GENERALIZED NET MODEL OF LECTURERS’ EVALUATION OF STUDENT WORK WITH INTUITIONISTIC FUZZY ESTIMATIONS

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Abstract: A generalized net is used to construct a model which describes the process of evaluation by lecturers of the work of students. The evaluations utilize the theory of intuitionistic fuzzy sets.

Keywords: Generalized nets, Intuitionistic Fuzzy Sets, University, E-Learning.

Introduction
In a series of research papers, the authors have studied some of the most important processes of the functioning of universities (see [5]). In [4] the process of evaluation of the tasks solved by students is described by Generalized Nets (GNs, see [3]). In [6] describes the process of evaluation by lecturers of the tasks presented by students.

The evaluations corresponding to the students background about some theme are represented in intuitionistic fuzzy form (for the concept of Intuitionistic Fuzzy Set (IFS, see [1, 2]). The present paper a generalized net is used to construct a model which describes of the process of evaluation by lecturers.
Let us have \( n \) students. Let us also have \( m \) lecturers. The evaluations corresponding to the lecturers are represented by intuitionistic fuzzy estimations. They have the form \( \langle \mu_j, \nu_j \rangle \), \( \mu_j \) and \( \nu_j \) determine the degrees of strictness and non-strictness of the estimation of the \( j \)-th lecturer, \( j = 1, 2, \ldots, m \).

1 DETERMINE OF THE EVALUATIONS

Let \( s^j \) be the \( i \)-th student, who is taught by the \( j \)-th lecturer, \( i = 1, 2, \ldots, n, j = 1, 2, \ldots, m \). The \( i \)-th student obtains average evaluations \( o^\text{avr}_i \).

Of course, the way of evaluation of the different estimations will vary, but the authors consider that the evaluations of the lecturers of the tasks solved by students can be obtained, in general, by two ways:

**Version 1.**

The \( i \)-th student \( s^j \) obtains three estimations from three different lecturers. Let us note these estimations by \( o^1_{s^j,i}, o^2_{s^j,i+1} \) and \( o^3_{s^j,i+2} \), \( i = 1, 2, \ldots, n, j = 1, 2, \ldots, m \).

The estimation process has the following steps:

1. Each lecturer estimates the results of all his/her students.
   
   As a consequence, the \( i \)-th student \( s^j \) obtains estimation \( o^1_{s^j,i} \).  

2. The first lecturer estimates the results of the students of the second lecturer, he – of the third one, etc., \( m \)-th lecturer – of the first lecturer’s students.
   
   As a consequence, the \( i \)-th student \( s^j \) obtains estimation \( o^2_{s^j,i+1} \).  

3. The first lecturer estimates the results of the students of the third lecturer, he – of the fourth one, etc., \( (m-1) \)-th lecturer – of the first lecturer’s students, \( m \)-th lecturer – of the second lecturer’s students,
   
   As a consequence, the \( i \)-th student \( s^j \) obtains estimation \( o^3_{s^j,i+2} \).  

4. Order the estimations \( o^1_{s^j,i}, o^2_{s^j,i+1} \) and \( o^3_{s^j,i+2} \) of the \( i \)-th student following their order of strictness.

5. Averaging of the estimations \( o^1_{s^j,i}, o^2_{s^j,i+1} \) and \( o^3_{s^j,i+2} \) for the \( i \)-th student \( s^j \) and obtaining of the estimation \( o^\text{avr}_i \):

   \[
   o^\text{avr}_i = \frac{o^1_{s^j,i} + o^2_{s^j,i+1} + o^3_{s^j,i+2}}{3}.
   \]

6. Calculation of the intuitionistic fuzzy estimation for each lecturer on the basis of estimations \( o^1_{s^j,i}, o^2_{s^j,i+1} \) and \( o^3_{s^j,i+2} \) for \( i \)-th student \( s^j \).

In this case the evaluation of the \( j \)-th lecturer is
\[ <\mu_j, v_j> = \left\{ \frac{x_j}{n_j}, \frac{y_j}{n_j} \right\} \]

where:

- \( x_j \) is the number of the estimations, given by the \( j \)-th lecturer, that are higher than the average estimation of the \( i \)-th student \( o_{i,\text{avr}}^j \);
- \( y_j \) is the number of estimations, given by the \( j \)-th lecturer, that are smaller than the average estimation of the \( i \)-th student \( o_{i,\text{avr}}^j \);
- \( r_j \) is the number of the students estimated by the \( j \)-th lecturer.

Therefore, the degree of uncertainty here is determined by the number of the evaluations which are equal to the average estimation \( o_{i,\text{avr}}^j \) for the \( i \)-th student.

