On intuitionistic fuzzy trees

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\section{Introduction}

The concept of Intuitionistic Fuzzy Graph (IFG) was introduced in 1994 in [11]. It was an object of some subsequent extensions (see [4, 6, 12]), representations (see [2, 3, 5]) and applications (see [5]).

In [8] we discussed an intuitionistic fuzzy version of the special particular case of a graph – the tree, called an \textit{Intuitionistic Fuzzy Tree (IFTree)}. In [9] we gave the index matrix (IM, see [1, 7]) interpretation and gave an example for an application of the IFTrees.

Here we shall introduce operation "substitution" of a leaf of an IFTrees with whole IFTrees.

2 Definition and properties of intuitionistic fuzzy trees

Let a set \( E \) be fixed. An IFS \( A \) in \( E \) is an object of the following form:

\[
A = \{ (x, \mu_A(x), \nu_A(x)) | x \in E \},
\]

where functions \( \mu_A : E \to [0,1] \) and \( \nu_A : E \to [0,1] \) determine the degree of membership and the degree of non-membership of the element \( x \in E \), respectively, and for every \( x \in E \):

\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1.
\]

Let the oriented graph \( G = (V,A) \) be given, where \( V \) is a set of vertices and \( A \) is a set of arcs. Every graph arc connects one or two graph vertices (see, e.g., [10]).

Following [11], we shall note that the set

\[
A^* = \{ (\langle v, w \rangle, \mu_A(v, w), \nu_A(v, w)) | \langle v, w \rangle \in V \times V \}
\]

is called an IFG if the functions \( \mu_A : V \times V \to [0,1] \) and \( \nu_A : V \times V \to [0,1] \) define the respective degrees of membership and non-membership of the element \( \langle v, w \rangle \in V \times V \) and for all \( \langle v, w \rangle \in V \times V \):

\[
0 \leq \mu_A(v, w) + \nu_A(v, w) \leq 1.
\]

The above definition can be transformed directly to the case of IFTree, but we will extend the new object.

Let us have a (fixed) set of vertices \( V \). An IFTree \( T \) (over \( V \)) will be the ordered pair \( T = (V^*, A^*) \), where

\[
V \subseteq V,
\]

\[
V^* = \{ (v, \mu_V(v), \nu_V(v)) | v \in V \},
\]

\[
A \subseteq V \times V,
\]

\[
A^* = \{ (g, \mu_A(g), \nu_A(g)) | (\exists v, w \in V)(g = \langle v, w \rangle \in A) \},
\]

where

\( \mu_V(v) \) and \( \nu_V(v) \) are degrees of membership and non-membership of the element \( v \in V \) to \( V \) and

\[
0 \leq \mu_V(v) + \nu(v) \leq 1.
\]

\[
\mu_A(g) \quad \nu_A(g)
\]

\[
\forall v, w \in V.
\]

b) strong

\[
\nu_A(g) = \mu_A(g)
\]

\[
\forall v, w \in V.
\]

\[
\text{average}
\]

\[
\mu_A(g) = \overline{\mu_A(g)}
\]

\[
\forall v, w \in V.
\]

Let \( T \) be an IFTree and let for

\[
\text{Theorem:}
\]

\[
\mu_A(g) \quad \nu_A(g)
\]

\[
\forall v, w \in V.
\]
IFTrees with fuzzy trees

The degree of vertices and edges (see, e.g., [7])$\{\}

$\times V \rightarrow [0, 1]$ of the element

Let two IFTrees $T_1 = (V_1^*, A_1^*)$ and $T_2 = (V_2^*, A_2^*)$ be given. We define:

$T_1 \cup T_2 = (V_1^* \cup V_2^*, A_1^* \cup A_2^*)$,

$T_1 \cap T_2 = (V_1^* \cap V_2^*, A_1^* \cap A_2^*)$.

Let

$\mathcal{P}(X) = \{Y \mid Y \subset X\}$,

and let for $T = (V^*, A^*)$

$T_{full} = (E(V), E(A))$,

$T_{empty} = (O(V), O(A))$,

where

$E(V) = \{(v, 1, 0) \mid v \in V\}$,

$O(V) = \{(v, 0, 1) \mid v \in V\}$,

$E(A) = \{(g, 1, 0) \mid (\exists v, w \in V)(g = (v, w)) \in V \times V\}$,

$O(A) = \{(g, 0, 1) \mid (\exists v, w \in V)(g = (v, w)) \in V \times V\}$.

