

## On intuitionistic fuzzy pairs

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**Abstract:** The concept of an intuitionistic fuzzy pair is introduced and described formally. Relations, operations and operators over these pairs are defined.

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## 1 Introduction

The first researches, related to Intuitionistic Fuzzy Sets (IFSs), started in 1983 and from the beginning, the concept of Intuitionistic Fuzzy Pair (IFP) has been in use. The authors of the present paper and a lot of other researchers working in the area of intuitionistic fuzziness, have used this concept without a special definition in a plethora of publications, under various terms: *intuitionistic fuzzy pair*, *intuitionistic fuzzy couple*, *intuitionistic fuzzy value*, and others. With the present paper, we propose to the researchers in the area to stick to only one term for that concept. After a long discussion we decided to offer the term *intuitionistic fuzzy pair*.

Here, we give a formal definition of IFP and collect definitions of all operations, relations and operators, defined over IFPs during the last already 30 years, using our books [1, 2, 10].

## 2 Definition and geometrical interpretations of an IFP

The Intuitionistic Fuzzy Pair (IFP) is an object with the form  $\langle a, b \rangle$ , where  $a, b \in [0, 1]$  and  $a + b \leq 1$ , that is used as an evaluation of some object or process and which components ( $a$  and  $b$ ) are interpreted as degrees of membership and non-membership, or degrees of validity and non-validity, or degree of correctness and non-correctness, etc. The geometrical interpretations

of the IFPs are the same as of the IFSs. Two of them are shown on Fig. 1–4. The first one (which is analogous to the standard fuzzy set interpretation) is shown on Fig. 1.

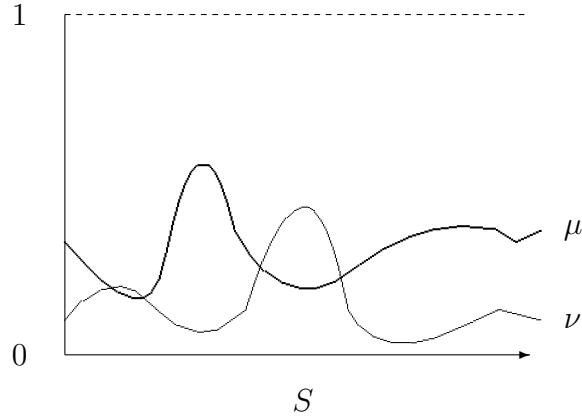


Fig. 1.

Its analogue is given in Fig. 2.

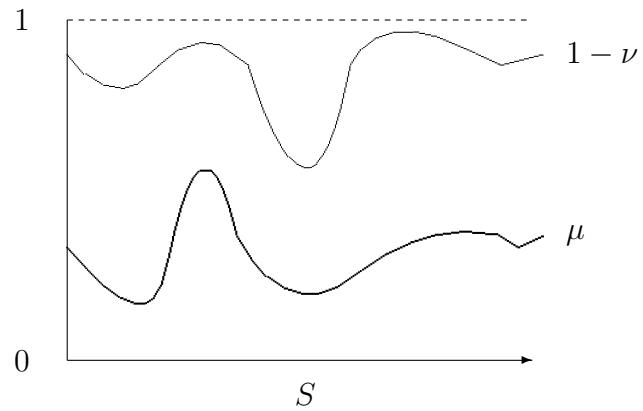


Fig. 2.

Therefore, we can map to every proposition  $p \in S$  a unit segment of the form:

$$\left. \begin{array}{c} b \\ a \\ p \end{array} \right\}$$

For the needs of the discussion below, we define the notion of Intuitionistic Fuzzy Tautological Pair (IFTP) by:

$x$  is an IFTP if and only if  $a \geq b$ ,

while  $p$  is a Tautological Pair (TP) iff  $a = 1$  and  $b = 0$ .

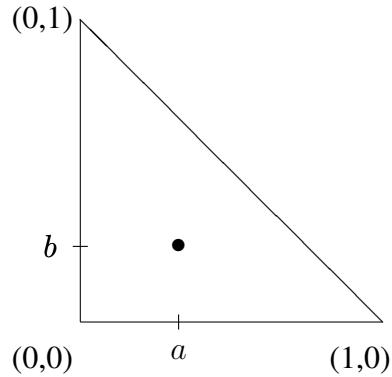


Fig. 3.

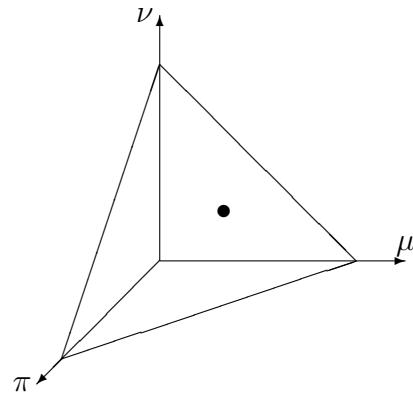


Fig. 4.

### 3 Relations over IFPs

Let us have two IFPs  $x = \langle a, b \rangle$  and  $y = \langle c, d \rangle$ . We define the relations

$$\begin{aligned}
 x <_{\square} y &\quad \text{iff} \quad a < c \\
 x <_{\diamond} y &\quad \text{iff} \quad b > d \\
 x < y &\quad \text{iff} \quad a < c \text{ and } b > d \\
 x \leq_{\square} y &\quad \text{iff} \quad a \leq c \\
 x \leq_{\diamond} y &\quad \text{iff} \quad b \geq d \\
 x \leq y &\quad \text{iff} \quad a \leq c \text{ and } b \geq d \\
 x >_{\square} y &\quad \text{iff} \quad a > c \\
 x >_{\diamond} y &\quad \text{iff} \quad b < d \\
 x > y &\quad \text{iff} \quad a > c \text{ and } b < d \\
 x \geq_{\square} y &\quad \text{iff} \quad a \geq c \\
 x \geq_{\diamond} y &\quad \text{iff} \quad b \leq d \\
 x \geq y &\quad \text{iff} \quad a \geq c \text{ and } b \leq d \\
 x =_{\square} y &\quad \text{iff} \quad a = c \\
 x =_{\diamond} y &\quad \text{iff} \quad b = d \\
 x = y &\quad \text{iff} \quad a = c \text{ and } b = d
 \end{aligned}$$

## 4 Operations over IFPs

In some definitions below, we use the functions  $\text{sg}$  and  $\overline{\text{sg}}$ , as defined by,

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

$$\overline{\text{sg}}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

Let us have two IFPs  $x = \langle a, b \rangle$  and  $y = \langle c, d \rangle$ .