**Version 2.**

The \( i \)-th student \( s_i^j \) obtains \( m \) estimations from the \( m \) different lecturers. Let us mark them by \( o_{s_i^j}^k \), where \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \), \( k = 1, 2, \ldots, m \).

The estimation process has the following steps:

1. Each lecturer estimates the results of all his/her own students.
   
   As a consequence, the \( i \)-th student \( s_i^j \) obtains estimation \( o_{s_i^j}^k \), where \( k = j \).

2. Each lecturer estimates the results of all students whom he/she does not teach.
   
   As a consequence, the \( i \)-th student \( s_i^j \) obtains \( m-1 \) new estimations \( o_{s_i^j}^k \), where \( k = 1, 2, \ldots, j-1, j+1, \ldots, m \).

3. Order the estimations (\( m \) in numbers) \( o_{s_i^j}^1 \), \( o_{s_i^j}^2 \), \ldots, \( o_{s_i^j}^m \) obtained from the different lecturers by the \( i \)-th student, following their order of strictness.

4. Averaging of the estimations \( o_{s_i^j}^1 \), \( o_{s_i^j}^2 \), \ldots, \( o_{s_i^j}^m \) from the \( i \)-th student \( s_i^j \) to obtain estimation \( o_{i,\text{avr}}^j \):

   \[ o_{i,\text{avr}}^j = \frac{o_{s_i^j}^1 + o_{s_i^j}^2 + \ldots + o_{s_i^j}^m}{m} \]

5. Calculation of the intuitionistic fuzzy estimation for each lecturer on the basis of estimations \( o_{s_i^j}^1 \), \( o_{s_i^j}^2 \), \ldots, \( o_{s_i^j}^m \) for the \( i \)-th student \( s_i^j \).

   In this case the evaluation of the \( j \)-th lecturer is

   \[ <\mu_j, v_j> = \left\{ \frac{x_j}{n}, \frac{y_j}{n} \right\} \]

where:
\(x_j\) is the number of the estimations, given by the \(j\)-th lecturer, that are higher than the average estimation of the \(i\)-th student \(o_{i^{avr}}^j\).

\(y_j\) is the number of estimations, given by the \(j\)-th lecturer, that are smaller than the average estimation of the \(i\)-th student \(o_{i^{avr}}^j\).

Therefore, the degree of uncertainty here is determined by the number of the evaluations which are equal to the average estimation \(o_{i^{avr}}^j\) for \(i\)-th student.

2 A GN-model

The GN-model (see Fig. 1) contains 7 transitions and 16 places, collected in two groups and related to the two types of the tokens that will enter respective types of places: \(\alpha\)-tokens and \(\alpha\)-places represent the lecturers and their activities, \(\beta\)-tokens and \(\beta\)-places represent the students and their solutions of the problems.

For brevity, we shall use the notation \(\alpha\)- and \(\beta\)-tokens instead of \(\alpha_i\)- and \(\beta_j\)-tokens, where \(i, j\) are numerations of the respective tokens.

In the beginning \(\alpha\)- and \(\beta\)-tokens stay, respectively, in places \(a_3\) and \(b_3\) with initial characteristics:

\[
x_0^\alpha = \text{“name, speciality and score of a lecturer”},
\]

\[
x_0^\beta = \text{“name, speciality and current evaluations of a student”}.
\]

If we would like the model to be more detailed, the first and the latest characteristics can have, e.g., the following larger forms

\[
x_0^\alpha = \text{“name, speciality and score of a lecturer}
\text{ variant of evaluation that the lecturer uses”},
\]

\[
x_0^\beta = \text{“name, speciality and current evaluations of a student,}
\text{name of the student’s lecturer who will give the problems and/or examine the student”}.
\]

All \(\alpha\)-tokens and all \(\beta\)-tokens have equal priorities, but the priority of \(\alpha\)-tokens is higher than the priority of \(\beta\)-tokens.

The new lecturers and students enter the net via places \(a_1\) and \(b_1\) respectively.

The forms of the transitions are the following.

\[
Z_1 = \{\{a_1, a_3, a_9\}, \{a_2, a_3\},
\]

where:

\[
\begin{array}{c|cc}
   & a_2 & a_3 \\
  a_1 & false & true \\
  a_3 & W_{3,2}^a & W_{3,3}^a \\
  a_9 & false & true
\end{array}
\]

\(W_{3,2}^a = \text{“The lecturer must examine”},\)

\(W_{3,3}^a = \neg W_{3,2}^a.\)

where \(\neg P\) is the negation of predicate \(P.\)

The \(\alpha\)-tokens do not obtain any characteristic in place \(a_3\) and they obtain the characteristic

\[\text{“list of the problems that the student must solve”}\]
in place $a_2$.

$$Z_2 = \langle \{ b_1, b_5, b_7 \}, \{ b_2, b_3 \},
\begin{array}{c|cc}
b_1 & \text{false} & \text{true} \\
b_2 & W_{3,2}^b & W_{3,3}^b \\
b_3 & \text{false} & \text{true} \\
\end{array} \rangle,$$

where:

$W_{3,2}^b = \text{"The student must have examination"}$,

$W_{3,3}^b = -W_{3,2}^b$.

