19th Int. Workshop on IFSs, Burgas, 4–6 June 2015
Notes on Intuitionistic Fuzzy Sets
ISSN 1310–4926
Vol. 21, 2015, No. 2, 63–70

Temporal intuitionistic fuzzy topology in Šostak's sense

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Abstract: In this study, we extended the concept of Šostak's sense topological spaces to temporal intuitionistic fuzzy sets and investigated some properties of this space. Also, we gave topological instant closure and instant interior operators for temporal intuitionistic fuzzy topological space (briefly ST-TIFS).

Keywords: Temporal intuitionistic fuzzy set, Šostak topology.

AMS Classification: 03E72, 46S40

1 Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 and has been well understood and used in various aspects of science and technology such as engineering and medicine. The theory of fuzzy set is one of the most important inventions of our time. On the other hand, as a natural generalization of fuzzy set, intuitionistic fuzzy set (IFS for short) was introduced by Atanassov in 1986. His definition was found to be useful to deal with vagueness of knowledge. In the concept of intuitionistic fuzzy set, each element has two degrees named degree of membership and degree of non-membership to IFS respectively [1]. The concept of fuzzy topological space is defined by Chang in 1968 as a collection of fuzzy sets. Fuzzifying of topology concept was made by Šostak in 1985. In his definition, openness and closeness of fuzzy sets are graded among 0 and 1. In 1996, D. Çoker and M. Demirci introduced the concept of intuitionistic fuzzy set in Šostak's sense and gave fundamental definitions and properties of it.

Temporal intuitionistic fuzzy set (TIFS) is defined by Atanassov in 1991. In his definition, membership and non-membership degree of an element change with both of the element and time moment. This is one of the most important extensions of IFS. Because, real world situations are generally spatio-temporal. Thus, by the theory of TIFS, real world situations like weather, medicine, economy, image-video processing can be handled more realistic and effective. As stated in [7]; it is well-known that time is monotone and time is a fundamental issue for modeling dynamic information. In recent years, some fundamental concepts are defined by several authors A. I. Ban 1993, R. Parvathi and S. P. Geetha. In 2014, Çuvalcıoğlu and S. Yılmaz defined level operators on TIFSs. The concept of temporal intuitionistic fuzzy is very untouched area and the most fundamental concepts have not been defined yet. One of these concepts is topology on TIFSs.

The rest of the paper will be designed as follows: In section 2, we gave some basic definitions about IFS and TIFS. In section 3, we defined temporal intuitionistic fuzzy topology in Šostak's sense in which the topologies on TIFSs are defined with degree of openness and degree of non-openness. We also give definition of closeness and non-closeness degree of a TIFS. At the end of the section 3, we give the definition of instant interior and instant closure of an TIFS and prove some fundamental properties of them.

2 Preliminaries

Definition 2.1: An intuitionistic fuzzy set in a non-empty set X given by a set of ordered triples $A=\{(x,\mu_A(x),\eta_A(x)):x\in X\}$ where $\mu_A(x):X\to I,\eta_A(x):X\to I$ for $I=[0,1]$ are functions defined such that $0\le\mu(x)+\eta(x)\le 1$ for all $x\in X$. For $x\in X$, $\mu_A(x)$ and $\eta_A(x)$ represent the degree of membership and degree of non-membership of x to A respectively. For each $x\in X$; intuitionistic fuzzy index of x in A can be defined as follows $\pi_A(x)=1-\mu_A(x)-\eta_A(x)$ and π_A is called degree of hesitation or indeterminacy. [1]

Definition 2.2: Let $A,B\in IFS(X)$. Then;

