Intuitionistic fuzzy regular weakly generalized connected spaces

P. Rajarajeswari[1] and L. Senthil Kumar[2,*]

[1] Department of Mathematics, Chikkanna Government Arts College
Tirupur-641 602, Tamilnadu, India
E-mail: `p.rajarajeswari29@gmail.com`

[2] Department of Mathematics, SVS College of Engineering
Coimbatore-642 109, Tamilnadu, India
E-mail: `senthilmaths29@gmail.com`
Corresponding author

Abstract: The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy regular weakly generalized connected spaces. Some of their properties are explored.

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1 Introduction

Fuzzy set (FS), proposed by Zadeh [14] in 1965, as a framework to encounter uncertainty, vagueness and partial truth, represents a degree of membership for each member of the universe of discourse to a subset of it. Turnali and Coker [13] have introduced and investigated connectedness in intuitionistic fuzzy topological spaces in the year 2000. Later intuitionistic fuzzy rg-connectedness was introduced by Thakur and Rekka Chaturvedi [12] in 2006. Recently many fuzzy topological concepts such as fuzzy connectedness and fuzzy compactness have been generalized in intuitionistic fuzzy topological spaces.

In this paper we have introduced intuitionistic fuzzy regular weakly generalized connected spaces. Also we have provided some characterizations of intuitionistic fuzzy regular weakly generalized connectedness.

2 Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A={⟨ x, μ_A (x), ν_A(x) ⟩ / x ∈ X} where the functions μ_A(x): X → [0, 1] and ν_A(x): X → [0, 1] denote the degree of membership (namely μ_A(x)) and the degree of non-membership (namely ν_A(x)) of each element x ∈ X to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each x ∈ X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form A = {⟨ x, μ_A(x), ν_A(x)⟩ / x∈ X} and B = {⟨ x, μ_B(x), ν_B(x) ⟩ / x ∈ X}. Then
(a) A ⊆ B if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all x ∈ X
(b) A = B if and only if A ⊆ B and B ⊆ A
(c) A^c = {⟨ x, ν_A(x), μ_A(x) ⟩ / x ∈ X}
(d) A ∩ B = {⟨ x, μ_A(x) ∧ μ_B(x), ν_A(x) ∨ ν_B(x) ⟩ / x ∈ X}
(e) A ∪ B = {⟨ x, μ_A(x) ∨ μ_B(x), ν_A(x) ∧ ν_B(x) ⟩ / x ∈ X}.
 For the sake of simplicity, we shall use the notation A = ⟨ x, μ_A, ν_A ⟩ instead of A = {⟨ x, μ_A(x), ν_A(x)⟩ / x ∈X}. Also for the sake of simplicity, we shall use the notation A =⟨ x, (μ_A, μ_B), (ν_A, ν_B) ⟩ instead of A =⟨ x, (A/μ_A, B/μ_B), (A/ν_A, B/ν_B)⟩.
 The intuitionistic fuzzy sets $0_\sim$ = {⟨ x, 0, 1 ⟩ / x ∈ X} and $1_\sim$ = {⟨ x, 1, 0 ⟩ / x ∈ X} are respectively the empty set and the whole set of X.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:
(a) $0_\sim$, $1_\sim$ ∈ τ
(b) $G_1 \cap G_2$ ∈ τ for any G_1, G_2 ∈ τ
(c) ∪ G_i ∈ τ for any arbitrary family {G_i / i∈ J} ⊆ τ.
 In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.
 The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and A = ⟨ x, μ_A, ν_A ⟩ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by
int(A) = ∪ {G / G is an IFOS in X and G ⊆ A}
cl(A) = ∩ {K / K is an IFCS in X and A ⊆ K}.
 Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$ [14].

Definition 2.5: An IFS A = {⟨ x, μ_A (x), ν_A(x) ⟩ / x ∈ X} in an IFTS (X, τ) is said to be an
(a) [4] *intuitionistic fuzzy semi closed set* (IFSCS in short) if int(cl (A)) ⊆ A
(b) [4] *intuitionistic fuzzy α-closed set* (IFαCS in short) if cl(int(cl(A))) ⊆ A
(c) [4] *intuitionistic fuzzy pre-closed set* (IFPCS in short) if cl(int(A)) ⊆ A
(d) [4] *intuitionistic fuzzy regular closed set* (IFRCS in short) if cl(int(A)) = A

(e) [12] *intuitionistic fuzzy generalized closed set* (IFGCS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS

(f) [10] *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if scl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS

(g) [8] *intuitionistic fuzzy α generalized closed set* (IFαGCS in short) if αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IαFGOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6: [5] An IFS A = $\{\langle$ x, μ_A (x), ν_A(x) $\rangle$ / x $\in$ X$\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy regular weakly generalized closed set* (IFRWGCS in short) if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is an IFROS in X.
The family of all IFRWGCSs of an IFTS (X, τ) is denoted by IFRWGCS(X).

