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Operation division by n over intuitionistic fuzzy sets

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The present remark is a continuation of [1, 3]. In the beginning, the necessary concepts from intuitionistic fuzzy set theory will be given.

Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Different relations and operations are introduced over the IFSs. Some of them are the following

$$\begin{array}{lll} A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\ A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)), \\ \overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\ A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\ A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\ A + B & = & \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\}, \\ A.B & = & \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle | x \in E\}. \end{array}$$

In [2] Supriya Kumar De, Ranjit Biswas and Akhil Ranjan Roy introduced two operations which are related to the last two above ones:

$$\begin{array}{lll} n.A & = & \{\langle x, 1 - (1 - \mu_A(x))^n, (\nu_A(x))^n\rangle \mid x \in E\}, \\ A^n & = & \{\langle x, (\mu_A(x))^n, 1 - (1 - \nu_A(x))^n\rangle \mid x \in E\}, \end{array}$$

where n is a natural number.

In [3] we defined operstor "extraction" over a given IFS. Now, we will introduce a new operator, defined over IFS, that will be an analogous as of operations "extraction" as well as of opertion "multiplication of an IFS with $\frac{1}{n}$" or "division of an IFS with the natural number n". It has the form for every IFS A and for every natural number $n \geq 1$:

$$\frac{1}{n}A = \{\langle x, 1 - \sqrt[n]{1 - \mu_A(x)}, \sqrt[n]{\nu_A(x)}\rangle | x \in E\}.$$

First, we must check that in a result of the operation we obtain an IFS. Really, for given IFS A, for each $x \in E$, and for each $n \geq 1$:

$$1 - \sqrt[n]{1 - \mu_A(x)} + \sqrt[n]{\nu_A(x)} \leq 1,$$

because from $\nu_A(x) \leq 1 - \mu_A(x)$ it follows that

$$\sqrt[n]{\nu_A(x)} \leq \sqrt[n]{1 - \mu_A(x)}.$$

Obviously, for every natural number $n \geq 1$:

$$\frac{1}{n}O^* = O^*,$$

$$\frac{1}{n}U^* = U^*,$$

$$\frac{1}{n}E^* = E^*.$$

By similar to the above way we can prove the following assertions.

Theorem 1: For every IFS A and for every natural number $n \geq 1$:

(a) $\frac{1}{n}(nA) = A$,

(b) $n(\frac{1}{n}A) = A$.

Theorem 2: For every IFS A and for every two natural numbers $m, n \geq 1$:

$$\frac{1}{m}(\frac{1}{n}A) = \frac{1}{mn}A = \frac{1}{n}(\frac{1}{m}A).$$

Theorem 3: For every two IFSs A and B and for every natural number $n \geq 1$:

(a) $\frac{1}{n}(A \cap B) = \frac{1}{n}A \cap \frac{1}{n}B$,

(b) $\frac{1}{n}(A \cup B) = \frac{1}{n}A \cup \frac{1}{n}B$.

Proof: We shall prove (a) and (b) is proved analogically.

$$\frac{1}{n}(A \cap B) = \frac{1}{n}(\{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\})$$
$$= \{\langle x, 1 - \sqrt[n]{1 - \min(\mu_A(x), \mu_B(x))}, \sqrt[n]{\max(\nu_A(x), \nu_B(x))}\rangle | x \in E\}$$
$$= \{\langle x, 1 - \sqrt[n]{\max(1 - \mu_A(x), 1 - \mu_B(x))}, \max(\sqrt[n]{\nu_A(x)}, \sqrt{\nu_B(x)})\rangle | x \in E\}$$
$$= \{\langle x, 1 - \max(\sqrt[n]{1 - \mu_A(x)}, \sqrt{1 - \mu_B(x)}), \max(\sqrt[n]{\nu_A(x)}, \sqrt{\nu_B(x)})\rangle | x \in E\}$$
$$= \{\langle x, \min(1 - \sqrt[n]{1 - \mu_A(x)}, 1 - \sqrt{1 - \mu_B(x)}), \max(\sqrt[n]{\nu_A(x)}, \sqrt{\nu_B(x)})\rangle | x \in E\}$$
$$= \frac{1}{n}A \cap \frac{1}{n}B.$$

Theorem 4: For every two IFSs A and B and for every natural number $n \geq 1$:

$$\frac{1}{n}(A + B) = \frac{1}{n}A + \frac{1}{n}B.$$

The simplest modal operators defined over IFSs (see, e.g., [1]) are:

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\};$$
$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.$$

They are analogous of the modal logic operators *"necessity"* and *"possibility"*. For them it is valid

Theorem 5: For every IFS A and for every natural number $n \geq 1$:

(a) $\Box \frac{1}{n}A = \frac{1}{n}\Box A$,

(b) $\Diamond \frac{1}{n}A = \frac{1}{n}\Diamond A$.

In IFSs theory some level operators are defined. Two of them are:

$$P_{\alpha,\beta}(A) = \{\langle x, \max(\alpha, \mu_A(x)), \min(\beta, \nu_A(x))\rangle | x \in E\},$$
$$Q_{\alpha,\beta}(A) = \{\langle x, \min(\alpha, \mu_A(x)), \max(\beta, \nu_A(x))\rangle | x \in E\},$$

where $\alpha + \beta \leq 1$. For them it is valid

Theorem 6: For every IFS A, for every natural number $n \geq 1$ and for every $\alpha, \beta \in [0, 1]$, so that $\alpha + \beta \leq 1$:

(a) $P_{\alpha,\beta}(\frac{1}{n}A) = \frac{1}{n}P_{1-(1-\alpha)^n, \beta^n}A$,

(b) $Q_{\alpha,\beta}(\frac{1}{n}A) = \frac{1}{n}Q_{1-(1-\alpha)^n, \beta^n}A$.

References

[1] K. Atanassov, Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[2] S.K. De, R. Biswas and A. R. Roy, Some operations on intuitionistic fuzzy sets, *Fuzzy sets and Systems,* Vol. 114, 2000, No. 4, 477-484.

[3] R. Riećan and K. Atanassov, n-extraction operation over intuitionistic fuzzy sets. Notes on Intuitionistic Fuzzy Sets, Vol. 12, 2006, No. 4, 38-40.