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On IF-numbers

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Abstract: In the paper analogously to the notion of fuzzy numbers ([10, 11, 12, 13, 14, 18], the notion of the IF-number is introduced, using a new approach and it is studied. Especially it is proved that the space of all IF-numbers with a convenient metric function is a complete metric space.
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1 Introduction

Various results of classical theories can be trasformed to some fuzzy generalizations [19, 20]). So in [10], and [7] the Kurzweil - Henstock integration theory (e.g. [7, 17]) was generalized to the fuzzy case.

Of course, a special problem presents the studium of intuitionistic fuzzy sets theory ([1, 2]). Since in [10] and [18] an integration theory has been based on well constructed theory of fuzzy numbers, our aim in this paper is a construction of something similar in the IF-case.

In a series of papers, see, e.g. [3, 4, 5, 6, 8], the concet of an IF-number is introduced and some of its basic properties are studied. Here, we shall give a new approach for defining of the notion of an IF-number and study some properties of the set $\mathcal{D}$ of all IF-numbers.

2 Preliminaries

First we shall define the notion of an IF-number. A maping $\alpha : \mathbb{R} \to [0,1]$ is called a fuzzy number, if

1. α is normal, i.e. $\alpha(r) = 1$ for some $r \in \mathbb{R}$.
2. α is convex, i.e. $\alpha(\lambda u + (1-\lambda)v) \geq \min\{\alpha(u), \alpha(v)\}$ for all $u, v \in \mathbb{R}$ and $\alpha \in [0,1]$.
3. α is semicontinuous, i.e. for every $\lambda \in [0,1]$ the set $\{x \in \mathbb{R}; \alpha(x) \geq \lambda\}$ is closed.
4. $cl([\alpha]_0) = cl(\{x \in \mathbb{R}; \alpha(x) > 0\}$ is compact.

If α is a fuzzy number and $\lambda \in \mathbb{R}$, then $\{x; \alpha(x) \geq \lambda\}$ is an interval $[\alpha_{\lambda,1}, \alpha_{\lambda,2}]$ (see [13]). In [12] the following assertion has been proved:

Lemma 1. *Let* $\alpha_1, \alpha_2 : [0,1] \to \mathbb{R}$ *satisfy the following properties:*

1. α_1 *is increasing,* α_2 *is decreasing,*

2. $\alpha_1(1) \leq \alpha_2(1)$*,*

3. α_1, α_2 *are left continuous on* $(0,1]$*, and right continuous at* 0*.*

Then there exists exactly one fuzzy number α *such that*

$$\{x; \alpha(x) \geq \lambda\} = [\alpha_1(\lambda), \alpha_2(\lambda)]$$

for each $\lambda \in [0,1]$*.*

Let α, β be fuzzy numbers and $\lambda \in [0,1]$. Put

$$\gamma_1 = \alpha_1 + \beta_1, \gamma_2 = \alpha_2 + \beta_2.$$

Then by Lemma 1 there exists exactly one fuzzy number γ such that

$$\{x; \gamma(x) \geq \lambda\} = [\alpha_1(\lambda) + \beta_1(\lambda), \alpha_2(\lambda) + \beta_2(\lambda)]$$

The fuzzy number γ will be denoted by

$$\gamma = \alpha + \beta.$$

3 An IF-number

Let again α, β be fuzzy numbers and $\lambda \in [0,1]$. Then we define

$$\rho_\lambda(\alpha,\beta) = \max\{\mid \alpha_{\lambda,1} - \beta_{\lambda,1} \mid, \mid \alpha_{\lambda,2} - \beta_{\lambda,2} \mid\}$$

and

$$\hat{\rho}(\alpha,\beta) = \sup\{\rho_\lambda(\alpha,\beta); \lambda > 0\}.$$

If $A = (\mu_A, \nu_A)$ is an IF set, then it is an IF-number, iff μ_A, ν_A are fuzzy numbers.

Denote by $\mathcal{D}$ the family of all IF-numbers. If A, $B \in \mathcal{D}$, then we define the distance

$$d(A,B) = \hat{\rho}(\mu_A,\mu_B) + \hat{\rho}(\nu_A,\nu_B).$$

Theorem. The function $d : \mathcal{D} \times \mathcal{D} \to R$ is a metric, the couple $(\mathcal{D}, d)$ is a complete metric space.

Proof. Let $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B), C = (\mu_C, \nu_C)$. Then

$$d(A,A) = \hat{\rho}(\mu_A,\mu_A) + \hat{\rho}(\nu_A,\nu_A) = 0$$

and

$$d(A,B) = \hat{\rho}(\mu_A,\mu_B) + \hat{\rho}(\nu_A,\nu_B) = 0$$

implies

$$\hat{\rho}(\mu_A,\mu_B) = 0, \quad \hat{\rho}(\nu_A,\nu_B) = 0.$$

Therefore

$$A = (\mu_A,\nu_A) = (\mu_B,\nu_B) = B.$$

Evidently,

$$d(A,B) = d(B,A)$$

and

$$d(A,C) = \hat{\rho}(\mu_A,\mu_C) + \hat{\rho}(\nu_A,\nu_C) \le$$
$$\le \hat{\rho}(\mu_A,\mu_B) + \hat{\rho}(\nu_A,\nu_B) + \hat{\rho}(\mu_B,\mu_C) + \hat{\rho}(\nu_B,\nu_C) = d(A,B) + d(B,C).$$

Now let $(A_n)_n$ be Cauchy, $A_n = (\mu_{A_n}, \nu_{A_n})_n$. Then also $(\mu_{A_n})_n, (\nu_{A_n})_n$ are Cauchy, hence they have limits. Denote them μ_A, resp. ν_A. Hence

$$\lim_{n\to\infty} \hat{\rho}(\mu_{A_n},\mu_A) = 0, \lim_{n\to\infty} \hat{\rho}(\nu_{A_n},\nu_A) = 0$$

Since $A_n = (\mu_{A_n}, \nu_{A_n}) \in \mathcal{D}$ we have

$$\mu_{A_n} + \nu_{A_n} \le 1, n = 1, 2, \cdots$$

Therefore

$$\mu_A + \nu_A = \lim_{n\to\infty} \mu_{A_n} + \lim_{n\to\infty} \nu_{A_n} = \lim_{n\to\infty} (\mu_{A_n} + \nu_{A_n}) \le 1,$$

hence $A \in \mathcal{D}$. □

4 Conclusion

Analagously to the notion of the fuzzy number we have introduced the notion of IF-numbers. Simultaneously we have shown two applications. Of course, it is possible to hope to find some other applications on mathematical analysis on $\mathcal{D}$ similarly to those appearing in [4, 9, 15, 16].

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