Definition 2.7: [5] An IFS A is said to be an *intuitionistic fuzzy regular weakly generalized open set* (IFRWGOS in short) in (X, τ) if the complement A^c is an IFRWGCS in X.
The family of all IFRWGOSs of an IFTS (X, τ) is denoted by IFRWGO(X).

Result 2.8: [5] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFRWGCS but the converses need not be true in general.

Definition 2.9: [6] Let (X, τ) be an IFTS and A = $\langle$x, μ_A, $\nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by
rwgint(A) = $\cup$ {G / G is an IFRWGOS in X and G $\subseteq$ A}
rwgcl(A) = $\cap$ {K / K is an IFRWGCS in X and A $\subseteq$ K}.

Definition 2.10: [3] Let f be a mapping from an IFS X to an IFS Y. If B = $\{\langle$y, μ_B(y), ν_B(y)$\rangle$ / y$\in$Y$\}$ is an IFS in Y, then the pre-image of B under f denoted by f^{-1}(B), is the IFS in X defined by f^{-1}(B) = $\{\langle$ x, f^{-1}(μ_B(x)), f^{-1} (ν_B(x)) $\rangle$ / x $\in$ X$\}$. If A = $\{\langle$ x, λ_A(x), ν_A(x) $\rangle$ / x $\in$ X$\}$ is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by f (A) = $\{\langle$y, f (λ_A(y)), f_(ν_A(y)) $\rangle$ / y $\in$ Y$\}$ where f_(ν_A) = 1 $-$ f(1 $-$ ν_A).

Definition 2.11: [7] A mapping f: (X, τ) $\to$ (Y, σ) is called an *intuitionistic fuzzy regular weakly generalized continuous* (IFRWG continuous in short) if f^{-1}(B) is an IFRWGCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.12: [6] A mapping f: (X, τ) $\to$ (Y,σ) is called an *intuitionistic fuzzy regular weakly generalized irresolute* (IFRWG irresolute in short) if f^{-1}(B) is an IFRWGCS in (X, τ) for every IFRWGCS B of (Y, σ).

Definition 2.13: A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ from an IFTS (X, τ) into an IFTS (Y, σ) is said to be an

(a) [11] intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X.

(b) [4] intuitionistic fuzzy semi closed mapping (IFSCM for short) if f(A) is an IFSCS in Y for every IFCS A in X.

(c) [4] intuitionistic fuzzy pre-closed mapping (IFPCM for short) if f(A) is an IFPCS in Y for every IFCS A in X.

(d) [4] intuitionistic fuzzy α-closed mapping (IFαCM for short) if f(A) is an IFαCS in Y for every IFCS A in X.

(e) [9] intuitionistic fuzzy α-generalized closed mapping (IFαGCM for short) if f(A) is an IFαGCS in Y for every IFCS A in X.

Definition 2.14: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $IF_{rw}T_{1/2}$ space if every IFRWGCS in X is an IFCS in X.

Definition 2.15: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $IF_{rwg}T_{1/2}$ space if every IFRWGCS in X is an IFPCS in X.

Definition 2.16: [10] Two IFSs are said to be q-coincident (AqB in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ and $\nu_A(x) < \mu_B(x)$. For any two IFSs A and B of X, AqB if and only if $A \subseteq B^c$.

Definition 2.17: [12] An IFTS (X, τ) is said to be intuitionistic fuzzy C_5-connected space if the only intuitionistic fuzzy sets which are both IFOS and IFCS are $0_\sim$ and $1_\sim$.

Definition 2.18: [12] An IFTS (X, τ) is said to be intuitionistic fuzzy GO-connected space if the only intuitionistic fuzzy sets which are both IFGOS and IFGCS are $0_\sim$ and $1_\sim$.

3 Intuitionistic fuzzy regular weakly generalized connected spaces

In this section we have introduced intuitionistic fuzzy regular weakly generalized connected (IFRWG connected in short) space and studied some of its properties.

