Generalized Net Models
of Crossover Operators in Genetic Algorithms

T. Pencheva¹, O. Roeva¹ and A. Shannon²

¹Centre of Biomedical Engineering – Bulgarian Academy of Sciences
105 Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
E-mail: tania.pencheva@clbme.bas.bg, olympia@clbme.bas.bg

²Raffles College of Design and Commerce, North Sydney, 2060, Australia
Warrane College, University of New South Wales, Kensington, 1465, Australia
E-mails: tonySHANNON@raffles.edu.au, tony@warrane.unsw.edu.au

Abstract: The apparatus of Generalized nets (GN) is here applied to a description of different techniques of crossover, which is one of the basic genetic algorithm operators. Presented here are GN models which describe three crossover techniques, namely one-point, two-point crossover as well as the “cut and splice” technique. The resulting GN models can be considered as separate modules, but they can also be accumulated into a GN model to describe a whole genetic algorithm.

Keywords: Generalized net, Genetic algorithms, Crossover.

Introduction

Genetic algorithm (GA) is a concept that promises a lot. The GA is a parallel, global search technique that emulates natural genetic operators [8, 9]. Because it simultaneously evaluates many points in the parameter space, it is more likely to converge towards the global solution. A GA requires only information concerning the quality of the solution produced by each parameter set. Since GAs do not require any problem-specific information, they are more flexible and adaptable than most search methods. Some short remarks about GAs will be presented below.

The GA maintains a population of chromosomes (solutions) with associated fitness values. Parents are selected to mate, on the basis of their fitness, producing offspring via a reproductive plan. Consequently highly fit solutions are given more opportunities to reproduce, so that offspring inherit characteristics from each parent. After an initial population is randomly generated, the algorithm evolves through basic operators selection (equates to survival of the fittest), crossover (represents mating between individuals) and mutation (introduces random modifications).

The key idea of the selection operator is to give a preference to better individuals, allowing them to pass on their genes to the next generation. The goodness of each individual depends on its fitness, which may be determined by an objective function or by a subjective judgment.
**Crossover operator** is a prime factor of GA that distinguishes them from other optimization techniques. Two individuals are chosen from the population using the selection operator. A crossover site along the bit strings is randomly chosen and the values of the two strings are exchanged up to this point. The two new offspring created from this mating are put into the next generation of the population. By recombining portions of good individuals, this process is likely to create even better individuals.

Based on the **mutation operator**, with some low probability, a portion of new individuals will have some of their bits flipped. Its purpose is to maintain diversity within the population and inhibit premature convergence. Mutation alone induces a random walk through the search space.

The effects of the basic genetic operators can be summarized as follows:
- using selection alone will tend to fill the population with copies of the best individual from the population;
- using selection and crossover operators will tend to cause the algorithms to converge on a good but sub-optimal solution;
- using mutation alone induces a random walk through the search space;
- using selection and mutation creates a parallel, noise-tolerant, hill-climbing algorithm.

Based on the operators’ key features and advantages, recently GAs have established themselves as an important component from the instrumentation of artificial intelligence. GA are quite popular and are applied in many domains such as industrial design, scheduling, network design, routing, time series prediction, database mining, control systems, artificial life systems, as well as in many fields of the science \[4, 7\]. From the other hand, up to now the apparatus of Generalized nets (GN) has been used as a tool for parallel processes description in several areas – economics, transport, medicine, computer technologies etc \[2\]. That is why the idea of using GN for description of GA has been intuitively appeared. Up to the moment a few GN models regarding GA performance are developed \[1, 3, 10, 11\]. In \[1, 3\] a GN model for GA learning is proposed. The GN model in \[11\] describes GA search procedure. The model simultaneously evaluates several fitness functions, ranks the individuals according to their fitness and has the opportunity to choice the best fitness function regarding to specific problem domain. The GN model in \[10\] describes the selection of GA operators. The model has the possibility to test different groups of the defined GA operators and to choose the most appropriate combination between them. The developed GN execute GA and realizes tuning of genetic operators, as well as of the fitness function, regarding to the considered problem.

In this paper the apparatus of GN is applied for a description of different techniques of one of the basic genetic algorithm operators, namely crossover. GN models for description of three crossover techniques are here developed.