First, we define analogues of operations “conjunction” and “disjunction”:

$$\begin{aligned} x \&_1 y &= x \cap y = \langle \min(a, c), \max(b, d) \rangle \\ x \vee_1 y &= x \cup y = \langle \max(a, c), \min(b, d) \rangle \\ x \&_2 y &= x + y = \langle a + c - a.c, b.d \rangle \\ x \vee_2 y &= x.y = \langle a.c, b + d - b.d \rangle. \end{aligned}$$

Second, we define analogues of operations “implication” and “negation”. In Table 1 the results of implications  $x \rightarrow y$  are given. Implications  $\rightarrow_{139}, \dots, \rightarrow_{149}$  are given in [3, 4, 5] and  $\rightarrow_{150}, \dots, \rightarrow_{152}$  – in [7, 8, 9], all other are described in [2].

Table 1

$\rightarrow_1$	$\langle \max(b, \min(a, c)), \min(a, d) \rangle$
$\rightarrow_2$	$\langle \overline{\text{sg}}(a - c), d.\text{sg}(a - c) \rangle$
$\rightarrow_3$	$\langle 1 - (1 - c).\text{sg}(a - c), d.\text{sg}(a - c) \rangle$
$\rightarrow_4$	$\langle \max(b, c), \min(a, d) \rangle$
$\rightarrow_5$	$\langle \min(1, b + c), \max(0, a + d - 1) \rangle$
$\rightarrow_6$	$\langle b + ac, ad \rangle$
$\rightarrow_7$	$\langle \min(\max(b, c), \max(a, b), \max(c, d)), \max(\min(a, d), \min(a, b), \min(c, d)) \rangle$
$\rightarrow_8$	$\langle 1 - (1 - \min(b, c)).\text{sg}(a - c), \max(a, d).\text{sg}(a - c), \text{sg}(d - b) \rangle$
$\rightarrow_9$	$\langle b + a^2c, ab + a^2d \rangle$
$\rightarrow_{10}$	$\langle c.\overline{\text{sg}}(1 - a) + \text{sg}(1 - a).(\overline{\text{sg}}(1 - c) + b.\text{sg}(1 - c)), d.\overline{\text{sg}}(1 - a) + a.\text{sg}(1 - a).\text{sg}(1 - c) \rangle$
$\rightarrow_{11}$	$\langle 1 - (1 - c).\text{sg}(a - c), d.\text{sg}(a - c).\text{sg}(d - b) \rangle$
$\rightarrow_{12}$	$\langle \max(b, c), 1 - \max(b, c) \rangle$
$\rightarrow_{13}$	$\langle b + c - b.c, a.d \rangle$
$\rightarrow_{14}$	$\langle 1 - (1 - c).\text{sg}(a - c) - d.\overline{\text{sg}}(a - c).\text{sg}(d - b), d.\text{sg}(d - b) \rangle$
$\rightarrow_{15}$	$\langle 1 - (1 - \min(b, c)).\text{sg}(a - c).\text{sg}(d - b) - \min(b, c).\text{sg}(a - c).\text{sg}(d - b), 1 - (1 - \max(a, d)).\text{sg}(\overline{\text{sg}}(a - c) + \overline{\text{sg}}(d - b)) - \max(a, d).\overline{\text{sg}}(a - c).\overline{\text{sg}}(d - b) \rangle$
$\rightarrow_{16}$	$\langle \max(\overline{\text{sg}}(a), c), \min(\text{sg}(a), d) \rangle$
$\rightarrow_{17}$	$\langle \max(b, c), \min(a.b + a^2, d) \rangle$