1. $A\subset B\Leftrightarrow$ $\mu_A(x)\le\mu_B(x)$ and $\eta_A(x)\ge\eta_B(x)$ for $\forall x\in X$
2. $A=B\Leftrightarrow A\subset B$ and $B\subset A$
3. $A^c=\{(x,\eta_A(x),\mu_A(x)):x\in X\}$
4. $\bigcap A_i=\{(x,\wedge\mu_{A_i}(x),\vee\eta_{A_i}(x));x\in X\}$
5. $\bigcup A_i=\{(x,\vee\mu_{A_i}(x),\wedge\eta_{A_i}(x));x\in X\}$
6. $\underset{\sim}{0}=\{(x,0,1):x\in X\}$ and $\underset{\sim}{1}=\{(x,1,0):x\in X\}$. [2]

Definition 2.3: Let a and b be two real numbers in $[0,1]$ satisfying the inequality $a+b\le 1$. Then the pair $\langle a,b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1,b_1\rangle$ and $\langle a_2,b_2\rangle$ be two intuitionistic fuzzy pair. Then define

i. $\langle a_1,b_1\rangle\le\langle a_2,b_2\rangle\Leftrightarrow a_1\le a_2$ and $b_1\ge b_2$

ii. $\langle a_1,b_1\rangle=\langle a_2,b_2\rangle\Leftrightarrow a_1=a_2$ and $b_1=b_2$

iii. If $\{\langle a_i,b_i\rangle;i\in J\}$ is a family of intuitionistic fuzzy pairs, then $\vee\langle a_i,b_i\rangle=\langle\vee a_i,\wedge b_i\rangle$ and $\wedge\langle a_i,b_i\rangle=\langle\wedge a_i,\vee b_i\rangle$.

iv. The complement of $\langle a,b\rangle$ is defined by $\overline{\langle a,b\rangle}=\langle b,a\rangle$

v. $1^{\sim}=\langle 1,0\rangle$ and $0^{\sim}=\langle 0,1\rangle$. [4]

Definition 2.4: An intuitionistic fuzzy topology in Šostak's sense (briefly, S-IFS) on a nonempty set X is an IFF τ defined with $\tau(A)=(\mu_\tau(A),\eta_\tau(A))$ on X satisfying the following axioms:

a. $\tau(\underset{\sim}{0})=1^{\sim}$ and $\tau(\underset{\sim}{1})=1^{\sim}$

b. $\tau(A_1\cap A_2)\geq\tau(A_1)\wedge\tau(A_2)$ for any sets $A_1,A_2\in IFS(X)$

c. $\tau(\bigcup A_i)\geq\bigwedge_{i\in J}(\tau(A_i))$ for $\{A_i;i\in J\}\subseteq IFS(X)$.

The pair (X,τ) is called an intuitionistic fuzzy topological space in Šostak sense. For any $A\in IFS(X)$, the number $\mu_\tau(A)$ is called the openness degree of A, while $\eta_\tau(A)$ is called non-openness degree of A.[4]

Definition 2.5: Let E be an universe and T be a non-empty set. We call the elements of T "time moments". Based on the definition of IFS, a temporal intuitionistic fuzzy set (TIFS) is defined as the following:

$$A(T)=\{(x,\mu_A(x,t),\eta_A(x,t)):E\times T\}$$

where:

a. $A\subseteq E$ is a fixed set

b. $\mu_A(x,t)+\eta_A(x,t)\leq 1$ for every $(x,t)\in E\times T$

c. $\mu_A(x,t)$ and $\eta_A(x,t)$ are the degrees of membership and non-membership, respectively, of the element $x\in E$ at the time moment $t\in T$

For brevity, we write A instead of $A(T)$. The hesitation degree of a TIFS is defined as $\pi_A(x,t)=1-\mu_A(x,t)-\eta_A(x,t)$. Obviously, every ordinary IFS can be regarded as TIFS for which T is a singleton set. All operations and operators on IFS can be defined for TIFSs. [3]

