THE HOMOMORPHISM AND ANTI–HOMOMORPHISM OF LOWER LEVEL SUBGROUPS OF AN INTUITIONISTIC ANTIFUZZY SUBGROUP

N. PALANIAPPAN¹, K. ARJUNAN² AND M. S. ANITHA³

¹ Alagappa University, Karaikudi – 630003, Tamilnadu, India
palaniappan1950@yahoo.co.in

² Department of Mathematics, St. Michael College of Engineering & Technology,
Kalayarkoil – 630551, Tamilnadu, India
arjunan_1975@yahoo.co.in

³ Department of Mathematics, St. Michael College of Engineering & Technology,
Kalayarkoil – 630551, Tamilnadu, India.

ABSTRACT.

In this paper, we introduce some properties of an intuitionistic antifuzzy subgroup of a group with homomorphism and anti-homomorphism.

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INTRODUCTION.


1. PRELIMINARIES :

1.1 Definition :

An intuitionistic fuzzy subset (IFS) A in a set X is defined as an object of the form A = { (x, μA(x), νA(x) ) / x ∈ X }, where μA : X → [0,1] and νA : X → [0,1] define the degree of membership and the degree of non-membership of the element x ∈ X respectively and for every x ∈ X satisfying 0 ≤ μA(x) + νA(x) ≤ 1.

1.2 Definition:

Let G be a group. An intuitionistic fuzzy subset A of G is said to be an intuitionistic antifuzzy subgroup(AIFSG) of G if the following conditions are satisfied:

(i) μA(xy⁻¹) ≤ max{ μA(x), μA(y) },
(ii) νA(xy⁻¹) ≥ min{ νA(x), νA(y) }, for all x and y ∈ G.
1.3 Definition:
Let $A$ be an intuitionistic fuzzy subset of a set $X$. For $t \in [0, 1]$, the lower level subset of $A$ is the set,
$$A_t = \{ x \in X : \mu_A(x) \leq t \text{ and } \nu_A(x) \geq t \}.$$ 
This is called an intuitionistic fuzzy lower level subset of $A$.

1.4 Definition:
Let $A$ be an intuitionistic antifuzzy subgroup of a group $G$. The subgroup $A_t$ of $G$, for $t \in [0,1]$ such that $t \geq \mu_A(e)$ and $t \leq \nu_A(e)$ is called a lower level subgroup of $A$.

1.5 Definition:
If $(G, \cdot)$ and $(G', \cdot)$ are any two groups, then the function $f : G \to G'$ is called a group homomorphism if $f(xy) = f(x)f(y)$, for all $x$ and $y \in G$.

1.6 Definition:
If $(G, \cdot)$ and $(G', \cdot)$ are any two groups, then the function $f : G \to G'$ is called a group anti-homomorphism if $f(xy) = f(y)f(x)$, for all $x$ and $y \in G$.

1.7 Definition:
Let $X$ and $X'$ be any two sets. Let $f : X \to X'$ be any function and let $A$ be an intuitionistic fuzzy subset in $X$, $V$ be an intuitionistic fuzzy subset in $f(X) = X'$, defined by
$$\mu_V(y) = \inf_{x \in f^{-1}(y)} \mu_A(x) \quad \text{and} \quad \nu_V(y) = \sup_{x \in f^{-1}(y)} \nu_A(x), \quad \text{for all } x \in X \text{ and } y \in X'.$$

$A$ is called a preimage of $V$ under $f$ and is denoted by $f^{-1}(V)$.

1.1 Theorem:
Let $G, G'$ be any two groups with identity. Let $f : G \to G'$ be a homomorphism. Then,
(i) $f(1) = 1'$ where $1$ and $1'$ are the identities of $G$ and $G'$ respectively.
(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$.

Proof: It is trivial.

1.2 Theorem:
Let $G, G'$ be any two groups with identity. Let $f : G \to G'$ be an anti-homomorphism. Then,
(i) $f(1) = 1'$ where $1$ and $1'$ are the identity of $G$ and $G'$ respectively, and
(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$.

Proof: It is trivial.

SOME PROPOSITIONS:
1.1 Proposition:
Let $A$ be an intuitionistic antifuzzy subgroup of a group $G$. Then for $t \in [0,1]$ such that $t \geq \mu_A(e)$ and $t \leq \mu_A(e)$, $A_t$ is a subgroup of $G$.

1.2 Proposition:
The homomorphic image of an intuitionistic antifuzzy subgroup of a group $G$ is an intuitionistic antifuzzy subgroup of a group $G'$.

1.3 Proposition:
The homomorphic pre-image of an intuitionistic antifuzzy subgroup of a group $G'$ is an intuitionistic antifuzzy subgroup of a group $G$.

1.4 Proposition:
The anti-homomorphic image of an intuitionistic antifuzzy subgroup of a group $G$ is an intuitionistic antifuzzy subgroup of a group $G'$. 

15
1.5 Proposition:

The anti-homomorphic pre-image of an intuitionistic antifuzzy subgroup of a group $G'$ is an intuitionistic antifuzzy subgroup of a group $G$.