Theorem: $(\mathcal{P}(V), \cup, T_{empty})$ and $(\mathcal{P}(V), \cap, T_{full})$ are commutative monoids.
3 Index matrix interpretation of the intuitionistic fuzzy trees

Following [1] the basic definitions and properties related to IMs will be given.

Let \( I \) be a fixed set of indices and \( \mathcal{R} \) be the set of the real numbers. By an IM with index sets \( K \) and \( L \) \((K, L \subset I)\) we will consider the object:

\[
[K, L, \{a_{k_l, l_j}\}] = \begin{bmatrix}
  l_1 & l_2 & \cdots & l_n \\
  k_1 & a_{k_1, l_1} & a_{k_1, l_2} & \cdots & a_{k_1, l_n} \\
  k_2 & a_{k_2, l_1} & a_{k_2, l_2} & \cdots & a_{k_2, l_n} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  k_m & a_{k_m, l_1} & a_{k_m, l_2} & \cdots & a_{k_m, l_n}
\end{bmatrix}
\]

(or briefly: \([K, L, \{a_{k_l, l_j}\}]\)), where \( K = \{k_1, k_2, \ldots, k_m\} \), \( L = \{l_1, l_2, \ldots, l_n\} \), for \( 1 \leq i \leq m \), and for \( 1 \leq j \leq n \) : \( a_{k_l, l_j} \in \mathcal{R} \) – the set of the real numbers.

For any two IMs different operations and relations are defined in [1, 7]. Here we will give only one of them.

Let \( G = (V, A) \) be a given IFTree. We can construct its standard incidence matrix. After this, we can change the elements of the matrix with their degrees of membership and non-membership. Finally, numbering the rows and columns of the matrix with the identifiers of the IFTree vertices, we will obtain an IM.

![Image](image1.png)

Figure 1.

For example, if we have the IFTree from Fig. 1, we can construct the IM that corresponds to its incidence matrix:

\[
\begin{array}{c|cccc}
   & A & B & C & D \\
\hline
  A & (0, 1) & \mu & \mu & \mu \\
  B & (0, 1) & (0, 0) & \mu & \mu \\
  C & (0, 1) & (0, 0) & (0, 0) & \mu \\
  D & (0, 1) & (0, 0) & (0, 0) & (0, 0)
\end{array}
\]

Having this matrix we can modify the elements by:

\[
\begin{array}{c|c}
  & B \\
\hline
  A & (0, 1) \\
  B & (0, 0) \\
  C & (0, 0) \\
  D & (0, 0)
\end{array}
\]

Finally, we can index the rows and columns of the matrix:

\[
\begin{array}{c|c}
  & B \\
\hline
  A & (\mu) \\
  B & (0, 0)
\end{array}
\]
Having in mind that arcs $AA, AC, AD, BB, CC, CD$ and $DD$ do not exist, we can modify the above IM to the form:

$$\begin{align*}
\{\{A, B, C, D\}, \{A, B, C, D\},
\begin{array}{c|cc}
A & B & C \\
\hline
A & \langle \mu(A, A), \nu(A, A) \rangle & \langle \mu(A, B), \nu(A, B) \rangle \\
B & \langle \mu(B, A), \nu(B, A) \rangle & \langle \mu(B, B), \nu(B, B) \rangle \\
C & \langle \mu(C, A), \nu(C, A) \rangle & \langle \mu(C, B), \nu(C, B) \rangle \\
D & \langle \mu(D, A), \nu(D, A) \rangle & \langle \mu(D, B), \nu(D, B) \rangle \\
\hline
C & \langle \mu(A, C), \nu(A, C) \rangle & \langle \mu(A, D), \nu(A, D) \rangle \\
D & \langle \mu(B, C), \nu(B, C) \rangle & \langle \mu(B, D), \nu(B, D) \rangle \\
\end{array}
\end{align*}$$

Now, we see, that all elements of the column indexed with $A$ and all elements of the rows indexed with $C$ and $D$ are $\langle 0, 1 \rangle$. Therefore, we can omit these two rows and the column and we will obtain the essentially simpler IM:

$$\begin{align*}
\{\{A, B, C, D\}, \{A, B, C, D\},
\begin{array}{c|cc}
A & B & C \\
\hline
A & \langle \mu(A, B), \nu(A, B) \rangle & \langle 0, 1 \rangle \\
B & \langle 0, 1 \rangle & \langle \mu(B, C), \nu(B, C) \rangle & \langle \mu(B, D), \nu(B, D) \rangle \\
C & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
D & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
\end{array}
\end{align*}$$

