Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 20, 2014, No. 3, 42–52

Common fixed point theorems in ϵ-chainable intuitionistic fuzzy metric spaces

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Abstract: In this paper, we prove a common fixed point by using a new notion of absorbing maps in ϵ-chainable intuitionistic fuzzy metric space with reciprocal continuity and semi-compatible maps. Ours result generalizes results of Ranadive et al. [10, 11], A. Jain et al. [6], Y. Bano et al. [4] and M. Verma et al. [13] in intuitionistic fuzzy metric spaces.
Keywords: Absorbing maps, Semi-compatible mapping, Reciprocal continuity, Intuitionistic fuzzy metric space.
AMS Classification: 54H25, 47H10.

1 Introduction

Zadeh [14] introduced the notion of fuzzy sets. Later many authors have extensively developed the theory of fuzzy sets and its application. The idea of fuzzy metric space introduced by Kramosil et al. [7] and modified by George et al. [5]. Atanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [1–4, 8, 9, 12, 13]. In 2004, Park [9] introduced a notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil et al. [7]. The aim of this paper is to introduce the new notion of absorbing maps, it is not necessary that absorbing maps commute at their coincidence points however if the mapping pair satisfy the contractive type condition then point wise absorbing maps not only commute at their coincidence points but it becomes a necessary condition for obtaining a common fixed point of mapping pair.

In this paper, we prove a common fixed point by using this new notion of absorbing maps in ϵ-chainable intuitionistic fuzzy metric space with reciprocal continuity and semi-compatible maps. Ours result generalizes results of Ranadive et al. [10, 11], A. Jain et al. [6], Y. Bano et al. [4] and M. Verma et al. [13] in intuitionistic fuzzy metric spaces.

2 Preliminaries

In this section, we recall some definitions and known results in intuitionistic fuzzy metric spaces.

Definition 2.1 [1, 12]: A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

(i) $a * 1 = a$

(ii) $a * b = b * a$

(iii) $a * b = c * d$, whenever $a = c$ and $b = d$

(iv) $a * (b * c) = (a * b) * c$.

Definition 2.2 [1, 12]: A binary operation $\Diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if it satisfies the following conditions

(a) $\Diamond$ is commutative and associative

(b) $\Diamond$ is continuous

(c) $a \Diamond 0 = a$ for all $a \in [0, 1]$

(d) $a \Diamond b = c \Diamond d$ whenever $a \geq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 2.3 [1, 12]: A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t-norm $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$

(IFM-2) $M(x, y, 0) = 0$

(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$

(IFM-4) $M(x, y, t) = M(y, x, t)$

(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(IFM-6) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous

(IFM-7) $\lim_{n\to\infty} M(x, y, t) = 1$

(IFM-8) $N(x, y, 0) = 1$

(IFM-9) $N(x, y, t) = 0$ if and only if $x = y$

(IFM-10) $N(x, y, t) = N(y, x, t)$

(IFM-11) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$

(IFM-12) $N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous

(IFM-13) $\lim_{n\to\infty} N(x, y, t) = 0$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4 [1, 12]: Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space, if X of the form $(X, M, 1 - M, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated, that is, $x \Diamond y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in X$. But the converse is not true.

Example 2.5 [1, 12]: Let (X, d) be a metric space. Denote $a * b = ab$ and $a \Diamond b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = t/t + d(x, y),$$
$$N_d(x, y, t) = d(x, y)/t + d(x, y).$$

Then (M_d, N_d) is an intuitionistic fuzzy metric on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.6 [1, 12]: Note the above example holds even with the t-norm $a * b = \min\{a, b\}$ and the t-conorm $a \Diamond b = \max\{a, b\}$ and hence (M_d, N_d) is an intuitionistic fuzzy metric with respect to any continuous t-norm and continuous t-conorm.