$\rightarrow_{18}$	$\langle \max(b, c), \min(1 - b, d) \rangle$
$\rightarrow_{19}$	$\langle \max(1 - \text{sg}(\text{sg}(a) + \text{sg}(1 - b)), c), \min(\text{sg}(1 - b), d) \rangle$
$\rightarrow_{20}$	$\langle \max(\overline{\text{sg}}(a), \text{sg}(c)), \min(\text{sg}(a), \overline{\text{sg}}(c)) \rangle$
$\rightarrow_{21}$	$\langle \max(b, c.(c + d)), \min(a.(a + b), d.(c^2 + d + c.d)) \rangle$
$\rightarrow_{22}$	$\langle \max(b, 1 - d), 1 - \max(b, 1 - d) \rangle$
$\rightarrow_{23}$	$\langle 1 - \min(\text{sg}(1 - b), \overline{\text{sg}}(1 - d)), \min(\text{sg}(1 - b), \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{24}$	$\langle \overline{\text{sg}}(a - c).\overline{\text{sg}}(d - b), \text{sg}(a - c).\text{sg}(d - b) \rangle$
$\rightarrow_{25}$	$\langle \max(b, \overline{\text{sg}}(a).\overline{\text{sg}}(1 - b), c.\overline{\text{sg}}(d).\overline{\text{sg}}(1 - c)), \min(a, d) \rangle$
$\rightarrow_{26}$	$\langle \max(\overline{\text{sg}}(1 - b), c), \min(\text{sg}(a), d) \rangle$
$\rightarrow_{27}$	$\langle \max(\overline{\text{sg}}(1 - b), \text{sg}(c)), \min(\text{sg}(a), \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{28}$	$\langle \max(\overline{\text{sg}}(1 - b), c), \min(a, d) \rangle$
$\rightarrow_{29}$	$\langle \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - c)), \min(a, \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{30}$	$\langle \max(1 - a, \min(a, 1 - d)), \min(a, d) \rangle$
$\rightarrow_{31}$	$\langle \overline{\text{sg}}(a + d - 1), d.\text{sg}(a + d - 1) \rangle$
$\rightarrow_{32}$	$\langle 1 - d.\text{sg}(a + d - 1), d.\text{sg}(a + d - 1) \rangle$
$\rightarrow_{33}$	$\langle 1 - \min(a, d), \min(a, d) \rangle$
$\rightarrow_{34}$	$\langle \min(1, 2 - a - d), \max(0, a + d - 1) \rangle$
$\rightarrow_{35}$	$\langle 1 - a.d, a.d \rangle$
$\rightarrow_{36}$	$\langle \min(1 - \min(a, d), \max(a, 1 - a), \max(1 - d, d)), \max(\min(a, d), \min(a, 1 - a), \min(1 - d, d)) \rangle$
$\rightarrow_{37}$	$\langle 1 - \max(a, d).\text{sg}(a + d - 1), \max(a, d).\text{sg}(a + d - 1) \rangle$
$\rightarrow_{38}$	$\langle 1 - a + (a^2.(1 - d)), a.(1 - a) + a^2.d \rangle$
$\rightarrow_{39}$	$\langle (1 - d).\overline{\text{sg}}(1 - a) + \text{sg}(1 - a).(\overline{\text{sg}}(d) + (1 - a).\text{sg}(d)), d.\overline{\text{sg}}(1 - a) + a.\text{sg}(1 - a).\text{sg}(d) \rangle$
$\rightarrow_{40}$	$\langle 1 - \text{sg}(a + d - 1), 1 - \overline{\text{sg}}(a + d - 1) \rangle$
$\rightarrow_{41}$	$\langle \max(\overline{\text{sg}}(a), 1 - d), \min(\text{sg}(a), d) \rangle$
$\rightarrow_{42}$	$\langle \max(\overline{\text{sg}}(a), \text{sg}(1 - d)), \min(\text{sg}(a), \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{43}$	$\langle \max(\overline{\text{sg}}(a), 1 - d), \min(\text{sg}(a), d) \rangle$
$\rightarrow_{44}$	$\langle \max(\overline{\text{sg}}(a), 1 - d), \min(a, d) \rangle$
$\rightarrow_{45}$	$\langle \max(\overline{\text{sg}}(a), \overline{\text{sg}}(d)), \min(a, \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{46}$	$\langle \max(b, \min(1 - b, c)), 1 - \max(b, c) \rangle$
$\rightarrow_{47}$	$\langle \overline{\text{sg}}(1 - b - c), (1 - c).\text{sg}(1 - b - c) \rangle$
$\rightarrow_{48}$	$\langle 1 - (1 - c).\text{sg}(1 - b - c), (1 - c).\text{sg}(1 - b - c) \rangle$
$\rightarrow_{49}$	$\langle \min(1, b + c), \max(0, 1 - b - c) \rangle$
$\rightarrow_{50}$	$\langle b + c - b.c, 1 - b - c + b.c \rangle$
$\rightarrow_{51}$	$\langle \min(\max(b, c), \max(1 - b, b), \max(c, 1 - c)), \max(1 - \max(b, c), \min(1 - b, b), \min(c, 1 - c)) \rangle$
$\rightarrow_{52}$	$\langle 1 - (1 - \min(b, c)).\text{sg}(1 - b - c), 1 - \min(b, c).\text{sg}(1 - b - c) \rangle$
$\rightarrow_{53}$	$\langle b + (1 - b)^2.c, (1 - b).b + (1 - b)^2.(1 - c) \rangle$
$\rightarrow_{54}$	$\langle c.\overline{\text{sg}}(b)) + \text{sg}(b).(\overline{\text{sg}}(1 - c) + b.\text{sg}(1 - c)), (1 - c).\overline{\text{sg}}(b) + (1 - b).\text{sg}(b).\text{sg}(1 - c) \rangle$

$\rightarrow_{55}$	$\langle 1 - \text{sg}(1 - b - c), 1 - \overline{\text{sg}}(1 - b - c) \rangle$
$\rightarrow_{56}$	$\langle \max(\overline{\text{sg}}(1 - b), c), \min(\text{sg}(1 - b), (1 - c)) \rangle$
$\rightarrow_{57}$	$\langle \max(\overline{\text{sg}}(1 - b), \text{sg}(c)), \min(\text{sg}(1 - b), \overline{\text{sg}}(c)) \rangle$
$\rightarrow_{58}$	$\langle \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - c)), 1 - \max(b, c) \rangle$
$\rightarrow_{59}$	$\langle \max(\overline{\text{sg}}(1 - b), c), (1 - \max(b, c)) \rangle$
$\rightarrow_{60}$	$\langle \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - c)), \min((1 - b), \overline{\text{sg}}(c)) \rangle$
$\rightarrow_{61}$	$\langle \max(c, \min(b, d)), \min(a, d) \rangle$
$\rightarrow_{62}$	$\langle \overline{\text{sg}}(d - b), a.\text{sg}(d - b) \rangle$
$\rightarrow_{63}$	$\langle 1 - (1 - b).\text{sg}(d - b), a.\text{sg}(d - b) \rangle$
$\rightarrow_{64}$	$\langle c + b.d, a.d \rangle$
$\rightarrow_{65}$	$\langle 1 - (1 - \min(b, c)).\text{sg}(d - b), \max(a, d).\text{sg}(d - b).\text{sg}(a - c) \rangle$
$\rightarrow_{66}$	$\langle c + d^2.b, b.d + d^2.a \rangle$
$\rightarrow_{67}$	$\langle b.\overline{\text{sg}}(1 - d) + \text{sg}(1 - d).(\overline{\text{sg}}(1 - b) + c.\text{sg}(1 - b)), a.\overline{\text{sg}}(1 - d) + d.\text{sg}(1 - d).\text{sg}(1 - b) \rangle$
$\rightarrow_{68}$	$\langle 1 - (1 - b).\text{sg}(d - b), a.\text{sg}(d - b).\text{sg}(a - c) \rangle$
$\rightarrow_{69}$	$\langle 1 - (1 - b).\text{sg}(d - b) - a.\overline{\text{sg}}(d - b).\text{sg}(a - c), a.\text{sg}(a - c) \rangle$
$\rightarrow_{70}$	$\langle \max(\overline{\text{sg}}(d), b), \min(\text{sg}(d), a) \rangle$
$\rightarrow_{71}$	$\langle \max(b, c), \min(c.d + d^2, a) \rangle$
$\rightarrow_{72}$	$\langle \max(b, c), \min(1 - c, a) \rangle$
$\rightarrow_{73}$	$\langle \max(1 - \max(\text{sg}(d), \text{sg}(1 - c)), b), \min(\text{sg}(1 - c), a) \rangle$
$\rightarrow_{74}$	$\langle \max(\text{sg}(b), \overline{\text{sg}}(d)), \min(\overline{\text{sg}}(b), \text{sg}(d)) \rangle$
$\rightarrow_{75}$	$\langle \max(c, b.(a + b)), \min(d.(c + d), a.(b^2 + a) + a.b) \rangle$
$\rightarrow_{76}$	$\langle \max(c, 1 - a), \min(1 - c, a) \rangle$
$\rightarrow_{77}$	$\langle (1 - \min(\overline{\text{sg}}(1 - a), \text{sg}(1 - c))), \min(\overline{\text{sg}}(1 - a), \text{sg}(1 - c)) \rangle$
$\rightarrow_{78}$	$\langle \max(\overline{\text{sg}}(1 - c), b), \min(\text{sg}(d), a) \rangle$
$\rightarrow_{79}$	$\langle \max(\overline{\text{sg}}(1 - c), \text{sg}(b)), \min(\text{sg}(d), \overline{\text{sg}}(1 - a)) \rangle$
$\rightarrow_{80}$	$\langle \max(\overline{\text{sg}}(1 - c), b), \min(d, a) \rangle$
$\rightarrow_{81}$	$\langle \max(\overline{\text{sg}}(1 - b), \overline{\text{sg}}(1 - c)), \min(d, \overline{\text{sg}}(1 - a)) \rangle$
$\rightarrow_{82}$	$\langle \max(1 - d, \min(d, 1 - a)), \min(d, a) \rangle$
$\rightarrow_{83}$	$\langle \overline{\text{sg}}(a + d - 1), a.\text{sg}(a + d - 1) \rangle$
$\rightarrow_{84}$	$\langle 1 - a.\text{sg}(a + d + 1), a.\text{sg}(a + d + 1) \rangle$
$\rightarrow_{85}$	$\langle 1 - d + d^2.(1 - a), d.(1 - d) + d^2. \rangle$
$\rightarrow_{86}$	$\langle (1 - a).\overline{\text{sg}}(1 - d) + \text{sg}(1 - d).\overline{\text{sg}}(a + \min(1 - d, \text{sg}(a))), a.\overline{\text{sg}}(1 - d) + d.\text{sg}(1 - d).\text{sg}(a) \rangle$
$\rightarrow_{87}$	$\langle \max(\overline{\text{sg}}(d), 1 - a), \min(\text{sg}(d), a) \rangle$
$\rightarrow_{88}$	$\langle \max(\overline{\text{sg}}(d), \text{sg}(1 - a)), \min(\text{sg}(d), \overline{\text{sg}}(1 - a)) \rangle$
$\rightarrow_{89}$	$\langle \max(\overline{\text{sg}}(d), 1 - a), \min(d, a) \rangle$
$\rightarrow_{90}$	$\langle \max(\overline{\text{sg}}(a), \overline{\text{sg}}(d)), \min(d, \overline{\text{sg}}(1 - a)) \rangle$
$\rightarrow_{91}$	$\langle \max(c, \min(1 - c, b)), 1 - \max(b, c) \rangle$
$\rightarrow_{92}$	$\langle \overline{\text{sg}}(1 - b - c), \min(1 - b, \text{sg}(1 - b - c)) \rangle$
$\rightarrow_{93}$	$\langle (1 - \min(1 - b, \text{sg}(1 - b - c)), \min(1 - b, \text{sg}(1 - b - c))) \rangle$