Finally, having in mind that there is no more a column indexed with $A$ and rows indexed with $C$ and $D$, we obtain as a final form of the IM:

$$\begin{align*}
\{\{A, B\}, \{B, C, D\},
\begin{array}{c|cc}
B & C & D \\
\hline
A & \langle \mu(A, B), \nu(A, B) \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\
B & \langle 0, 1 \rangle & \langle \mu(B, C), \nu(B, C) \rangle & \langle \mu(B, D), \nu(B, D) \rangle \\
\end{array}
\end{align*}$$
4 Operation “substitution of an IFTree’s leaf with an IFTree”

Let us have an IFTree $G = (V, A)$ and let $L$ be one of its leaves. Let $F = (W, B)$ be another IFTree so that

$$V \cap W = \{L\},$$
$$A \cup B = \emptyset.$$ 

Then, following definitions from [7], we can describe the result of operation “substitution of an IFTree’s leaf $L$ with the IFTree $F$. The result will have the form of the IFTree $(V \cup W, A \cup B)$.

For example, if $G$ is the IFTree from Fig. 1 and if we like to substitute its leaf $D$ with the IFTree $F$ from Fig. 2 that has the shorter IM-representation

![Diagram of IFTrees]

Figure 2.

$$\{D, E\}, \{E, F; G\},$$

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$F$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>$(\mu(D, E), \nu(D, E))$</td>
<td>$(0, 1)$</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$(0, 1)$</td>
<td>$(\mu(E, F), \nu(E, F))$</td>
<td>$(\mu(E, G), \nu(E, G))$</td>
</tr>
</tbody>
</table>

then, the result will be the IFTree from Fig. 3 and it will have the IM-representation

$$\{A, B, D, E\}, \{B, C, D, E, F, G\},$$

<table>
<thead>
<tr>
<th></th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$(\mu(A, B), \nu(A, B))$</td>
<td>$(0, 1)$</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>$B$</td>
<td>$(0, 1)$</td>
<td>$(\mu(B, C), \nu(B, C))$</td>
<td>$(\mu(B, D), \nu(B, D))$</td>
</tr>
<tr>
<td>$D$</td>
<td>$(0, 1)$</td>
<td>$(0, 1)$</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$(0, 1)$</td>
<td>$(0, 1)$</td>
<td>$(0, 1)$</td>
</tr>
</tbody>
</table>

182

P. R. Rakhmanov
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\[
\begin{array}{c|ccc}
  & E & F & G \\
  A & (0, 1) & (0, 1) & (0, 1) \\
  B & (0, 1) & (0, 1) & (0, 1) \\
  D & (\mu(D, E), \nu(D, E)) & (0, 1) & (0, 1) \\
  E & (0, 1) & (\mu(E, F), \nu(E, F)) & (\mu(E, G), \nu(E, G)) \\
\end{array}
\]

\[= \{(A, B, D, E), (B, C, D, E, F, G),
\]

\[
\begin{array}{c|ccc}
  & B & C & D \\
  A & (\mu(A, B), \nu(A, B)) & (0, 1) & (0, 1) \\
  B & (0, 1) & (\mu(B, C), \nu(B, C)) & (\mu(B, D), \nu(B, D)) \\
\end{array}
\]

\[
\begin{array}{c|ccc}
  & E & F & G \\
  D & (\mu(D, E), \nu(D, E)) & (0, 1) & (0, 1) \\
  E & (0, 1) & (\mu(E, F), \nu(E, F)) & (\mu(E, G), \nu(E, G)) \\
\end{array}
\]

Figure 3.

Acknowledgement

P. Rangasamy and K. Atanassov are grateful for the support provided by the Bulgarian-Indian bilateral project of the Bulgarian Ministry of Education and Science (Grant BIn-2/09) and Department of Science and Technology, India.
References


1 Introduction

Metaset is a graph-theoretic formalism, based on the intuitionistic fuzzy sets and their graphs, which are in the form of binary relations, fuzzy sets are a special case of the intuitionistic fuzzy sets.

The graph-theoretic construction of metaset is based on the idea of representing the set of objects by means of the set of nodes and the set of relations between them by means of the set of edges. The set of nodes and the set of edges form the graph that represents the set of objects of metaset.

The graph-theoretic approach to the representation of the metaset leads to the construction of a graph that represents the metaset.

Keywords

Developmental Computing, Metaset, Fuzzy Graphs, Fuzzy Sets, Computational Intelligence, M. Krawczak

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