Example 2.7: Let $X = N$ (set of natural numbers). Define $a * b = \max\{0, a + b - 1\}$ and $a \Diamond b = a + b - ab$ for all $a, b \in [0, 1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows;

$$M(x, y, t) = \begin{cases} x/y, & x \le y \\ y/x, & y \le x \end{cases},$$
$$N(x, y, t) = \begin{cases} y - x/y, & x \le y \\ x - y/x, & y \le x \end{cases},$$

for all $x, y, z \in X$ and $t > 0$. Then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space.

Definition 2.8 [1, 12]: Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space.

(a) A sequence $\{x_n\}$ in X is called Cauchy sequence if for each $t > 0$ and $P > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$

(b) A sequence $\{x_n\}$ in X is convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$ for each $t > 0$.

(c) An intuitionistic fuzzy metric space is said to be complete if every Cauchy sequence is convergent.

Definition 2.9 [10, 11]: Let A and S be mappings from an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ into itself. Then the mappings are said to be reciprocally continuous if

$$\lim_{n\to\infty} ASx_n = Az, \text{ and } \lim_{n\to\infty} SAx_n = Sz,$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} SAx_n = z, \text{ for some } z \in X.$$

Remark 2.10 [10, 11]: If A and S are both continuous then they are obviously reciprocally continuous. But the converse need not be true. We shall use the following lemmas to prove our next result without any further citation:

Lemma 2.11 [12]: In an intuitionistic fuzzy metric space X, $M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.

Lemma 2.12 [12]: Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. If there exists a constant $k \in (0, 1)$ such that $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$ and $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$ for every $t > 0$ and $n = 1, 2 \ldots,$ then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.13 [12]: Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. If there exists a constant $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$, for $x, y \in X$. Then $x = y$. Now, we introduce following in an intuitionistic fuzzy metric space

Definition 2.14: Let A and B are two self maps on an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ then f is called B-absorbing if there exists a positive integer $R > 0$, such that

$$M(Bx, BAx, t) \geq M(Bx, Ax, t/R),$$
$$N(Bx, BAx, t) \leq N(Bx, Ax, t/R)$$

for all $x \in X$. Similarly, B is called A-absorbing if there exists a positive integer $R > 0$ such that

$$M(Ax, ABx, t) \geq M(Ax, Bx, t/R),$$
$$N(Ax, ABx, t) \leq N(Ax, Bx, t/R)$$

for all $x \in X$. The map A is called point wise B-absorbing if forgiven $x \in X$, there exists a positive integer $R > 0$ such that

$$M(Bx, BAx, t) \geq M(Bx, Ax, t/R),$$
$$N(Bx, BAx, t) \leq N(Bx, Ax, t/R)$$

for all $x \in X$, Similarly, we can define point wise A-absorbing maps.

3 Main results

Theorem 3.1: Let A, B, S, T, L and M be self mappings of a complete ϵ-chainable intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$, defined by $a * b = \min\{a, b\}$ and $a \Diamond b = \max\{a, b\}$ where $a, b \in [0, 1]$ satisfying the conditions:

(1) $L(X) \subseteq ST(X)$, $M(X) \subseteq AB(X)$;

(2) M is ST absorbing;

(3) $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$;

(4) There exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(My, STy, t), M(ABx, My, t)\}$
$N(Lx, My, kt) \leq \max\{N(ABx, STy, t), N(My, STy, t), N(ABx, My, t)\}$

If $\{L, AB\}$ is reciprocally continuous semi-compatible maps. Then A, B, S, T, L and M have a unique fixed point in X.

Proof: Let $x_0 \in X$ be an arbitrary point. From (1), there exists $x_1, x_2 \in X$ such that $Lx_0 = STx_1 = y_0$ and $Mx_1 = ABx_2 = y_1$. Inductively, we can construct sequence $\{x_n\}$ and $\{y_n\}$ in X such that $Lx_{2n-2} = STx_{2n-1} = y_{2n-1}$ and $Mx_{2n-1} = ABx_{2n} = y_{2n}$ for $n = 1, 2, \ldots$