Definition 2.6: Let

$$A(T')=\{(x,\mu_A(x,t),\eta_A(x,t)):E\times T'\}$$

and

$$B(T'')=\{(x,\mu_B(x,t),\eta_B(x,t)):E\times T''\}$$

where T' and T'' have finite number of distinct time-elements or they are time intervals. Then;

$$A(T') \cup B(T'') = \{(x, \min(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)), \max(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t)))$$

and

$$A(T') \cup B(T'') = \{(x, \max(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)), \min(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t)))$$

Also from definition of subset in IFS theory, Subsets of TIFS can be defined as the following:

$A(T') \subseteq B(T'') \Leftrightarrow \bar{\mu}_A(x,t) \geq \bar{\mu}_B(x,t)$ and $\bar{\eta}_A(x,t) \leq \bar{\eta}_B(x,t)$ for every $(x,t) \in E \times (T' \cup T'')$

where

$$\bar{\mu}_A(x,t) = \begin{cases} \mu_A(x,t), & if\ t \in T' \\ 0, & if\ t \in T'' - T' \end{cases} \quad \text{and} \quad \bar{\mu}_B(x,t) = \begin{cases} \mu_B(x,t), & if\ t \in T'' \\ 0, & if\ t \in T' - T'' \end{cases}$$

$$\bar{\eta}_A(x,t) = \begin{cases} \eta_A(x,t), & if\ t \in T' \\ 1, & if\ t \in T'' - T' \end{cases} \quad \text{and} \quad \bar{\eta}_B(x,t) = \begin{cases} \eta_B(x,t), & if\ t \in T'' \\ 1, & if\ t \in T'' - T' \end{cases}$$

It is obviously seen that if $T' = T''$; $\bar{\mu}_A(x,t) = \mu_A(x,t)$, $\bar{\mu}_B(x,t) = \mu_B(x,t)$, $\bar{\eta}_A(x,t) = \eta_A(x,t)$, $\bar{\eta}_B(x,t) = \eta_B(x,t)$. [2]

Let J be an arbitrary index set. Then we define that $T = \bigcup_{i \in J} T_i$ where T_i is a time set for each $i \in J$. Thus, we can extend the definition of union and intersection of TIFSs family $F = \{A_i(T_i) = (x, \mu_{A_i}(x,t), \eta_{A_i}(x,t)) : x \in E \times T_i, i \in J\}$ as follows:

$$\bigcup_{i \in J} A(T_i) = \{(x, \max_{i \in J}(\bar{\mu}_{A_i}(x,t)), \min_{i \in J}(\bar{\eta}_{A_i}(x,t)); (x,t) \in E \times T)$$

$$\bigcap_{i \in J} A(T_i) = \{(x, \min_{i \in J}(\bar{\mu}_{A_i}(x,t)), \max_{i \in J}(\bar{\eta}_{A_i}(x,t)); (x,t) \in E \times T)$$

where $\bar{\mu}_{A_j}(x,t) = \begin{cases} \mu_{A_j}(x,t), & if\ t \in T_j \\ 0, & if\ t \in T - T_j \end{cases}$ and $\bar{\eta}_{A_j}(x,t) = \begin{cases} \eta_{A_j}(x,t), & if\ t \in T_j \\ 1, & if\ t \in T - T_j \end{cases}$.

3 Main results

Definition 3.1: $\underset{\sim}{0}^t$ and $\underset{\sim}{1}^t$ $\in TIFS(X)$ are defined as: $\underset{\sim}{0}^t = \{(x,1,0); x \in X\}$ and $\underset{\sim}{1}^t = \{(x,0,1); x \in X\}$ for each time moment *t*, i.e. $\mu_{\underset{\sim}{0}^t}(x,t) = 0$, $\eta_{\underset{\sim}{0}^t}(x,t) = 1$ and $\mu_{\underset{\sim}{1}^t}(x,t) = 1$, $\eta_{\underset{\sim}{1}^t}(x,t) = 0$ for each $(x,t) \in X \times T$.