Definition 3.2.1: An IFTS (X, τ) is said to be an IFRWG connected space if the only intuitionistic fuzzy sets which are both IFRWGOS and IFRGCS are $0_\sim$ and $1_\sim$.

Theorem 3.2.2: Every IFRWG connected space is an intuitionistic fuzzy C_5-connected space but not conversely.
Proof: Let (X, τ) be an IFRWG connected space. Suppose (X, τ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X, τ). That is A is both IFRWGOS and IFRWGCS in (X, τ).

This implies that (X, τ) is not an IFRWG connected space. Thus we get a contradiction. Hence (X, τ) must be an intuitionistic fuzzy C_5-connected space. $\square$

Example 3.2.3: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ be an IFT on X, where $T_1 = < x, (0.3, 0.4, 0.4), (0.6, 0.5, 0.6) >$ and $T_2 = < x, (0.5, 0.4, 0.7), (0.2, 0.3, 0.3) >$. Then (X, τ) is an intuitionistic fuzzy C_5-connected space. But (X, τ) is not an IFRWG connected space since the IFS $A = < x, (0.2, 0.3, 0.4), (0.8, 0.7, 0.6)>$ is both IFRWGOS and IFRWGCS in (X, τ).

Theorem 3.2.4: Every IFRWG connected space is an intuitionistic fuzzy GO-connected space but not conversely.
Proof: Let (X, τ) be an IFRWG connected space. Suppose (X, τ) is not an intuitionistic fuzzy GO-connected space, then there exists a proper IFS A which is both IFGOS and IFGCS in (X, τ). That is A is both IFRWGOS and IFRWGCS in (X, τ). This implies that (X, τ) is not an IFRWG connected space. Thus we get a contradiction. Therefore (X, τ) must be an intuitionistic fuzzy GO-connected space. $\square$

Example 3.2.5: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ be an IFT on X, where $T_1 = < x, (0.2, 0.4, 0.3), (0.6, 0.6, 0.7) >$ and $T_2 = < x, (0.3, 0.4, 0.7), (0.4, 0.3, 0.2) >$. Then (X, τ) is an intuitionistic fuzzy GO-connected space. But (X, τ) is not an IFRWG connected space since the IFS $A = < x, (0.1, 0.2, 0.4), (0.2, 0.6, 0.6) >$ is both IFRWGOS and IFRWGCS in (X, τ).

The following implications are true:

IFRWG connected

IFGO connected   IFC₅-connected
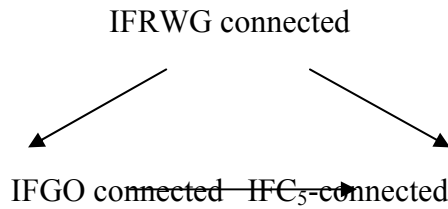
The reverse implications are not true in general.

Theorem 3.2.6: The IFTS (X, τ) is an IFRWG connected space if and only if there exists no non-zero IFRWGOSs A and B in (X, τ) such that $A = B^c$.
Proof:
Necessity: Let A and B be two IFRWGOSs in (X, τ) such that $A \neq 0_\sim$, $B \neq 0_\sim$ and $A = B^c$. Therefore B^c is an IFRWGCS. Since $A \neq 0_\sim$ and $B \neq 1_\sim$, B is a proper IFS which is both IFRWGOS and IFRWGCS in (X, τ). Hence (X, τ) is not an IFRWG connected space. But it is a contradiction to our hypothesis. Hence there exists no non-zero IFRWGOSs A and B in (X, τ) such that $A = B^c$.

Sufficiency: Let A be both IFRWGOS and IFRWGCS in (X, τ) such that $0_\sim \neq A \neq 1_\sim$. Now let $B = A^c$. Then B is an IFRWGOS and $B \neq 1_\sim$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Hence (X, τ) is an IFRWG connected space. $\square$

Theorem 3.2.7: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFRWG continuous mapping and (X, τ) is an IFRWG connected space, then (Y, σ) is an intuitionistic fuzzy C_5 connected space.

Proof: Let (X, τ) be an IFRWG connected space. Suppose (Y, σ) is not an intuitionistic fuzzy C_5-connected space, then there exists a proper IFS A which is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (Y, σ). Since f is an IFRWG continuous mapping, $f^{-1}(A)$ is both IFRWGOS and IFRWGCS in (X, τ). But it is a contradiction to our hypothesis. Hence (Y, σ) must be an intuitionistic fuzzy C_5-connected space. $\square$

Theorem 3.2.8: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFRWG irresolute mapping and (X, τ) is an IFRWG connected space, then (Y, σ) is an IFRWG connected space.