The $\beta$-tokens do not obtain any characteristic in places $b_2$ and $b_3$.

$$Z_3 = \langle \{ a_2, b_3 \}, \{ a_4, b_5 \},
\begin{array}{c|cc}
a_4 & b_1 \\
a_2 & \text{true} & \text{false} \\
b_2 & \text{false} & \text{true} \\
\end{array} \rangle.$$

The $\alpha$-tokens do not obtain any characteristic in place $a_4$, while $\beta$-tokens obtain characteristic “student's solutions of the problems” in place $b_4$.

$$Z_4 = \langle \{ a_4, a_5, b_4 \}, \{ a_5, a_6, b_5, b_6 \},$$
\[
\begin{array}{|c|c|c|c|}
\hline
 & a_5 & a_6 & b_5 & b_6 \\
\hline
a_4 & W_{4,5}^a & W_{4,6}^a & \text{false} & \text{false} \\
a_5 & W_{5,5}^a & W_{5,6}^a & \text{false} & \text{false} \\
b_4 & \text{false} & \text{false} & W_{4,5}^b & W_{4,6}^b \\
\hline
\end{array}
\]

where:
\[W_{4,5}^a = W_{5,5}^a = \text{“There are students whose research must be evaluated by the current lecturer”},\]
\[W_{4,6}^a = W_{5,6}^a = -W_{4,5}^a,\]
\[W_{4,5}^b = \text{“The lecturer that will examine the present research prefers First way for evaluation”},\]
\[W_{4,6}^b = \text{“The lecturer that will examine the present research prefers Second way for evaluation”}.\]

The \(\alpha\)-tokens do not obtain any characteristic in places \(a_5\) and \(a_6\).

The \(\beta\)-tokens enter one of the output places \(b_5\) or \(b_6\) of transition \(Z_4\) obtaining characteristic

\[\text{“estimation of the current student’s problems”}.\]

\[Z_5 = <\{a_6\}, \{a_7, a_8\},\]
\[\begin{array}{c|c}
\hline
 & a_7 & a_8 \\
\hline
a_6 & W_{6,7}^a & W_{6,8}^a \\
\hline
\end{array}>,\]

where:
\[W_{6,7}^a = \text{“Calculation of the intuitionistic fuzzy estimation for the lecturer prefers First way for evaluation”},\]
\[W_{6,8}^a = \text{“Calculation of the intuitionistic fuzzy estimation for the lecturer prefers Second way for evaluation”}.\]

The \(\alpha\)-tokens enter one of the two output places of transition \(Z_5\) obtaining characteristic

\[\text{“estimation of the lecturer’s score for the current examination”}.\]

This estimation for the \(j\)-th lecturer can be obtained in the following intuitionistic fuzzy form
\[<\mu_j, \nu_j> = \left\{ \begin{array}{c}
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
\end{array} \right\},\]
in place \(a_7\)

and
\[<\mu_j, \nu_j> = \left\{ \begin{array}{c}
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
x_j, y_j \\
\end{array} \right\},\]
in place \(a_8\),

where:
\(x_j\) is the number of the estimations, given by the \(j\)-th lecturer, that are higher than the average estimation of the \(i\)-th student;
\(y_j\) is the number of estimations, given by the \(j\)-th lecturer, that are smaller than the average estimation of the \(i\)-th student;
\(r_j\) is the number of the students estimated by the \(j\)-th lecturer.

\[
Z_6 = \langle \{ b_5, b_6 \}, \{ b_7 \}, < b_7 \mid \text{true}>, \ b_5 \mid \text{true}, \ b_6 \mid \text{true} >.
\]

The \(\beta\)-tokens obtain characteristic

“average estimation of the student”

in place \(b_7\).

\[
Z_7 = \langle \{ a_7, a_8 \}, \{ a_9 \}, < a_9 \mid \text{true}>, \ a_7 \mid \text{true}, \ a_8 \mid \text{true} >.
\]

The \(\alpha\)-tokens do not obtain any characteristic in place \(a_9\).

3 Conclusion

The paper describes a way of analyzing the estimations of the lecturer’s strictness since this is a personal characteristic which can be difficult to measure by traditional means. Having in mind the personal criteria of the lecturers, we can obtain more objective estimations of the students’ results, thus altering the lecturers’ estimations with the to accommodate their level of strictness. The present model is an element of a more general model describing different processes, flowing in a university.

4 References