Putting $x = x_{2n+1}$, $y = x_{2n}$ for $t > 0$ in (4); we get

$$M(Lx_{2n+1}, Mx_{2n}, kt) \geq \min\{M(ABx_{2n+1}, STx_{2n}, t), M(Mx_{2n}, STx_{2n}, t), M(ABx_{2n+1}, Mx_{2n}, t)\} \quad \text{(i)}$$
$$\begin{aligned} M(y_{2n+2}, y_{2n+1}, kt) &\geq \min\{M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n+1}, t)\} \\ &\geq \min\{M(y_{2n+1}, y_{2n}, t), 1\} \\ &= M(y_{2n+1}, y_{2n}, t) \end{aligned}$$

$$N(Lx_{2n+1}, Mx_{2n}, kt) \leq \max\{N(ABx_{2n+1}, STx_{2n}, t), N(Mx_{2n}, STx_{2n}, t), N(ABx_{2n+1}, Mx_{2n}, t)\} \quad \text{(i)}$$
$$\begin{aligned} N(y_{2n+2}, y_{2n+1}, kt) &\leq \max\{N(y_{2n+1}, y_{2n}, t), N(y_{2n+1}, y_{2n}, t), N(y_{2n+1}, y_{2n+1}, t)\} \\ &\leq \max\{N(y_{2n+1}, y_{2n}, t), 0\} \\ &= N(y_{2n+1}, y_{2n}, t) \end{aligned}$$

Similarly we put $x = x_{2n+2}$ and $y = x_{2n+1}$ in (4); we have

$$M(Lx_{2n+2}, Mx_{2n+1}, kt) \geq \min\{M(ABx_{2n+2}, STx_{2n+1}, t), M(Mx_{2n+1}, STx_{2n+1}, t), M(ABx_{2n+2}, Mx_{2n+1}, t)\} \quad \text{(ii)}$$
$$\begin{aligned} M(y_{2n+3}, y_{2n+2}, kt) &\geq \min\{M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n+2}, t)\} \\ &\geq \min\{M(y_{2n+2}, y_{2n+1}, t), 1\} \\ &= M(y_{2n+1}, y_{2n+2}, t) \end{aligned}$$

$$N(Lx_{2n+2}, Mx_{2n+1}, kt) \leq \max\{N(ABx_{2n+2}, STx_{2n+1}, t), N(Mx_{2n+1}, STx_{2n+1}, t), N(ABx_{2n+2}, Mx_{2n+1}, t)\} \quad \text{(ii)}$$
$$\begin{aligned} N(y_{2n+3}, y_{2n+2}, kt) &\leq \max\{N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+2}, y_{2n+2}, t)\} \\ &\leq \max\{N(y_{2n+1}, y_{2n+2}, t), 0\} \\ &= N(y_{2n+1}, y_{2n+2}, t) \end{aligned}$$

From (i) and (ii) we have

$$\begin{aligned} M(y_{n+1}, y_{n+2}, kt) &\geq M(y_n, y_{n+1}, t) \\ N(y_{n+1}, y_{n+2}, kt) &\leq N(y_n, y_{n+1}, t) \end{aligned} \quad \text{(iii)}$$

From (iii)

$$M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, t/k) \geq M(y_{n-2}, y_{n-1}, t/k^2) \geq \ldots \geq M(y_0, y_1, t/k^n) \to 1 \text{ as } n \to \infty$$
$$N(y_n, y_{n+1}, t) \leq N(y_{n-1}, y_n, t/k) \leq N(y_{n-2}, y_{n-1}, t/k^2) \leq \ldots \leq N(y_0, y_1, t/k^n) \to 0 \text{ as } n \to \infty$$

So,

$$M(y_n, y_{n+1}, t) \to 1 \text{ as } n \to \infty \text{ and for any } t > 0$$
$$N(y_n, y_{n+1}, t) \to 0 \text{ as } n \to \infty \text{ and for any } t > 0$$