Definition 3.2: An temporal intuitionistic fuzzy topology in Šostak's sense (briefly, ST-TIFS) on a non-empty set X is an IFF τ_t defined with $\tau_t(A) = (\mu_{\tau_t}(A), \eta_{\tau_t}(A))$ on X satisfying the following axioms for each time moment t:

I. $\tau_t\left(\underset{\sim}{0}^t\right)=1^\sim$ and $\tau_t\left(\underset{\sim}{1}^t\right)=1^\sim$,

II. $\tau_t\left(A_1\cap A_2\right)\geq\tau_t\left(A_1\right)\wedge\tau_t\left(A_2\right)$ for any sets $A_1,A_2\in TIFS\left(X\right)$,

III. $\tau_t\left(\bigcup A_i\right)\geq\bigwedge_{i\in J}\left(\tau_t\left(A_i\right)\right)$ for $\left\{A_i;i\in J\right\}\subseteq TIFS\left(X\right)$.

The pair $\left(X,\tau_t\right)$ is called temporal intuitionistic fuzzy topological space in Šostak sense. For any $A\in TIFS\left(X\right)$, the number $\mu_{\tau_t}\left(A\right)$ is called instant openness degree of A at time moment t, while $\eta_{\tau_t}\left(A\right)$ is called instant non-openness degree of A at time moment t. In this definition, it is worth to note that the instant openness and the instant non-openness degree change with depending on both time and TIFS.

Definition 3.3: IFF τ_t^* defined with $\tau_t^*\left(A\right)=\left(\mu_{\tau_t^*}\left(A\right),\eta_{\tau_t^*}\left(A\right)\right)$, if it satisfies the following axioms for each time moment t

I. $\tau_t^*\left(\underset{\sim}{0}^t\right)=1^\sim$ and $\tau_t^*\left(\underset{\sim}{1}^t\right)=1^\sim$,

II. $\tau_t^*\left(A_1\cup A_2\right)\geq\tau_t\left(A_1\right)\wedge\tau_t\left(A_2\right)$ for any sets $A_1,A_2\in TIFS\left(X\right)$,

III. $\tau_t^*\left(\bigcap_{i\in J}A_i\right)\geq\bigwedge_{i\in J}\left(\tau_t^*\left(A_i\right)\right)$ for $\left\{A_i;i\in J\right\}\subseteq TIFS\left(X\right)$.

Then the number $\mu_{\tau_t^*}\left(A\right)$ is called instant closeness degree of A at time moment t, while $\eta_{\tau_t^*}\left(A\right)$ is called instant non-closeness degree of A at time moment t.

It is worth to note that for singleton time set $\left(X,\tau_t\right)$ is an intuitionistic fuzzy topology on $TIFS\left(X\right)$ in Šostak's sense.

Proposition 3.4: Let $\left(X,\tau_t\right)$ be a ST-TIFS on X and time set T. Then $\left(X,\wedge\tau_t\right)$ defined by $\wedge\tau_t\left(A\right)=\left(\min_{t\in T}\mu_{\tau_t}\left(A\right),\max_{t\in T}\eta_{\tau_t}\left(A\right)\right)$ is an intuitionistic fuzzy topology on $TIFS\left(X\right)$ in Šostak's sense.

Proof: **1.** From the definition, $\wedge\tau_t\left(\underset{\sim}{0}\right)=\left(\min_{t\in T}\mu_{\tau_t}\left(\underset{\sim}{0}\right),\max_{t\in T}\eta_{\tau_t}\left(\underset{\sim}{0}\right)\right)$. Since $\left(X,\tau_t\right)$ is ST-TIFS, for each $t\in T$, $\mu_{\tau_t}\left(\underset{\sim}{0}\right)=1$ and $\eta_{\tau_t}\left(\underset{\sim}{0}\right)=0$. So it is clear that $\wedge\tau_t\left(\underset{\sim}{0}\right)=1^\sim$.