$\rightarrow_{94}$	$\langle c + (1 - c)^2.b, (1 - c).c + (1 - c)^2.(1 - b) \rangle$
$\rightarrow_{95}$	$\langle \min(b, \overline{\text{sg}}(c)) + \text{sg}(c).(\overline{\text{sg}}(1 - b) + \min(c, \text{sg}(1 - b))), (\min(1 - b, \overline{\text{sg}}(c)) + \min(1 - c, \text{sg}(c), \text{sg}(1 - b))) \rangle$
$\rightarrow_{96}$	$\langle \max(\overline{\text{sg}}(1 - c), b), \min(\text{sg}(1 - b), 1 - c) \rangle$
$\rightarrow_{97}$	$\langle \max(\overline{\text{sg}}(1 - c), \text{sg}(b)), \min(\text{sg}(1 - c), \overline{\text{sg}}(b)) \rangle$
$\rightarrow_{98}$	$\langle \max(\overline{\text{sg}}(1 - c), b), 1 - \max(b, c) \rangle$
$\rightarrow_{99}$	$\langle \max(\overline{\text{sg}}(1 - c), \overline{\text{sg}}(1 - b)), \min(1 - c, \overline{\text{sg}}(b)) \rangle$
$\rightarrow_{100}$	$\langle \max(\min(b, \text{sg}(a)), c), \min(a, \text{sg}(b), d) \rangle$
$\rightarrow_{101}$	$\langle \max(\min(b, \text{sg}(a)), \min(c, \text{sg}(d))), \min(a, \text{sg}(b), \text{sg}(c), d) \rangle$
$\rightarrow_{102}$	$\langle \max(b, \min(c, \text{sg}(d))), \min(a, \text{sg}(c), d) \rangle$
$\rightarrow_{103}$	$\langle \max(\min(1 - a, \text{sg}(a)), 1 - d), \min(a, \text{sg}(1 - a), d) \rangle$
$\rightarrow_{104}$	$\langle \max(\min(1 - a, \text{sg}(a)), \min(1 - d, \text{sg}(d))), \min(a, \text{sg}(1 - a), d, \text{sg}(1 - d)) \rangle$
$\rightarrow_{105}$	$\langle \max(1 - a, \min(1 - d, \text{sg}(d))), \min(a, d, \text{sg}(1 - d)) \rangle$
$\rightarrow_{106}$	$\langle \max(\min(b, \text{sg}(1 - b)), c), \min(1 - b, \text{sg}(b), 1 - c) \rangle$
$\rightarrow_{107}$	$\langle \max(\min(b, \text{sg}(1 - b)), \min(c, \text{sg}(1 - c))), \min(1 - b, \text{sg}(b), 1 - c, \text{sg}(c)) \rangle$
$\rightarrow_{108}$	$\langle \max(b, \min(c, \text{sg}(1 - c))), \min(1 - b, 1 - c, \text{sg}(c)) \rangle$
$\rightarrow_{109}$	$\langle b + \min(\overline{\text{sg}}(1 - a), c), a.b + \min(\overline{\text{sg}}(1 - a), d) \rangle$
$\rightarrow_{110}$	$\langle \max(b, c), \min(a.b + \overline{\text{sg}}(1 - a), d) \rangle$
$\rightarrow_{111}$	$\langle \max(b, c.d + \overline{\text{sg}}(1 - c)), \min(a.b + \overline{\text{sg}}(1 - a), d.(c.d + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{112}$	$\langle b + c - b.c, a.b + \overline{\text{sg}}(1 - a).d \rangle$
$\rightarrow_{113}$	$\langle b + c.d - b.(c.d + \overline{\text{sg}}(1 - c)), (a.b + \overline{\text{sg}}(1 - a)).(d.(c.d + \overline{\text{sg}}(1 - c)) + \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{114}$	$\langle 1 - a + \min(\overline{\text{sg}}(1 - a), 1 - d), a.(1 - a) + \min(\overline{\text{sg}}(1 - a), d) \rangle$
$\rightarrow_{115}$	$\langle 1 - \min(a, d), \min(a.(1 - a) + \overline{\text{sg}}(1 - a), d) \rangle$
$\rightarrow_{116}$	$\langle \max(1 - a, (1 - d).d + \overline{\text{sg}}(d)), \min(a.(1 - a) + \overline{\text{sg}}(1 - a), d.((1 - d).d + \overline{\text{sg}}(d)) + \overline{\text{sg}}(1 - d)) \rangle$
$\rightarrow_{117}$	$\langle 1 - a - d + a.d, (a.(1 - a) + \overline{\text{sg}}(1 - a)).d \rangle$
$\rightarrow_{118}$	$\langle 1 - a + (1 - d).d - (1 - a).((1 - d).d + \overline{\text{sg}}(d)), (a.(1 - a) + \overline{\text{sg}}(1 - a)).d.((1 - d).d + \overline{\text{sg}}(d)) + \overline{\text{sg}}(1 - d) \rangle$
$\rightarrow_{119}$	$\langle b + \min(\overline{\text{sg}}(b), c), (1 - b).b + \min(\overline{\text{sg}}(b), 1 - c) \rangle$
$\rightarrow_{120}$	$\langle \max(b, c), \min((1 - b).b + \overline{\text{sg}}(b), 1 - c) \rangle$
$\rightarrow_{121}$	$\langle \max(b, c.(1 - c) + \overline{\text{sg}}(1 - c)), \min((1 - b).b + \overline{\text{sg}}(b), (1 - c).(c.(1 - c) + \overline{\text{sg}}(1 - c))) + \overline{\text{sg}}(c) \rangle$
$\rightarrow_{122}$	$\langle b + c - b.c, ((1 - c).b + \overline{\text{sg}}(b)).(1 - c) \rangle$
$\rightarrow_{123}$	$\langle b + c.(1 - c) - (b.(c.(1 - c) + \overline{\text{sg}}(1 - c))), ((1 - b).b + \overline{\text{sg}}(b)).(((1 - c).(c.(1 - c) + \overline{\text{sg}}(1 - c))) + \overline{\text{sg}}(c)) \rangle$
$\rightarrow_{124}$	$\langle c + \min(\overline{\text{sg}}(1 - d), b), c.d + \min(\overline{\text{sg}}(1 - d), a) \rangle$
$\rightarrow_{125}$	$\langle \max(b, c), \min(c.d + \overline{\text{sg}}(1 - d), a) \rangle$
$\rightarrow_{126}$	$\langle \max(c, a.b + \overline{\text{sg}}(1 - b)), \min(c.d + \overline{\text{sg}}(1 - d), a.(a.b + \overline{\text{sg}}(1 - b)) + \overline{\text{sg}}(1 - a)) \rangle$