For each $\varepsilon > 0$ and each $t > 0$, we can choose $n_0 \in N$ such that

$$M(y_n, y_{n+1}, t) > 1 - \varepsilon$$
$$N(y_n, y_{n+1}, t) < \varepsilon$$

for all $n > n_0$. For $m, n \in N$, we suppose $m \geq n$. Then we have that

$$\begin{aligned} M(y_n, y_m, t) &\geq \min\{M(y_n, y_{n+1}, t/m-n), M(y_{n+1}, y_{n+2}, t/m-n), \ldots, M(y_{m-1}, y_m, t/m-n)\} \\ &> \min\{(1-\varepsilon), 1-\varepsilon), \ldots, 1-\varepsilon)\} \geq 1-\varepsilon \end{aligned}$$

$$N(y_n, y_m, t) \le \max\{N(y_n, y_{n+1}, t/m-n), N(y_{n+1}, y_{n+2}, t/m-n), \ldots, N(y_{m-1}, y_m, t/m-n)\}$$
$$< \max\{\varepsilon, \varepsilon, \ldots, \varepsilon\} \le \varepsilon,$$

and hence $\{y_n\}$ is a Cauchy sequence in X.

Since X is complete therefore $\{y_n\} \to z$ in X and its subsequences $\{ABx_{2n}\}$, $\{Mx_{2n-1}\}$, $\{STx_{2n-1}\}$ and $\{Lx_{2n-2}\}$ also converges to z. Since X is ϵ-chainable, there exists ϵ-chain from x_n to x_{n+1}, that is, there exists a finite sequence, $x_n = y_1, y_2, \ldots, y_l = x_{n+1}$ such that

$$M(y_i, y_{i-1}, t) > 1 - \varepsilon, \text{ for all } t > 0 \text{ and } i = 1, 2, \ldots$$
$$N(y_i, y_{i-1}, t) < \varepsilon, \text{ for all } t > 0 \text{ and } i = 1, 2, \ldots$$

Thus, we have

$$M(x_n, x_{n+1}, t) \ge \min\{M(y_1, y_2, t/l), M(y_2, y_3, t/l), \ldots, M(y_{l-1}, y_l, t/l)\}$$
$$> \min\{(1 - \varepsilon), 1 - \varepsilon), \ldots, 1 - \varepsilon)\} \ge 1 - \varepsilon,$$
$$N(x_n, x_{n+1}, t) \le \max\{N(y_1, y_2, t/l), N(y_2, y_3, t/l), \ldots, N(y_{l-1}, y_l, t/1)\}$$
$$< \max\{\varepsilon, \varepsilon, \ldots, \varepsilon\} \le \varepsilon$$

For $m > n$,

$$M(x_n, x_m, t) \ge \min\{M(x_n, x_{n+1}, t/m-n), M(x_{n+1}, x_{n+2}, t/m-n), \ldots, M(x_{m-1}, x_m, t/m-n)\}$$
$$> \min\{(1 - \varepsilon), 1 - \varepsilon), \ldots, 1 - \varepsilon)\} \ge 1 - \varepsilon$$
$$N(x_n, x_m, t) \le \max\{N(x_n, x_{n+1}, t/m-n), N(x_{n+1}, x_{n+2}, t/m-n), \ldots, N(x_{m-1}, x_m, t/m-n)\}$$
$$< \max\{\varepsilon, \varepsilon, \ldots, \varepsilon\} \le \varepsilon$$

and so $\{x_n\}$ is a Cauchy sequence in X and hence there exists $x \in X$ such that $x_n \to z$. By the reciprocally continuity and Semi-compatibility of maps (L, AB), we have $\lim_{n\to\infty} L(AB)x_{2n} = Lz$, $\lim_{n\to\infty} AB(L)x_{2n} = ABz$ and $\lim_{n\to\infty} L(AB)x_{2n} = ABz$, which implies that $Lz = ABz$.