**Crossover techniques**

In GA, crossover is a genetic operator used to vary the programming of a chromosome (or chromosomes) from one generation to the next. It is analogous to reproduction and biological crossover, upon which GA are based. Many crossover techniques exist for organisms which
use different data structures to store themselves. Here the focus is pointed on one-point, two-point crossover as well as “cut and splice” technique.

One-point crossover
A single crossover point on both parents’ organism strings is selected. All data beyond that point in either organism string is swapped between the two parent organisms. The resulting organisms are the children.

Two-point crossover
Two-point crossover calls for two points to be selected on the parent organism strings. Everything between the two points is swapped between the parent organisms, rendering two child organisms.

“Cut and splice” technique
Another crossover variant, the “cut and splice” technique, results in a change in length of the children strings. The reason for this difference is that each parent string has a separate choice of crossover point.

GN models of crossover techniques

GN model of one-point crossover
The generalized net model described one-point crossover is presented in Fig. 1.

![Fig. 1. GN model of one-point crossover](image)

The token $\alpha$ enters GN in place $l_1$ with an initial characteristic “parameters of GA”. Some of the most common considered parameters of GA are: number of individuals (NIND), maximal number of generations (MAXGEN), number of variables (NVAR), crossover probability (XOVR), mutation probability (MUTR), generation gap (how many new individuals are created, GGAP), type of selection, crossover and mutation function for individuals etc. The token $\beta$ enters GN in place $l_2$ with an initial characteristic “pool of possible parents”. Tokens $\alpha$ and $\beta$ are combined and appear as a token $\gamma$ in place $l_3$ with a characteristic “parent 1” and as a token $\delta$ in place $l_4$ with a characteristic “parent 2”. The form of the first transition of the GN model is as follows:
Each of tokens $\gamma$ and $\delta$ is split into two new tokens, respectively $\gamma_1$ and $\gamma_2$, $\delta_1$ and $\delta_2$, with characteristics as follows:

- $\gamma_1$ in place $l_5$ - “first string of parent 1”;
- $\gamma_2$ in place $l_6$ - “second string of parent 1”;
- $\delta_1$ in place $l_7$ - “first string of parent 2”;
- $\delta_2$ in place $l_8$ - “second string of parent 2”.

The form of the second transition of the GN model is as follows:

$$Z_2 = \langle \{l_3, l_4, l_5, l_6, l_7, l_8\}, r_2, \land (l_3, l_4) >$$

where $W_1$ is an operation “one-point crossover”.

Tokens $\gamma_1$ and $\delta_2$ are further combined in a token $\varepsilon$ in place $l_9$ with a characteristic “offspring 1”. By analogy, tokens $\delta_1$ and $\gamma_2$ are further combined in a token $\sigma$ in place $l_{10}$ with a characteristic “offspring 2”. The form of the third transition of the GN model is as follows:

$$Z_3 = \langle \{l_5, l_6, l_7, l_8\}, \{l_9, l_{10}\}, r_3, \land (l_5, l_6, l_7, l_8) >$$

Tokens $\varepsilon$ and $\sigma$ are then combined in a token $\eta$ in place $l_{11}$ with a characteristic “new population”. The form of the fourth transition of the GN model is as follows:

$$Z_4 = \langle \{l_9, l_{10}\}, \{l_{11}\}, l_{11}, \land (l_9, l_{10}) >$$
In place $l_{11}$ the new population is obtained after the one-point crossover operation. Presented in such way separate module, realized the population crossover, could be included in a GN model for description of GA performance. Then, the next transitions will realize the mutation operator for the population from place $l_{11}$ and so on, following the steps of genetic algorithm.

*GN model of two-point crossover*

The generalized net model described two-point crossover is presented in Fig. 2.