1.6 Proposition:

The homomorphic image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group $G$ is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group $G'$.

Proof:

Let $G$ and $G'$ be any two groups.
Let $f : G \to G'$ be a homomorphism.
That is $f(xy) = f(x)f(y)$ for all $x$ and $y \in G$.
Let $V = f(A)$, where $A$ is an intuitionistic antifuzzy subgroup of a group $G$.
Clearly $V$ is an intuitionistic antifuzzy subgroup of a group $G'$.
Let $x$ and $y \in G$, implies $f(x)$ and $f(y)$ in $G'$.
Clearly $A_t$ is a lower level subgroup of $A$.
That is $\mu_A(x) \leq t$ and $\nu_A(x) \geq t$; $\mu_A(y) \leq t$ and $\nu_A(y) \geq t$.

We have to prove that $f(A_t)$ is a lower level subgroup of $V$.
Now,
$$\mu_V(f(x)) \leq \mu_A(x) \leq t \text{ and } \nu_V(f(x)) \geq \nu_A(x) \geq t;$$
$$\mu_V(f(y)) \leq \mu_A(y) \leq t \text{ and } \nu_V(f(y)) \geq \nu_A(y) \geq t;$$
which implies that $\mu_V(f(x)f(y)^{-1}) \leq t$.
And,
$$\nu_V(f(x)) \geq \nu_A(x) \geq t \text{ and } \nu_V(f(y)) \geq \nu_A(y) \geq t;$$
$$\nu_V(f(x)f(y)^{-1}) = \nu_V(f(x)f(y)^{-1}), \text{ as } f \text{ is a homomorphism}$$
Hence $f(A_t)$ is a lower level subgroup of an intuitionistic antifuzzy subgroup $V$ of a group $G'$.

1.7 Proposition:

The homomorphic pre-image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group $G'$ is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group $G$.

Proof:

Let $G$ and $G'$ be any two groups.
Let $f : G \to G'$ be a homomorphism.
That is $f(xy) = f(x)f(y)$ for all $x$ and $y \in G$.
Let $V = f(A)$, where $V$ is an intuitionistic antifuzzy subgroup of a group $G'$.
Clearly $A$ is an intuitionistic antifuzzy subgroup of a group $G$.
Let $f(x)$ and $f(y) \in G'$, implies $x$ and $y$ in $G$.
Clearly $f(A_t)$ is a lower level subgroup of $V$.
That is $\mu_V(f(x)) \leq t$ and $\nu_V(f(x)) \geq t$; $\mu_V(f(y)) \leq t$ and $\nu_V(f(y)) \geq t$;
\[ \mu_V( f(x)(f(y))^{-1} ) \leq t \text{ and } \nu_V( f(x)(f(y))^{-1} ) \geq t. \]

We have to prove that \( A_t \) is a lower level subgroup of \( A \).

Now, \( \mu_A(x) = \mu_V( f(x) ) \leq t \), implies that \( \mu_A(x) \leq t \):

\[ \mu_A(y) = \mu_V( f(y) ) \leq t, \text{ implies that } \mu_A(y) \leq t; \text{ and} \]
\[ \mu_A( xy^{-1} ) = \mu_V( f(x)f(y^{-1}) ), \]
\[ = \mu_V( f(x)f(y^{-1}) ), \text{ as } f \text{ is a homomorphism} \]
\[ = \mu_V( f(x)(f(y))^{-1} ), \text{ as } f \text{ is a homomorphism} \]
\[ \leq t, \]

which implies that \( \mu_A(xy^{-1}) \leq t. \)

And, \( \nu_A(x) = \nu_V( f(x) ) \geq t \), implies that \( \nu_A(x) \geq t \):

\[ \nu_A(y) = \nu_V( f(y) ) \geq t, \text{ implies that } \nu_A(y) \geq t; \text{ and} \]
\[ \nu_A( xy^{-1} ) = \nu_V( f(x)f(y^{-1}) ), \]
\[ = \nu_V( f(x)f(y^{-1}) ), \text{ as } f \text{ is a homomorphism} \]
\[ = \nu_V( f(x)(f(y))^{-1} ), \text{ as } f \text{ is a homomorphism} \]
\[ \geq t, \]

which implies that \( \nu_A(xy^{-1}) \geq t. \)

Therefore \( \mu_A(xy^{-1}) \leq t \text{ and } \nu_A(xy^{-1}) \geq t. \)

Hence \( A_t \) is a lower level subgroup of an intuitionistic antifuzzy subgroup \( A \) of a group \( G \).

1.8 Proposition:

The anti-homomorphic image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group \( G \) is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group \( G^1 \).

Proof:

Let \( G \) and \( G^1 \) be any two groups.

Let \( f : G \rightarrow G^1 \) be an anti-homomorphism.

That is \( f(xy) = f(y)f(x) \) for all \( x \) and \( y \in G \).

Let \( V = f(A) \), where \( A \) is an intuitionistic antifuzzy subgroup of a group \( G \).

Clearly \( V \) is an intuitionistic antifuzzy subgroup of a group \( G^1 \).