Step (1): By putting $x = z$, $y = x_{2n+1}$ in (4), we get

$$M(Lz, Mx_{2n+1}, kt)$$
$$\ge \min\{M(ABz, STx_{2n+1}, t), M(Mx_{2n+1}, STx_{2n+1}, t), M(ABz, Mx_{2n+1}, t)\}$$

$$N(Lz, Mx_{2n+1}, \mathrm{kt})$$
$$\le \max\{N(ABz, STx_{2n+1}, t), N(Mx_{2n+1}, STx_{2n+1}, t), N(ABz, Mx_{2n+1}, t)\}$$

Letting $n \to \infty$; we get

$$M(Lz, z, kt) \ge \min\{M(Lz, z, t), M(z, z, t), M(Lz, z, t)\}$$
$$N(Lz, z, kt) \le \max\{N(Lz, z, t), N(z, z, t), N(Lz, z, t)\}$$

$$M(Lz, z, kt) \ge M(Lz, z, t)$$
$$N(Lz, z, kt) \le N(Lz, z, t).$$

Thus we get $Lz = z = ABz$.

Step (2): By putting $x = Bz$, $y = x_{2n+1}$ in (4), we get

$$M(L(Bz), Mx_{2n+1}, kt)$$
$$\ge \min\{M(AB(Bz), STx_{2n+1}, t), M(Mx_{2n+1}, STx_{2n+1}, t), M(AB(Bz), Mx_{2n+1}, t)\}$$

$$N(L(Bz), Mx_{2n+1}, kt)$$
$$\le \max\{N(AB(Bz), STx_{2n+1}, t), N(Mx_{2n+1}, STx_{2n+1}, t), N(AB(Bz), Mx_{2n+1}, t))\}$$

Since $AB = BA$, $LB = BL$; therefore $AB(Bz) = B(ABz) = Bz$ and $L(Bz) = B(Lz) = Bz$.

Letting $n \to \infty$, we get

$$M(Bz, z, kt) \geq \min\{M(Bz, z, t), M(z, z, t), M(Bz, z, t)\}$$
$$N(Bz, z, kt) \leq \max\{N(Bz, z, t), N(z, z, t), N(Bz, z, t)\}$$

$$M(Bz, z, kt) \geq M(Bz, z, t)$$
$$N(Bz, z, kt) \leq N(Bz, z, t)$$

Hence by Lemma 2.13 $Lz = Az = Bz = z$.
Since $L(X) \subseteq ST(X)$, there exists $u \in X$ such that $z = Lz = STu$.

Step (3): By putting $x = x_{2n}$, $y = u$ in (4), we get

$$M(Lx_{2n}, Mu, kt) \geq \min\{M(ABx_{2n}, STu, t), M(Mu, STu, t), M(ABx_{2n}, Mu, t)\}$$
$$N(Lx_{2n}, Mu, kt) \leq \max\{N(ABx_{2n}, STu, t), N(Mu, STu, t), N(ABx_{2n}, Mu, t)\}$$

Letting $n \to \infty$, we get

$$M(z, Mu, kt) \geq \min\{M(z, z, t), M(Mu, z, t), M(z, Mu, t)\}$$
$$N(\mathrm{z}, \mathrm{Mu}, \mathrm{kt}) \leq \max\{N(z, z, t), N(Mu, z, t), N(z, Mu, t)\}$$

$$M(z, Mu, kt) \geq M(z, Mu, t)$$
$$N(z, Mu, kt) \leq N(z, Mu, t)$$

$$z = Mu = STu.$$

Since M is ST-absorbing then we have;

$$M(STu, STMu, t) \geq M(STu, Mu, t/R) = 1$$
$$N(STu, STMu, t) \leq N(STu, Mu, t/R) = 0$$

$$STMu = STu \Rightarrow z = STz.$$

Step (4): By putting $x = x_{2n}$, $y = z$ in (4), we get

$$M(Lx_{2n}, Mz, kt) \geq \min\{M(ABx_{2n}, STz, t), M(Mz, STz, t), M(ABx_{2n}, Mz, t)\}$$
$$N(Lx_{2n}, Mz, kt) \leq \max\{N(ABx_{2n}, STz, t), N(Mz, STz, t), N(ABx_{2n}, Mz, t)\}$$

Letting $n \to \infty$, we get

$$M(z, Mz, kt) \geq \min\{M(z, z, t), M(Mz, z, t), M(z, Mz, t)\}$$
$$N(z, Mz, kt) \leq \max\{N(z, z, t), N(Mz, z, t), N(z, Mz, t)\}$$

$$M(z, Mz, kt) \geq M(z, Mz, t)$$
$$N(z, Mz, kt) \leq N(z, Mz, t)$$

Hence by Lemma 2.13 $z = Mz = STz$.