$\rightarrow_{127}$	$\langle b + c - b.c, (c.d + \bar{sg}(1-d)).a \rangle$
$\rightarrow_{128}$	$\langle c + a.b - c.(a.b + \bar{sg}(1-b)), (c.d + \bar{sg}(1-d)).(a.(a.b + \bar{sg}(1-b)) + \bar{sg}(1-a)) \rangle$
$\rightarrow_{129}$	$\langle 1 - d + \min(\bar{sg}(1-d), 1-a), d.(1-d) + \min(\bar{sg}(1-d), a) \rangle$
$\rightarrow_{130}$	$\langle 1 - \min(d, a), \min(d.(1-d) + \bar{sg}(1-d), a) \rangle$
$\rightarrow_{131}$	$\langle \max(1-d, (1-a).a + \bar{sg}(a)), \min(d.(1-d) + \bar{sg}(1-d), a.((1-a).a + \bar{sg}(a)) + \bar{sg}(1-a)) \rangle$
$\rightarrow_{132}$	$\langle 1 - a.d, (d.(1-d) + \bar{sg}(1-d)).a \rangle$
$\rightarrow_{133}$	$\langle 1 - d + (1-a).a - (1-d).((1-a).a + \bar{sg}(a)), (d.(1-d) + \bar{sg}(1-d)).(a.((1-a).a + \bar{sg}(a)) + \bar{sg}(1-a)) \rangle$
$\rightarrow_{134}$	$\langle c + \min(\bar{sg}(c), b), (1-c).c + \min(\bar{sg}(c), (1-b)) \rangle$
$\rightarrow_{135}$	$\langle \max(b, c), \min((1-c).c + \bar{sg}(c), 1-b) \rangle$
$\rightarrow_{136}$	$\langle \max(c, (b.(1-b) + \bar{sg}(1-b))), \min((1-c).c + \bar{sg}(c), (1-b).(b.(1-b) + \bar{sg}(1-b)) + \bar{sg}(b)) \rangle$
$\rightarrow_{137}$	$\langle b + c - b.c, ((1-c).c + \bar{sg}(c)).(1-b) \rangle$
$\rightarrow_{138}$	$\langle c + b.(1-b) - c.(b.(1-b) + \bar{sg}(1-b)), ((1-c).c + \bar{sg}(c)).((1-b).(b.(1-b) + \bar{sg}(1-b)) + \bar{sg}(b)) \rangle$
$\rightarrow_{139}$	$\langle \frac{b+c}{2}, \frac{a+d}{2} \rangle$
$\rightarrow_{140}$	$\langle \frac{b+c+\min(b,c)}{3}, \frac{a+d+\max(a,d)}{3} \rangle$
$\rightarrow_{141}$	$\langle \frac{b+c+\max(b,c)}{3}, \frac{a+d+\min(a,d)}{3} \rangle$
$\rightarrow_{142}$	$\langle \frac{3-a-d-\max(a,d)}{3}, \frac{a+d+\max(a,d)}{3} \rangle$
$\rightarrow_{143}$	$\langle \frac{1-a+c+\min(1-a,c)}{3}, \frac{2+a-c-\min(1-a,c)}{3} \rangle$
$\rightarrow_{144}$	$\langle \frac{1+b-d+\min(b,1-d)}{3}, \frac{2-b+d+\min(b,1-d)}{3} \rangle$
$\rightarrow_{145}$	$\langle \frac{b+c+\min(b,c)}{3}, \frac{3-b-c-\min(b,c)}{3} \rangle$
$\rightarrow_{146}$	$\langle \frac{3-a-d-\min(a,d)}{3}, \frac{a+d+\min(a,d)}{3} \rangle$
$\rightarrow_{147}$	$\langle \frac{1-a+c+\max(1-a,c)}{3}, \frac{2+a-c-\max(1-a,c)}{3} \rangle$
$\rightarrow_{148}$	$\langle \frac{1+b-d+\max(b,1-d)}{3}, \frac{2-b+d-\max(b,1-d)}{3} \rangle$
$\rightarrow_{149}$	$\langle \frac{b+c+\max(b,c)}{3}, \frac{3-b-c-\max(b,c)}{3} \rangle$
$\rightarrow_{150,\lambda}$	$\langle \frac{b+c+\lambda-1}{2\lambda}, \frac{a+d+\lambda-1}{2\lambda}, \text{where } \lambda \geq 1 \rangle$
$\rightarrow_{151,\gamma}$	$\langle \frac{b+c+\gamma}{2\gamma+1}, \frac{a+d+\gamma-1}{2\gamma+1}, \text{where } \gamma \geq 1 \rangle$
$\rightarrow_{152,\alpha,\beta}$	$\langle \frac{b+c+\alpha-1}{\alpha+\beta}, \frac{a+d+\beta-1}{\alpha+\beta}, \text{where } \alpha \geq 1, \beta \in [0, \alpha] \rangle$
$\rightarrow_{153,\varepsilon,\eta}$	$\langle x, \min(1, \max(\mu_B(x), \nu_A(x) + \varepsilon)), \max(0, \min(\nu_B(x), \mu_A(x) - \eta)) \rangle$