2. Since $\left(X,\tau_t\right)$ is ST-TIFS, for each $t\in T$, $\tau_t\left(A_1\cap A_2\right)\geq\tau_t\left(A_1\right)\wedge\tau_t\left(A_2\right)$ for any sets $A_1,A_2\in TIFS\left(X\right)$. So $\tau_t\left(A_1\cap A_2\right)\geq\tau_t\left(A_1\right)\wedge\tau_t\left(A_2\right)$(I).

On the other hand,

$$\wedge\tau_t\left(A_1\cap A_2\right)=\left(\min_{t\in T}\mu_{\tau_t}\left(A_1\cap A_2\right),\max_{t\in T}\eta_{\tau_t}\left(A_1\cap A_2\right)\right)\text{(II)}.$$

From (I) , (II) and Defition 2.6,

$$\wedge\tau_t\left(A_1\cap A_2\right)\geq\left\langle\min\left(\mu_{\tau_t}\left(A_1\right),\mu_{\tau_t}\left(A_2\right)\right),\max\left(\eta_{\tau_t}\left(A_1\right),\eta_{\tau_t}\left(A_2\right)\right)\right\rangle.$$

Therefore $\wedge\tau_t\left(A_1\cap A_2\right)\geq\left(\wedge\tau_t\left(A_1\right)\right)\wedge\left(\wedge\tau_t\left(A_2\right)\right)$.

3. Since (X,τ_t) is ST-TIFS, $\tau_t\left(\bigcup A_i\right) \ge \bigwedge_{i\in J}\left(\tau_t(A_i)\right)$ for $\{A_i; i\in J\} \subseteq TIFS(X)$. On the other hand, $\wedge\tau_t\left(\bigcup A_i\right) = \left\langle \underset{t\in T}{\wedge}\mu_{\tau_t}\left(\bigcup A_i\right), \underset{t\in T}{\vee}\eta_{\tau_t}\left(\bigcup A_i\right)\right\rangle$. $\underset{t\in T}{\wedge}\mu_{\tau_t}\left(\bigcup A_i\right) \ge \bigwedge_{i\in J}\left(\underset{t\in T}{\wedge}\mu_{\tau_t}(A_i)\right)$ and $\underset{t\in T}{\vee}\eta_{\tau_t}\left(\bigcup A_i\right) \ge \bigvee_{i\in J}\left(\underset{t\in T}{\wedge}\eta_{\tau_t}(A_i)\right)$. Therefore $\wedge\tau_t\left(\bigcup A_i\right) \ge \bigwedge_{i\in J}\left(\wedge\tau_t(A_i)\right)$. □

Example 3.5: Let $X = \{x_1, x_2\}$ and $T = \{t_1, t_2, t_3\}$. Then let define A_1, A_2 and $A_3 \in TIFS(X)$ as:

A_1	x_1	x_2
t_1	(0.23, 0.35)	(0.32, 0.51)
t_2	(0.71, 0.1)	(0.43, 0.25)
t_2	(0.01, 0.95)	(0.93, 0)

and

A_2	x_1	x_2
t_1	(0.33, 0.33)	(0.77, 0.14)
t_2	(0.8, 0.2)	(0.51, 0.35)
t_2	(0.11, 0.45)	(0.25, 0.25)

Here, the first component of the intuitionistic fuzzy pair in each cell shows membership degree of element and second component shows degree of non-membership at time moment t_i where $i = \{1,2,3\}$. Define ST-TIFS, $\tau_t : TIFS(X) \to I \times I$, $\tau_{t_i}(A) = \left(\mu_{\tau_{t_i}}(A), \eta_{\tau_{t_i}}(A)\right)$ where;

$$\mu_{\tau_{t_i}}(A) = \begin{cases} 1, & A = \underset{\sim}{0}^t, \underset{\sim}{1}^t \\ \left(\mu_{A_1}(x,t_i)\right)^{\frac{1}{2^i}}, & A = A_1, \\ \left(\mu_{A_2}(x,t_i)\right)^{\frac{1}{3^i}}, & A = A_2, \\ \left(\max\left\{\mu_{A_1}(x,t_i), \mu_{A_2}(x,t_i)\right\}\right)^{\frac{1}{2^i}} & A = A_1 \vee A_2 \\ 0, & \text{otherwise;} \end{cases}$$