Step (5): By putting $x = x_{2n}$, $y = Tz$ in (4), we get

$$M(Lx_{2n}, M(Tz), kt)$$
$$\geq \min\{M(ABx_{2n}, ST(Tz), t), M(M(Tz), ST(Tz), t), M(ABx_{2n}, M(Tz), t)\}$$
$$N(Lx_{2n}, M(Tz), kt)$$
$$\leq \max\{N(ABx_{2n}, ST(Tz), t), N(M(Tz), ST(Tz), t), N(ABx_{2n}, M(Tz), t)\}$$

Since $ST = TS$, and $MT = TM$; therefore

$$M(Tz) = T(Mz) = Tz, ST(Tz) = T(STz) = Tz.$$

Letting $n \to \infty$, we get

$$M(z, Tz, kt) \geq \min\{M(z, Tz, t), M(Tz, Tz, t), M(z, Tz, t)\}$$
$$N(z, Tz, kt) \leq \max\{N(z, Tz, t), N(Tz, Tz, t), N(z, Tz, t)\}$$

$$M(z, Tz, kt) \geq M(z, Tz, t)$$
$$N(z, Tz, kt) \leq N(z, Tz, t)$$

Hence by Lemma 2.13 $z = Tz = Mz = Sz$.
Therefore $z = Az = Bz = Sz = Tz = Lz = Mz$.
Therefore z is a fixed point of A, B, S, T, L and M.

Uniqueness: Let w be another fixed point of A, B, S, T, L and M ; therefore putting $x = z$ and $y = w$ in (4), we have

$$M(Lz, Mw, kt) \geq \min\{M(ABz, STw, t), M(Mw, STw, t), M(ABz, Mw, t)\}$$
$$N(Lz, Mw, kt) \leq \max\{N(ABz, STw, t), N(Mw, STw, t), N(ABz, Mw, t)\}$$

$$M(z, w, kt) \geq \min\{M(z, w, t), M(w, w, t), M(z, w, t)\}$$
$$N(z, w, kt) \leq \max\{N(z, w, t), N(w, w, t), N(z, w, t)\}$$

$$M(z, w, kt) \geq M(z, w, t)$$
$$N(z, w, kt) \leq N(z, w, t)$$

Hence by Lemma 2.13 $z = w$. Hence z is a unique fixed point in X.
This completes the proof. □

Remark 3.2: Theorem 3.1 follows on same lines if we replace condition (4) by any of the following conditions:

(5) There exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(ABx, My, t)\}$
$N(Lx, My, kt) \leq \max\{N(ABx, STy, t), N(ABx, My, t)\}$

(6) There exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t), M(Lx, STy, t)\}$
$N(Lx, My, kt) \leq \max\{N(ABx, STy, t), N(Lx, ABx, t), N(My, STy, t), N(Lx, STy, t)\}$

(7) There exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t),$
$M(Lx, STy, t), M(ABx, My, 2t)\}$
$N(Lx, My, kt) \leq \max\{N(ABx, STy, t), N(Lx, ABx, t), N(My, STy, t),$
$N(Lx, STy, t), N(ABx, My, 2t)\}$

(8) There exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq M(ABx, STy, t)$,
$N(Lx, My, kt) \leq N(ABx, STy, t)$.

(9) There exists $k \in (0, 1)$ such that for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t),$
$M(Lx, STy, t), M(ABx, My, (2-\alpha)t)\}$.

$N(Lx, My, kt) \leq \max\{N(ABx, STy, t), N(Lx, ABx, t), N(My, STy, t),$
$N(Lx, STy, t), N(ABx, My, (2 - \alpha)t)\}$.