Proof: Suppose (Y, σ) is not an IFRWG connected space, then there exists a proper IFS A such that A is both IFRWGOS and IFRWGCS in (Y, σ). Since f is an IFRWG irresolute mapping, $f^{-1}(A)$ is both IFRWGOS and IFRWGCS in (X, τ). But this is a contradiction to our hypothesis. Hence (Y, σ) must be an IFRWG connected space. $\square$

Definition 3.2.9: An IFTS (X, τ) is an intuitionistic fuzzy C_5-connected between two IFSs A and B if there is no intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and EqB.

Definition 3.2.10: An IFTS (X, τ) is an IFRWG connected between two IFSs A and B if there is no IFRWGOS E in (X, τ) such that $A \subseteq E$ and EqB.

Example 3.2.11: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, T_1, 1_\sim\}$ be an IFT on X, where $T_1 = < x, (0.2, 0.3, 0.3), (0.7, 0.6, 0.7) >$. Then the IFTS (X, τ) is IFRWG connected between the two IFSs $A = <x, (0.5, 0.3, 0.7), (0.4, 0.6, 0.3)>$ and $B = <x, (0.6, 0.7, 0.8), (0.3, 0.2, 0.2)>$.

Theorem 3.2.12: If an IFTS (X, τ) is an IFRWG connected between two IFSs A and B, then it is an intuitionistic fuzzy C_5-connected between A and B but the converse may not be true in general.

Proof: Suppose (X, τ) is not an intuitionistic fuzzy C_5-connected between A and B, then there exists an intuitionistic fuzzy open set E in (X, τ) such that $A \subseteq E$ and EqB. Since every intuitionistic fuzzy open set is an IFRWGOS, there exists an IFRWGOS E in (X, τ) such that $A \subseteq E$ and EqB. This implies (X, τ) is not an IFRWG connected between A and B. Thus we get a contradiction to our hypothesis. Therefore the IFTS (X, τ) must be an intuitionistic fuzzy C_5-connected between A and B. $\square$

Example 3.2.13: Let $X = \{a, b, c\}$ and $\tau = \{0_\sim, T_1, 1_\sim\}$ be an IFT on X, where $T_1 = < x, (0.2, 0.4, 0.5), (0.3, 0.6, 0.4) >$. Then (X, τ) is an intuitionistic fuzzy C_5-connected between the IFSs $A = < x, (0.3, 0.4, 0.5), (0.7, 0.6, 0.5) >$ and $B = < x, (0.2, 0.3, 0.2), (0.5, 0.4, 0.7) >$. But (X, τ) is not an IFRWG connected between A and B, since the IFS $E = < x, (0.4, 0.4, 0.6), (0.6, 0.5, 0.4) >$ is an IFRWGOS such that $A \subseteq E$ and $E \subseteq B^c$.

Theorem 3.2.14: If an IFTS (X, τ) is IFRWG connected between A and B and $A \subseteq A_1$, $B \subseteq B_1$, then (X, τ) is an IFRWG connected between A_1 and B_1.

Proof: Suppose that (X, τ) is not an IFRWG connected between A_1 and B_1, then by definition, there exists an IFRWGOS E in (X, τ) such that $A_1 \subseteq E$ and EqB_1. This implies EqB_1^c and $A_1 \subseteq E$. That is $A \subseteq A_1 \subseteq E$. Hence $A \subseteq E$. Since $E \subseteq B_1^c$, $B_1^c \subseteq E^c$, That is $B \subseteq B1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore (X, τ) is not an IFRWG connected between A and B. thus we get a contradiction to our hypothesis. Hence X must be IFRWG connected between A_1 and B_1. $\qquad\square$

Theorem 3.2.15: Let (X, τ) be an IFTS and A and B be IFSs in (X, τ). If AqB, then X is an IFRWG connected between A and B.

Proof: Suppose (X, τ) is not IFRWG connected between A and B. Then there exists an IFRWGOS E in (X, τ) such that $A \subseteq E \subseteq E \subseteq B^c$. This implies that $A \subseteq B^c$. That is AqB. But this is a contradiction to our hypothesis. Hence X is must be an IFRWG connected between A and B. $\qquad\square$

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