In Table 2 the results of negations  $\neg x$  are given. Implications  $\neg_{35}, \dots, \neg_{41}$  are given in [3, 4, 5] and  $\neg_{42}, \dots, \neg_{45}$  – in [7, 8, 9], all other are described in [2].

Table 2

$\neg_1$	$\langle x, b, a \rangle$
$\neg_2$	$\langle x, \bar{sg}(a), sg(a) \rangle$
$\neg_3$	$\langle x, b, a.b + a^2 \rangle$
$\neg_4$	$\langle x, b, 1-b \rangle$

$\neg_5$	$\langle x, \overline{\text{sg}}(1-b), \text{sg}(1-b) \rangle$
$\neg_6$	$\langle x, \overline{\text{sg}}(1-b), \text{sg}(a) \rangle$
$\neg_7$	$\langle x, \overline{\text{sg}}(1-b), a \rangle$
$\neg_8$	$\langle x, 1-a, a \rangle$
$\neg_9$	$\langle x, \overline{\text{sg}}(a), a \rangle$
$\neg_{10}$	$\langle x, \overline{\text{sg}}(1-b), 1-b \rangle$
$\neg_{11}$	$\langle x, \text{sg}(b), \overline{\text{sg}}(b) \rangle$
$\neg_{12}$	$\langle x, b.(b+a), \min(1, a.(b^2 + a + b.a)) \rangle$
$\neg_{13}$	$\langle x, \text{sg}(1-a), \overline{\text{sg}}(1-a) \rangle$
$\neg_{14}$	$\langle x, \text{sg}(b), \overline{\text{sg}}(1-a) \rangle$
$\neg_{15}$	$\langle x, \overline{\text{sg}}(1-b), \overline{\text{sg}}(1-a) \rangle$
$\neg_{16}$	$\langle x, \overline{\text{sg}}(a), \overline{\text{sg}}(1-a) \rangle$
$\neg_{17}$	$\langle x, \overline{\text{sg}}(1-b), \overline{\text{sg}}(b) \rangle$
$\neg_{18}$	$\langle x, b.\text{sg}(a), a.\text{sg}(b) \rangle$
$\neg_{19}$	$\langle x, b.\text{sg}(a), 0 \rangle$
$\neg_{20}$	$\langle x, b, 0 \rangle$
$\neg_{21}$	$\langle x, \min(1-a, \text{sg}(a)), \min(a, \text{sg}(1-a)) \rangle$
$\neg_{22}$	$\langle x, \min(1-a, \text{sg}(a)), 0 \rangle$
$\neg_{23}$	$\langle x, 1-a, 0 \rangle$
$\neg_{24}$	$\langle x, \min(b, \text{sg}(1-b)), \min(1-b, \text{sg}(b)) \rangle$
$\neg_{25}$	$\langle x, \min(b, \text{sg}(1-b)), 0 \rangle$
$\neg_{26}$	$\langle x, b, a.b + \overline{\text{sg}}(1-a) \rangle$
$\neg_{27}$	$\langle x, 1-a, a.(1-a) + \overline{\text{sg}}(1-a) \rangle$
$\neg_{28}$	$\langle x, b, (1-b).b + \overline{\text{sg}}(b) \rangle$
$\neg_{29}$	$\langle x, \max(0, b.a + \overline{\text{sg}}(1-b)), \min(1, a.(b.a + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(1-a)) \rangle$
$\neg_{30}$	$\langle x, a.b, a.(a.b + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(1-a) \rangle$
$\neg_{31}$	$\langle x, \max(0, (1-a).a + \overline{\text{sg}}(a)), \min(1, a.((1-a).a + \overline{\text{sg}}(a)) + \overline{\text{sg}}(1-a)) \rangle$
$\neg_{32}$	$\langle x, (1-a).a, a.((1-a).a + \overline{\text{sg}}(a)) + \overline{\text{sg}}(1-a) \rangle$
$\neg_{33}$	$\langle x, b.(1-b) + \overline{\text{sg}}(1-b), (1-b).(b.(1-b) + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(b) \rangle$
$\neg_{34}$	$\langle x, b.(1-b), (1-b).(b.(1-b) + \overline{\text{sg}}(1-b)) + \overline{\text{sg}}(b) \rangle$
$\neg_{35}$	$\langle \frac{b}{2}, \frac{1+a}{2} \rangle$
$\neg_{36}$	$\langle \frac{b}{3}, \frac{2+a}{3} \rangle$
$\neg_{37}$	$\langle \frac{2b}{3}, \frac{2a+1}{3} \rangle$
$\neg_{38}$	$\langle \frac{1-a}{3}, \frac{2+a}{3} \rangle$
$\neg_{39}$	$\langle \frac{b}{3}, \frac{3-b}{3} \rangle$
$\neg_{40}$	$\langle \frac{2-2a}{3}, \frac{1+2a}{3} \rangle$
$\neg_{41}$	$\langle \frac{2b}{3}, \frac{3-2b}{3} \rangle$
$\neg_{42,\lambda}$	$\langle \frac{b+\lambda-1}{2\lambda}, \frac{a+\lambda}{2\lambda}, \text{ where } \lambda \geq 1 \rangle$
$\neg_{43,\gamma}$	$\langle \frac{b+\gamma}{2\gamma+1}, \frac{a+\gamma}{2\gamma+1}, \text{ where } \gamma \geq 1 \rangle$
$\neg_{44,\alpha,\beta}$	$\langle \frac{b+\alpha-1}{\alpha+\beta}, \frac{a+\beta}{\alpha+\beta}, \text{ where } \alpha \geq 1, \beta \in [0, \alpha] \rangle$
$\neg_{45,\varepsilon,\eta}$	$\langle \min(1, \nu_A(x) + \varepsilon), \max(0, \mu_A(x) - \eta) \rangle$