Corollary 3.3: Let A, T, L and M be self mappings of a complete ϵ-chainable intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$, defined by $a * b = \min\{a, b\}$ and $a \Diamond b = \max\{a, b\}$ where $a, b \in [0, 1]$ satisfy the conditions:

(10) $L(X) \subseteq T(X)$, $M(X) \subseteq A(X)$;

(11) M is T-absorbing;

(12) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(Ax, Ty, t), M(My, Ty, t), M(Ax, My, t)\}$
$N(Lx, My, kt) \leq \max\{N(Ax, Ty, t), N(My, Ty, t), N(Ax, My, t)\}$

If $\{L, A\}$ is reciprocally continuous semi-compatible maps. Then A, T, L and M have a unique fixed point in X.

Proof: Take $B = S = I$ (identity mapping) in Theorem 3.1. □

Corollary 3.4: Let L and M be self mappings of a complete ϵ-chainable intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $a * b = \min\{a, b\}$ and $a \Diamond b = \max\{a, b\}$ where $a, b \in [0, 1]$ satisfy the conditions:

(13) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(x, y, t), M(My, y, t), M(x, My, t)\}$

Then L and M have a unique fixed point in X.

Proof: Take $A = B = S = T = I$ (identity mapping). □

Remark 3.5: Corollary 3.4 follows on same lines if we replace condition (13) by any of the following conditions:

(14) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(x, y, t), M(x, My, t)\}$
$N(Lx, My, kt) \leq \max\{N(x, y, t), N(x, My, t)\}$

(15) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(x, y, t), M(Lx, x, t), M(My, y, t), M(Lx, y, t)\}$
$N(Lx, My, kt) \leq \max\{N(x, y, t), N(Lx, x, t), N(My, y, t), N(Lx, y, t)\}$

(16) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq \min\{M(x, y, t), M(Lx, x, t), M(My, y, t), M(Lx, y, t), M(x, My, 2t)\}$
$N(Lx, My, kt) \leq \max\{N(x, y, t), N(Lx, x, t), N(My, y, t), N(Lx, y, t), N(x, My, 2t)\}$

(17) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$M(Lx, My, kt) \geq M(x, y, t)$,
$N(Lx, My, kt) \leq N(x, y, t)$.

(18) There exists $k \in X\,(0, 1)$ such that for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$,

$$M(Lx, My, kt) \geq \min\{M(x, y, t), M(Lx, x, t), M(My, y, t), M(Lx, y, t), M(x, My, (2-\alpha)t)\}$$
$$N(Lx, My, kt) \leq \max\{N(x, y, t), N(Lx, x, t), N(My, y, t), N(Lx, y, t), N(x, My, (2-\alpha)t)\}$$

Corollary 3.6: Let L be self mappings of a complete ϵ-chainable intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $a * b = \min\{a, b\}$ and $a \Diamond b = \max\{a, b\}$ where $a, b \in [0, 1]$ satisfy the conditions:

(19) There exists $k \in X(0, 1)$ such that for all $x, y \in X$ and $t > 0$, $M(Lx, y, kt) \geq M(x, y, t)$.

Then L has a unique fixed point in X.

Proof: Take $M = I$ (identity mapping) in Corollary 3.4. □

Corollary 3.7: Let L be self mappings of a complete ϵ-chainable intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $a * b = \min\{a, b\}$ and $a \Diamond b = \max\{a, b\}$ where $a, b \in [0, 1]$ satisfy the conditions:

(20) There exists $k \in X(0, 1)$ such that for all $x, y \in X$ and $t > 0$, $M(Lx, Ly, kt) \geq M(x, y, t)$.

Then L has a unique fixed point in X.

Proof: Take $L = M = I$ in Corollary 3.4. □

Remark 3.8: By Corollaries 3.3, 3.4, 3.6, 3.7 and Remarks 3.2 and 3.5, it easily follows that our results generalizes results of Ranadive et al. [10, 11], A. Jain et al. [6], Y. Bano et al. [4] and M. Verma et al. [13] in intuitionistic fuzzy metric spaces.

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