---

**Fig. 2 GN model of two-point crossover**

The token $\alpha$ enters GN in place $l_1$ with an initial characteristic “parameters of GA” and the token $\beta$ enters GN in place $l_2$ with an initial characteristic “pool of possible parents”, as described in one-point crossover. Tokens $\alpha$ and $\beta$ are combined and appear as a token $\gamma$ in place $l_3$ with a characteristic “parent 1” and as a token $\delta$ in place $l_4$ with a characteristic “parent 2”. The form of the first transition of the GN model is as follows:

$$
Z_1 = \langle \{l_1, l_2\}, \{l_3, l_4\}, \quad \begin{array}{c|c}
  l_1 & l_4 \\
  \text{true} & \text{true} \\
\end{array} , \land(l_1, l_2) > \\
\begin{array}{c|c}
  l_2 & \text{true} \\
  \text{true} & \text{true} \\
\end{array}
$$

In the case of two-point crossover, each of tokens $\gamma$ and $\delta$ is split into three new tokens, respectively $\gamma_1$, $\gamma_2$ and $\gamma_3$, $\delta_1$, $\delta_2$ and $\delta_3$, with characteristics as follows:

- $\gamma_1$ in place $l_5$ - “first string of parent 1”;
- $\gamma_2$ in place $l_6$ - “second string of parent 1”;
- $\gamma_3$ in place $l_7$ - “third string of parent 1”;
- $\delta_1$ in place $l_8$ - “first string of parent 2”;
- $\delta_2$ in place $l_9$ - “second string of parent 2”;
- $\delta_3$ in place $l_{10}$ - “third string of parent 2”.

The form of the second transition of the GN model is as follows:
where \( W_1 \) is an operation “two-point crossover”.

Tokens \( \gamma_1, \delta_2 \) and \( \gamma_3 \) are further combined in a token \( \varepsilon \) in place \( l_{11} \) with a characteristic “offspring 1”. By analogy, tokens \( \delta_1, \gamma_2 \) and \( \delta_3 \) are further combined in a token \( \sigma \) in place \( l_{12} \) with a characteristic “offspring 2”. The form of the third transition of the GN model is as follows:

\[
Z_3 = <\{l_5, l_6, l_7, l_8, l_9, l_{10}\}, \{l_{11}, l_{12}\}, \land(l_5, l_6, l_7, l_8, l_9, l_{10}) >
\]

\[
r_3 = \begin{array}{cccccc}
l_5 & l_6 & l_7 & l_8 & l_9 & l_{10} \\
\varepsilon & \text{false} & \text{false} & \text{false} & W_1 & W_1 & W_1
\end{array}
\]

Tokens \( \varepsilon \) and \( \sigma \) are then combined in a token \( \eta \) in place \( l_{13} \) with a characteristic “new population”. The form of the fourth transition of the GN model is as follows:

\[
Z_4 = <\{l_{11}, l_{12}\}, \{l_{13}\}, \land(l_{11}, l_{12}) >
\]

\[
r_4 = \begin{array}{cc}
l_{11} & l_{12} \\
\varepsilon & \text{false}
\end{array}
\]

By analogy with GN model described one-point crossover, in place \( l_{13} \) the new population is obtained after the two-point crossover operation.

**GN model of “cut and splice” technique**

The GN model described “cut and splice” technique will not be explicitly presented here due to the fact that it will be equal to the GN model described one-point crossover. The tokens, characteristics, as well as the transitions will be absolutely the same. The main difference between these crossover techniques is the length of the obtained offspring individuals, because of the different crossover point in both parents. While in the one-point crossover the obtained offspring individuals are with a length equal to their parents, in the case of “cut and
splice” technique the obtained offspring individuals are with different length. However, this fact will not reflect to the structure of GN model, described these techniques.

Analysis and conclusions

Generalized nets models are here developed and applied to a description of different techniques for one of the basic genetic algorithm operators, namely crossover. Presented here GN models describe three techniques - one-point, two-point crossover as well as “cut and splice” technique. Due to the identical logic of the different techniques of crossover operators, the GN models of one- and two-point crossovers are quite similar - they differ just in the number of strings received from parents after the choice of one- or two-point crossover. The GN models to describe the “cut and splice” technique is equivalent to the GN model described for the one-point crossover. The main difference among these crossover techniques is the length of the obtained offspring individuals. However, this fact is not reflected in the structure of the GN model which describes these techniques. The resulting GN models of the three crossover techniques can be considered as separate modules, but they can also be aggregated into one GN model for a description of the whole genetic algorithm.

References