The relations between the implications and negations are shown in Table 3.

Table 3

$\neg_1$	$\rightarrow_1, \rightarrow_4, \rightarrow_5, \rightarrow_6, \rightarrow_7, \rightarrow_{10}, \rightarrow_{13}, \rightarrow_{61}, \rightarrow_{63}, \rightarrow_{64}, \rightarrow_{66}, \rightarrow_{67}, \rightarrow_{68}, \rightarrow_{69}, \rightarrow_{70}, \rightarrow_{71}, \rightarrow_{72}, \rightarrow_{73}, \rightarrow_{78}, \rightarrow_{80}, \rightarrow_{124}, \rightarrow_{125}, \rightarrow_{127}$
$\neg_2$	$\rightarrow_2, \rightarrow_3, \rightarrow_8, \rightarrow_{11}, \rightarrow_{16}, \rightarrow_{20}, \rightarrow_{31}, \rightarrow_{32}, \rightarrow_{37}, \rightarrow_{40}, \rightarrow_{41}, \rightarrow_{42}$
$\neg_3$	$\rightarrow_9, \rightarrow_{17}, \rightarrow_{21}$
$\neg_4$	$\rightarrow_{12}, \rightarrow_{18}, \rightarrow_{22}, \rightarrow_{46}, \rightarrow_{49}, \rightarrow_{50}, \rightarrow_{51}, \rightarrow_{53}, \rightarrow_{54}, \rightarrow_{91}, \rightarrow_{93}, \rightarrow_{94}, \rightarrow_{95}, \rightarrow_{96}, \rightarrow_{98}, \rightarrow_{134}, \rightarrow_{135}, \rightarrow_{137}$
$\neg_5$	$\rightarrow_{14}, \rightarrow_{15}, \rightarrow_{19}, \rightarrow_{23}, \rightarrow_{47}, \rightarrow_{48}, \rightarrow_{52}, \rightarrow_{55}, \rightarrow_{56}, \rightarrow_{57}$
$\neg_6$	$\rightarrow_{24}, \rightarrow_{26}, \rightarrow_{27}, \rightarrow_{65}$
$\neg_7$	$\rightarrow_{25}, \rightarrow_{28}, \rightarrow_{29}, \rightarrow_{62}$
$\neg_8$	$\rightarrow_{30}, \rightarrow_{33}, \rightarrow_{34}, \rightarrow_{35}, \rightarrow_{36}, \rightarrow_{38}, \rightarrow_{39}, \rightarrow_{76}, \rightarrow_{82}, \rightarrow_{84}, \rightarrow_{85}, \rightarrow_{86}, \rightarrow_{87}, \rightarrow_{89}, \rightarrow_{129}, \rightarrow_{130}, \rightarrow_{132}$
$\neg_9$	$\rightarrow_{43}, \rightarrow_{44}, \rightarrow_{45}, \rightarrow_{83}$
$\neg_{10}$	$\rightarrow_{58}, \rightarrow_{59}, \rightarrow_{60}, \rightarrow_{92}$
$\neg_{11}$	$\rightarrow_{74}, \rightarrow_{97}$
$\neg_{12}$	$\rightarrow_{75}$
$\neg_{13}$	$\rightarrow_{77}, \rightarrow_{88}$
$\neg_{14}$	$\rightarrow_{79}$
$\neg_{15}$	$\rightarrow_{81}$
$\neg_{16}$	$\rightarrow_{90}$
$\neg_{17}$	$\rightarrow_{99}$
$\neg_{18}$	$\rightarrow_{100}$
$\neg_{19}$	$\rightarrow_{101}$
$\neg_{20}$	$\rightarrow_{102}, \rightarrow_{108}$
$\neg_{21}$	$\rightarrow_{103}$
$\neg_{22}$	$\rightarrow_{104}$
$\neg_{23}$	$\rightarrow_{105}$
$\neg_{24}$	$\rightarrow_{106}$
$\neg_{25}$	$\rightarrow_{107}$
$\neg_{26}$	$\rightarrow_{109}, \rightarrow_{110}, \rightarrow_{111}, \rightarrow_{112}, \rightarrow_{113}$
$\neg_{27}$	$\rightarrow_{114}, \rightarrow_{115}, \rightarrow_{116}, \rightarrow_{117}, \rightarrow_{118}$
$\neg_{28}$	$\rightarrow_{119}, \rightarrow_{120}, \rightarrow_{121}, \rightarrow_{122}, \rightarrow_{123}$
$\neg_{29}$	$\rightarrow_{126}$
$\neg_{30}$	$\rightarrow_{128}$
$\neg_{31}$	$\rightarrow_{131}$
$\neg_{32}$	$\rightarrow_{133}$
$\neg_{33}$	$\rightarrow_{136}$
$\neg_{34}$	$\rightarrow_{138}$
$\neg_{35}$	$\rightarrow_{139}$
$\neg_{36}$	$\rightarrow_{140}$

$\neg_{37}$	$\rightarrow_{141}$
$\neg_{38}$	$\rightarrow_{142}, \rightarrow_{143}$
$\neg_{39}$	$\rightarrow_{144}, \rightarrow_{145}$
$\neg_{40}$	$\rightarrow_{146}, \rightarrow_{147}$
$\neg_{41}$	$\rightarrow_{148}, \rightarrow_{149}$
$\neg_{42}$	$\rightarrow_{149}$
$\neg_{43}$	$\rightarrow_{150}$
$\neg_{44}$	$\rightarrow_{151}$
$\neg_{45}$	$\rightarrow_{152}$

## 5 Operators over IFPs

There are three types of operators over IFPs. The first of them is of modal type.

