangle \geq \langle c,d\rangle \text{ if and only if } a \geq c \text{ and } b \leq d,$$

we can prove the validity of the following

Theorem 1. For every $a, b, c, d \in [0,1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\lambda \geq 1$:

$$\langle a,b\rangle \to^2_{151,\gamma} \langle c,d\rangle \geq \langle a,b\rangle \to^1_{151,\gamma} \langle c,d\rangle, \quad (1)$$

$$\langle a,b\rangle \to^3_{151,\gamma} \langle c,d\rangle \geq \langle a,b\rangle \to^4_{151,\gamma} \langle c,d\rangle, \quad (2)$$

$$\langle a,b\rangle \to^2_{151,\gamma} \langle c,d\rangle \geq \langle a,b\rangle \to^3_{151,\gamma} \langle c,d\rangle, \quad (3)$$

$$\langle a,b\rangle \to^1_{151,\gamma} \langle c,d\rangle \geq \langle a,b\rangle \to^4_{151,\gamma} \langle c,d\rangle. \quad (4)$$

Proof. For example, let us check the validity of the fourth inequality.

First, we see, that

$$1 - a + c + \gamma - b - c - \gamma = 1 - a - b \geq 0,$$

$$1 - b - c + \gamma - a + c - \gamma = 1 - a - b \geq 0.$$

Therefore,

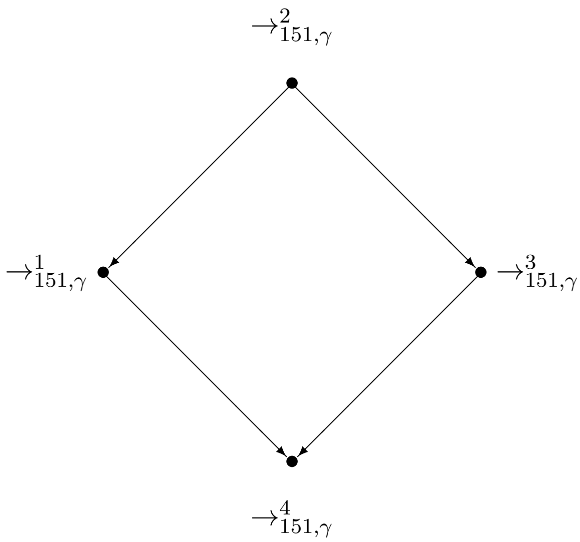
$$\langle \frac{1 - a + c + \gamma}{2\gamma + 1}, \frac{a - c + \gamma}{2\gamma + 1} \rangle \geq \langle \frac{b + c + \gamma}{2\gamma + 1}, \frac{1 - b - c + \gamma}{2\gamma + 1} \rangle,$$

i.e.,

$$\langle a, b \rangle \rightarrow^{1}_{151,\gamma} \langle c, d \rangle \geq \langle a, b \rangle \rightarrow^{4}_{151,\gamma} \langle c, d \rangle.$$

Hence, (4) is valid. (1) – (3) are proved by analogical manner. □

Now, we can construct the following diagram.



We show that the new implications can be represented by a part of the above operators.

Theorem 2. For every $a, b, c, d \in [0, 1]$, so that $a + b \leq 1$ and $c + d \leq 1$ and for every $\gamma \geq 1$:

$$\langle a, b \rangle \rightarrow^{1}_{151,\gamma} \langle c, d \rangle$$

$$= \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Diamond\neg\langle a,b\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Box\langle c,d\rangle,$$

$$\langle a,b\rangle \to^{2}_{151,\gamma} \langle c,d\rangle$$

$$= \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Diamond\neg\langle a,b\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Diamond\langle c,d\rangle,$$

$$\langle a,b\rangle \to^{3}_{151,\gamma} \langle c,d\rangle$$

$$= \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Box\neg\langle a,b\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Diamond\langle c,d\rangle,$$

$$\langle a,b\rangle \to^{4}_{151,\gamma} \langle c,d\rangle$$

$$= \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Box\neg\langle a,b\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Box\langle c,d\rangle.$$