Let \( x \) and \( y \in G \), implies \( f(x) \) and \( f(y) \) in \( G^1 \).

Clearly \( A_t \) is a lower level subgroup of \( A \).

That is \( \mu_A(x) \leq t \text{ and } \nu_A(x) \geq t; \mu_A(y) \leq t \text{ and } \nu_A(y) \geq t; \mu_A( y^{-1}x ) \leq t \text{ and } \nu_A( y^{-1}x ) \geq t. \)

We have to prove that \( f(A_t) \) is a lower level subgroup of \( V \).

Now, \( \mu_V( f(x) ) \leq \mu_A(x) \leq t, \text{ implies that } \mu_V( f(x) ) \leq t; \text{ and} \)
\[ \mu_V( f(y) ) \leq \mu_A(y) \leq t, \text{ implies that } \mu_V( f(y) ) \leq t; \text{ and} \]
\[ \mu_V( f(x)(f(y))^{-1} ) = \mu_V( f(x)f(y^{-1}) ), \text{ as } f \text{ is an anti-homomorphism} \]
\[ = \mu_V( f(y^{-1}x ) ), \text{ as } f \text{ is an anti-homomorphism} \]
\[ \leq \mu_A( y^{-1}x ) \leq t, \]

which implies that \( \mu_V( f(x)(f(y))^{-1} ) \leq t. \)

And, \( \nu_V( f(x) ) \geq \nu_A(x) \geq t, \text{ implies that } \nu_V( f(x) ) \geq t; \text{ and} \)
\[ \nu_V( f(y) ) \geq \nu_A(y) \geq t, \text{ implies that } \nu_V( f(y) ) \geq t; \text{ and} \]
\[ \nu_V( f(x)(f(y))^{-1} ) = \nu_V( f(x)f(y^{-1}) ), \text{ as } f \text{ is an anti-homomorphism} \]
\[ = \nu_V( f(y^{-1}x ) ), \text{ as } f \text{ is an anti-homomorphism} \]
\[ \geq \nu_A( y^{-1}x ) \geq t, \]

which implies that \( \nu_V( f(x)(f(y))^{-1} ) \geq t. \)
Therefore $\mu_V(f(x)(f(y))^{-1}) \leq t$ and $\nu_V(f(x)(f(y))^{-1}) \geq t$.

Hence $f(A_t)$ is a lower level subgroup of an intuitionistic antifuzzy subgroup $V$ of a group $G^l$.

1.9 Proposition :

The anti-homomorphic pre-image of a lower level subgroup of an intuitionistic antifuzzy subgroup of a group $G^l$ is a lower level subgroup of an intuitionistic antifuzzy subgroup of a group $G$.

Proof:

Let $G$ and $G^l$ be any two groups.

Let $f: G \rightarrow G^l$ be an anti-homomorphism.

That is $f(xy) = f(y)f(x)$, for all $x$ and $y \in G$.

Let $V=f(A)$, where $V$ is an intuitionistic antifuzzy subgroup of a group $G^l$.

Clearly $A$ is an intuitionistic antifuzzy subgroup of a group $G$.

Let $f(x)$ and $f(y) \in G^l$, implies $x$ and $y$ in $G$.

Clearly $f(A_t)$ is a lower level subgroup of $V$.

That is $\mu_V(f(x)) \leq t$ and $\nu_V(f(x)) \geq t$; $\mu_V(f(y)) \leq t$ and $\nu_V(f(y)) \geq t$;

$\mu_V((f(y))^{-1}f(x)) \leq t$ and $\nu_V((f(y))^{-1}f(x)) \geq t$.

We have to prove that $A_t$ is a lower level subgroup of $A$.

Now, $\mu_A(x) = \mu_V(f(x)) \leq t$, implies that $\mu_A(x) \leq t$: $\mu_A(y) = \mu_V(f(y)) \leq t$, implies that $\mu_A(y) \leq t$; and $\mu_A(xy^{-1}) = \mu_V(f(xy^{-1}))$,

$= \mu_V((f(y))^{-1}f(x))$, as $f$ is an anti-homomorphism

$= \mu_V((f(y))^{-1}f(x))$, as $f$ is an anti-homomorphism

$\leq t$,

which implies that $\mu_A(xy^{-1}) \leq t$.

And, Now, $\nu_A(x) = \nu_V(f(x)) \geq t$, implies that $\nu_A(x) \geq t$: $\nu_A(y) = \nu_V(f(y)) \geq t$, implies that $\nu_A(y) \geq t$; and $\nu_A(xy^{-1}) = \nu_V(f(xy^{-1}))$,

$= \nu_V((f(y))^{-1}f(x))$, as $f$ is an anti-homomorphism

$= \nu_V((f(y))^{-1}f(x))$, as $f$ is an anti-homomorphism

$\geq t$,

which implies that $\nu_A(xy^{-1}) \geq t$.

Therefore $\mu_A(xy^{-1}) \leq t$ and $\nu_A(xy^{-1}) \geq t$.

Hence $A_t$ is a lower level subgroup of an intuitionistic antifuzzy subgroup $A$ of a group $G$.

BIBLIOGRAPHY