Let as above,  $x = \langle a, b \rangle$  be an IFP and let  $\alpha, \beta \in [0, 1]$ . Then the modal type of operators defined over  $x$  have the forms:

$$\begin{aligned}
\Box x &= \langle a, 1 - a \rangle \\
\Diamond x &= \langle 1 - b, b \rangle \\
D_\alpha(x) &= \langle a + \alpha.(1 - a - b), b + (1 - \alpha).(1 - a - b) \rangle \\
F_{\alpha,1-\alpha}(x) &= \langle a + \alpha.(1 - a - b), b + \beta.(1 - a - b) \rangle, \text{ where } \alpha + \beta \leq 1 \\
G_{\alpha,\beta}(x) &= \langle \alpha.a, \beta.b \rangle \\
H_{\alpha,\beta}(x) &= \langle \alpha.a, b + \beta.(1 - a - b) \rangle \\
H_{\alpha,\beta}^*(x) &= \langle \alpha.a, b + \beta.(1 - \alpha.a - b) \rangle \\
J_{\alpha,\beta}(x) &= \langle a + \alpha.(1 - a - b), \beta.b \rangle \\
J_{\alpha,\beta}^*(x) &= \langle a + \alpha.(1 - a - \beta.b), \beta.b \rangle \\
X_{\alpha,\beta,\gamma,\delta,\varepsilon,\eta}(x) &= \langle \alpha.a + \beta.(1 - a - \gamma.b), \delta.b + \varepsilon.(1 - \eta.a - b) \rangle \\
d_\alpha(x) &= \langle b + \alpha.(1 - a - b), a + (1 - \alpha).(1 - a - b) \rangle \\
f_{\alpha,1-\alpha}(x) &= \langle b + \alpha.(1 - a - b), a + \beta.(1 - a - b) \rangle, \text{ where } \alpha + \beta \leq 1 \\
g_{\alpha,\beta}(x) &= \langle \alpha.b, \beta.a \rangle \\
h_{\alpha,\beta}(x) &= \langle \alpha.b, a + \beta.(1 - a - b) \rangle \\
h_{\alpha,\beta}^*(x) &= \langle \alpha.b, a + \beta.(1 - \alpha.b - a) \rangle \\
j_{\alpha,\beta}(x) &= \langle b + \alpha.(1 - a - b), \beta.a \rangle \\
j_{\alpha,\beta}^*(x) &= \langle a + \alpha.(1 - \beta.a - b), \beta.a \rangle \\
x_{\alpha,\beta,\gamma,\delta,\varepsilon,\eta}(x) &= \langle \alpha.b + \beta.(1 - b - \gamma.a), \delta.a + \varepsilon.(1 - \eta.b - a) \rangle
\end{aligned}$$

The second type of operators is from another (similar to modal) type. Let  $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta \in [0, 1]$ .

$$\begin{aligned}
\boxplus x &= \langle \frac{a}{2}, \frac{b+1}{2} \rangle \\
\boxtimes x &= \langle \frac{a+1}{2}, \frac{b}{2} \rangle \\
\boxplus_\alpha x &= \langle \alpha.a, \alpha.b + 1 - \alpha \rangle \\
\boxtimes_\alpha x &= \langle \alpha.a + 1 - \alpha, \alpha.b \rangle \\
\boxplus_{\alpha,\beta} x &= \langle \alpha.a, \alpha.b + \beta \rangle, \text{ where } \alpha + \beta \leq 1 \\
\boxtimes_{\alpha,\beta} x &= \langle \alpha.a + \beta, \alpha.b \rangle, \text{ where } \alpha + \beta \leq 1
\end{aligned}$$

$$\begin{aligned}
\boxplus_{\alpha,\beta,\gamma}x &= \langle \alpha.a, \beta.b + \gamma \rangle, \text{ where } \max(\alpha, \beta) + \gamma \leq 1 \\
\boxtimes_{\alpha,\beta,\gamma}x &= \langle \alpha.a + \gamma, \beta.b \rangle, \text{ where } \max(\alpha, \beta) + \gamma \leq 1 \\
\bullet_{\alpha,\beta,\gamma,\delta}x &= \langle \alpha.a + \gamma, \beta.b + \delta \rangle, \text{ where } \max(\alpha, \beta) + \gamma + \delta \leq 1 \\
E_{\alpha,\beta}x &= \langle \beta(\alpha.a + 1 - \alpha), \alpha(\beta.b + 1 - \beta) \rangle (\text{ see [6]}) \\
\boxodot_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}x &= \langle \alpha.a - \varepsilon.b + \gamma, \beta.b - \zeta.a + \delta \rangle, \text{ where } \max(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \leq 1 \\
&\quad \text{and } \min(\alpha - \zeta, \beta - \varepsilon) + \gamma + \delta \geq 0
\end{aligned}$$

The third type of operators is from level type. They are two operators of this type, namely:

$$\begin{aligned}
P_{\alpha,\beta}x &= \langle \max(\alpha, a), \min(\beta, b) \rangle \\
Q_{\alpha,\beta}x &= \langle \min(\alpha, a), \max(\beta, b) \rangle,
\end{aligned}$$

for  $\alpha, \beta \in [0, 1]$  and  $\alpha + \beta \leq 1$ .

## 6 IF values

The authors' opinion is that concept of IF value must be used for results of estimation of IF-objects. For example, when we have a variable  $R$  and estimation function  $V$ , the result of applying of  $V$  over  $P$  will be the IF-value (or IF-estimation)  $\langle \mu(P), \nu(P) \rangle$ , where  $\mu(P)$  and  $\nu(P)$  are the degrees of validity and of non-validity of  $P$ .

## 7 Conclusion

In the present paper, we transform the definitions of the basic relations, operations and operators to the concept of IFP. In future, we will give definitions new operations and operators over IFPs.

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