and

$$\eta_{\tau_{t_i}}(A) = \begin{cases} 0, & A = \underset{\sim}{0}^t, \underset{\sim}{1}^t \\ \dfrac{\eta_{\tau_{t_i}}(A_1)^{\frac{1}{2^i}}}{2^i}, & A = A_1, \\ \dfrac{\left(\eta_{\tau_{t_i}}(A_2)\right)^{\frac{1}{4^i}}}{3^i}, & A = A_2, \\ \dfrac{1}{2.i}.\min\left\{\eta_{\tau_{t_i}}(A_1), \eta_{\tau_{t_i}}(A_2)\right\} & A = A_2 \vee A_2 \\ 1, & \text{otherwise;} \end{cases}$$

From the definition of ST-IFTS, it can easily be shown that (X,τ_t) is intuitionistic fuzzy topology in Šostak's sense on $TIFS(X)$. We demonstrate instant openness and instant non-openness degrees of $A_1, A_2, A_1 \vee A_2$ according to τ_t as below:

	$\mu_{\tau_{t_i}}(A_1)$	$\eta_{\tau_{t_i}}(A_1)$	$\mu_{\tau_{t_i}}(A_2)$	$\eta_{\tau_{t_i}}(A_2)$	$\mu_{\tau_{t_i}}(A_1 \vee A_2)$	$\eta_{\tau_{t_i}}(A_1 \vee A_2)$
t_1	0.47958	0.2958	0.68399	0.23805	0.57446	0.165
t_2	0.84507	0.14059	0.91049	0.078567	0.94574	0.025
t_2	0.56234	0.1242	0.99732	0	0.75888	0.05625

We can define closure and interior of a TIFS with respect to [4] as following:

Definition 3.6: Let (X,τ_t) be a ST-TIFS and $A \in TIFS(X)$. Then we define instant closure and instant interior of A at time moment t according to τ_t respectively as:

$$\mathrm{cl}^t(A) = \bigcap\{K \in TIFS(X); \tau_t^*(K) > \tilde{0}, A \subseteq K\}$$

and

$$\mathrm{int}^t(A) = \bigcup\{K \in TIFS(X); \tau_t(K) > \tilde{0}, K \subseteq A\}.$$

On the other hand, (α,β)-instant closure and (α,β)-instant interior of A are defined by:

$$\mathrm{cl}^t_{(\alpha,\beta)}(A) = \bigcap\{K \in TIFS(X); \tau_t^*(K) > \langle\alpha,\beta\rangle, A \subseteq K\}$$

and

$$\mathrm{int}^t_{(\alpha,\beta)}(A) = \bigcup\{K \in TIFS(X); \tau_t(K) > \langle\alpha,\beta\rangle, K \subseteq A\}$$

where $\alpha \in (0,1]$, $\beta \in [0,1)$ with $\alpha + \beta \le 1$.

Proposition 3.7: Let (X,τ_t) be a ST-TIFS on X and time set *T*. Then $\mathrm{cl}(A) = \underset{t\in T}{\wedge}\left(\mathrm{cl}^t(A)\right)$ $\mathrm{int}(A) = \underset{t\in T}{\vee}\left(\mathrm{int}^t(A)\right)$ are closure and interior of A according to $(X, \wedge\tau_t)$.