Proof. Let $a,b,c,d \in [0,1]$, so that $a+b \leq 1$ and $c+d \leq 1$ and let $\gamma \geq 1$. Then

$$\boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Diamond\neg\langle a,b\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Box\langle c,d\rangle$$

$$= \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Diamond\langle b,a\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \Box\langle c,d\rangle$$

$$= \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \langle 1-a,a\rangle @ \boxdot_{\frac{2}{2\gamma+1},\frac{2}{2\gamma+1},\frac{\gamma}{2\gamma+1},\frac{\gamma-1}{2\gamma+1}} \langle c,1-c\rangle$$

$$= \langle \frac{2(1-a)}{2\gamma+1} + \frac{\gamma}{2\gamma+1}, \frac{2a}{2\gamma+1} + \frac{\gamma-1}{2\gamma+1}\rangle$$

$$@\langle \frac{2c}{2\gamma+1} + \frac{\gamma}{2\gamma+1}, \frac{2(1-c)}{2\gamma+1} + \frac{\gamma-1}{2\gamma+1}\rangle$$

$$= \langle \frac{1}{2}\left(\frac{2(1-a)}{2\gamma+1} + \frac{\gamma}{2\gamma+1} + \frac{2c}{2\gamma+1} + \frac{\gamma}{2\gamma+1}\right),$$

$$\frac{1}{2}\left(\frac{2a}{2\gamma+1} + \frac{\gamma-1}{2\gamma+1} + \frac{2(1-c)}{2\gamma+1} + \frac{\gamma-1}{2\gamma+1}\right)\rangle$$

$$= \langle \frac{1}{2}.\frac{2-2a+\gamma+2c+\gamma}{2\gamma+1}, \frac{1}{2}.\frac{2a+\gamma-1+2-2c+\gamma-1}{2\gamma+1}\rangle$$

$$= \langle \frac{1-a+c+\gamma}{2\gamma+1}, \frac{a-c+\gamma}{2\gamma+1}\rangle$$

$$= \langle a,b\rangle \to^{1}_{151,\gamma} \langle c,d\rangle.$$

The proofs of the rest assertions are similar to the first one. $\Box$

3 Conclusion

In the next parts of the research, we will study from one side other properties of the new implications and from another – the modifications of the third Dworniczak's implication and their properties.

We finish with the following two open problems.

Open Problem 1: Can all intuitionistic fuzzy implications be represented by similar way by operator $\boxdot$?

Open Problem 2: Can the four new intuitionistic fuzzy implications be represented by similar way by operator $\boxplus$ and $\boxtimes$, as it is done in [6] for the introduced there four intuitionistic fuzzy implications?

References

[1] Atanassov, K. *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin, 2012.

[2] Atanassov, K., Szmidt E., Kacprzyk J. On intuitionistic fuzzy pairs. *Notes on Intuitionistic Fuzzy Sets*, Vol. 19, 2013, No. 3, 1–13.

[3] Atanassova, L., A new intuitionistic fuzzy implication, *Cybernetics and Information Technologies*, Vol. 9, 2009, No. 2, 21–25.

[4] Atanassova, L., On two modifications of the intuitionistic fuzzy implication $\rightarrow_{@}$, *Notes on Intuitionistic Fuzzy Sets*, Vol. 18, 2012, No. 2, 26–30.

[5] Atanassova, L., On the modal form of the intuitionistic fuzzy implications $\rightarrow'_{@}$ and $\rightarrow''_{@}$, *Issues in Intuitionistic Fuzzy Sets and Generalized Nets*, Vol. 10, 2013, 5–11.

[6] Atanassova, L., Remark on Dworniczak's intuitionistic fuzzy implications. Part 1, *Notes on Intuitionistic Fuzzy Sets*, Vol. 21, 2015, No. 3, 18–23.

[7] Dworniczak, P., Some remarks about the L. Atanassova's paper "A new intuitionistic fuzzy implication", *Cybernetics and Information Technologies*, Vol. 10, 2010, 3, 3–9.

[8] Dworniczak, P., On one class of intuitionistic fuzzy implications, *Cybernetics and Information Technologies*, Vol. 10, 2010, 4, 13–21.

[9] Dworniczak, P., On some two-parametric intuitionistic fuzzy implication, *Notes on Intuitionistic Fuzzy Sets*, Vol. 17, 2011, No. 2, 8–16.