Proof: Let $\underset{t\in T}{\vee}\left(\mathrm{int}^t(A)\right)$ denote with $\mathrm{int}^*(A)$**.** We must show that $\mathrm{int}^*(A)$ = $\mathrm{int}(A)$= $= \bigcup\{K \in TIFS(X); \wedge\tau_t(K) \ge 0^{\sim},\ K \supseteq A,\}$. From the Definition 2.3,

$$\mathrm{int}^*(A) = \left\langle \underset{K\supseteq A}{\vee} \mu_{\tau_t}(K), \underset{K\supseteq A}{\wedge} \mu_{\tau_t}(K) \right\rangle$$

where $\mu_{\tau_t}(K) > 0$**,** $\eta_{\tau_t}(K) < 1$ for each $t \in T$**.** Therefore it is clear that $\mathrm{int}^*(A) \ge \mathrm{int}(A)$(I). On the other hand, from the definition of $\wedge\tau_t$, $\wedge\tau_t(K) = \left(\min_{t\in T} \mu_{\tau_t}(K), \max_{t\in T} \eta_{\tau_t}(K)\right)$ for each $K \in TIFS(X)$.

Since $\min_{t\in T}\mu_{\tau_t}(K)>0$ and $\max_{t\in T}\eta_{\tau_t}(K)<1$ for each $K\in\{K\in TIFS(X);\wedge\tau_t(K)\geq 0^{\sim}, K\supseteq A\}$.

Each element of $\{K\in TIFS(X);\tau_t(K)>\tilde{0}, K\subseteq A\}$ is a member of

$$\{K\in TIFS(X);\wedge\tau_t(K)\geq 0^{\sim}, K\supseteq A\}.$$

Therefore $\mathrm{int}(A)\geq\mathrm{int}^*(A)$(II). From (I), (II) and Definition 2.3 and 2.6, it is obtained that $\mathrm{int}^*(A)=\mathrm{int}(A)$. ☐

Proposition 3.8. Let (X,τ_t) be a ST-TIFS on X and time set T. For each $A\in TIFS(X)$ and $t\in T$

1. $A\subseteq cl^t(A)\subseteq cl^t_{(\alpha,\beta)}(A)$
2. $\mathrm{int}^t(A)\subseteq\mathrm{int}^t_{(\alpha,\beta)}(A)\subseteq A$
3. $cl^t(A)=\bigcup_{(\alpha,\beta)\in(0,1)}\mathrm{int}^t_{(\alpha,\beta)}(A)$
4. $\mathrm{int}^t(A)=\bigcap_{(\alpha,\beta)\in(0,1)}\mathrm{int}^t_{(\alpha,\beta)}(A)$

Proof: It is clear from Definition 3.6. ☐

References

[1] Atanassov, K. T. (1986) Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20, 87–96.

[2] Atanassov, K. T. (2012) *On Intuitionistic Fuzzy Sets Theory*, Springer, Berlin.

[3] Atanassov, K. T. (1991) Temporal intuitionistic fuzzy sets, *Comptes Rendus de l'Academie Bulgare*, 7, 5–7.

[4] Çoker, D., & Demirci, M. (1996) An introduction to intuitionistic topological spaces in Šostak's sense, *BUSEFAL*, 67, 67–76.

[5] Chang, C. L. (1968) Fuzzy topological spaces, *J. Math Ana. Appl.*, 24, 182–190.

[6] Šostak, A. (1985) On a fuzzy topological structure, *Rend Circ. Mat. Palermo Supp.*, 11, 89–103.

[7] Parvathi, R., & Geetha, S. P. (2009) A note on properties of temporal intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets*, 15(1), 42–48.

[8] Ertürk, R., & Demirci, M. (1998) On the compactness in fuzzy topological spaces in Šostak's sense, *Matematicki Vesnik*, 50(3), 75–81.

[9] Ramadan, A. A., Abbas, S.E., & Abd El-latif A. A. (2005) Compactness in fuzzy topological spaces, *International Journal of Mathematics and Mathematical Sciences*, 1, 19–32.

[10] Yılmaz, S., & Çuvalcıoğlu, G. (2014) On level operators for temporal intuitionistic fuzzy sets, *Notes on Intuitionistic Fuzzy Sets*, 20(2), 6–15.