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n-EXTRACTION OPERATION OVER INTUITIONISTIC FUZZY SETS

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Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS) A in E is defined by (see, e.g., [1]):

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Let us define the *empty IFS,* the *totally uncertain IFS,* and the *unit IFS* (see [1]) by:

$$O^* = \{\langle x, 0, 1\rangle | x \in E\},$$

$$U^* = \{\langle x, 0, 0\rangle | x \in E\},$$

$$E^* = \{\langle x, 1, 0\rangle | x \in E\}.$$

Different relations and operations are introduced over the IFSs. Some of them are the following

$$\begin{array}{lcl}
A \subset B & \text{iff} & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)), \\
A = B & \text{iff} & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)), \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}, \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\
A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}, \\
A + B & = & \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle | x \in E\}, \\
A.B & = & \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle | x \in E\}.
\end{array}$$

In [2] Supriya Kumar De, Ranjit Biswas and Akhil Ranjan Roy introduced two operations which are related to the last two above ones:

$$\begin{array}{lcl} n.A & = & \{\langle x, 1-(1-\mu_A(x))^n, (\nu_A(x))^n\rangle \mid x \in E\}, \\ A^n & = & \{\langle x, (\mu_A(x))^n, 1-(1-\nu_A(x))^n\rangle \mid x \in E\}, \end{array}$$

where n is a natural number.

In this short remark we introduce a new operator, defined over IFSs. It is an analogous of operations "extraction" and has the form for every IFS A and for every natural number $n \geq 1$:

$$\sqrt[n]{A} = \{\langle \sqrt[n]{\mu_A(x)}, 1 - \sqrt[n]{1-\nu_A(x)}, \rangle | x \in E\}.$$

First, we must check that in a result of the operation we obtain an IFS. Really, for given IFS A, for each $x \in E$, and for each $n \geq 1$:

$$\sqrt[n]{\mu_A(x)} + 1 - \sqrt[n]{1-\nu_A(x)} \leq 1,$$

because from $\mu_A(x) \leq 1 - \nu_A(x)$ it follows that

$$\sqrt[n]{\mu_A(x)} \leq \sqrt[n]{1-\nu_A(x)}.$$

Obviously, for every natural number $n \geq 1$:

$$\sqrt[n]{O^*} = O^*,$$

$$\sqrt[n]{U^*} = U^*,$$

$$\sqrt[n]{E^*} = E^*.$$

By similar to the above way we can prove the following assertions.

Theorem 1: For every IFS A and for every natural number $n \geq 1$:

(a) $\sqrt[n]{A^n} = A$,

(b) $(\sqrt[n]{A})^n = A$.

Theorem 2: For every IFS A and for every two natural numbers $m, n \geq 1$:

$$\sqrt[m]{\sqrt[n]{A}} = \sqrt[mn]{A} = \sqrt[n]{\sqrt[m]{A}}.$$

Theorem 3: For every two IFSs A and B and for every natural number $n \geq 1$:

(a) $\sqrt[n]{A \cap B} = \sqrt[n]{A} \cap \sqrt[n]{B}$,

(b) $\sqrt[n]{A \cup B} = \sqrt[n]{A} \cup \sqrt[n]{B}$.

Proof: We shall prove (a) and (b) is proved analogically.

$$\sqrt[n]{A \cap B} = \sqrt[n]{\{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle | x \in E\}}$$

$$= \{\langle x, \sqrt[n]{\min(\mu_A(x), \mu_B(x))}, 1 - \sqrt[n]{1 - \max(\nu_A(x), \nu_B(x))}\rangle | x \in E\}$$

$$= \{\langle x, \min(\sqrt[n]{\mu_A(x)}, \sqrt[n]{\mu_B(x)}), 1 - \sqrt[n]{\min(1-\nu_A(x), 1-\nu_B(x))}\rangle | x \in E\}.$$

Let for arbitrary $x \in E : \nu_A(x) \leq \nu_B(x)$. Then

$$\sqrt[n]{\min(1-\nu_A(x), 1-\nu_B(x))} = \sqrt[n]{1-\nu_B(x)} = \min(\sqrt[n]{1-\nu_A(x)}, \sqrt[n]{1-\nu_B(x)}).$$

The same equality will be valid for $\nu_A(x) \geq \nu_B(x)$. Therefore

$$\sqrt[n]{A \cap B} = \{\langle x, \min(\sqrt[n]{\mu_A(x)}, \sqrt[n]{\mu_B(x)}), 1 - \min(\sqrt[n]{1-\nu_A(x)}, \sqrt[n]{1-\nu_B(x)})\rangle | x \in E\}$$

$$= \{\langle x, \min(\sqrt[n]{\mu_A(x)}, \sqrt[n]{\mu_B(x)}), \max(1 - \sqrt[n]{1-\nu_A(x)}, 1 - \sqrt[n]{1-\nu_B(x)})\rangle | x \in E\}$$

$$= \{\langle x, \sqrt[n]{\mu_A(x)}, 1 - \sqrt[n]{1-\nu_A(x)}\rangle | x \in E\} \cap \{\langle x, \sqrt[n]{\mu_B(x)}, 1 - \sqrt[n]{1-\nu_B(x)}\rangle | x \in E\}$$

$$= \sqrt[n]{A} \cap \sqrt[n]{B}.$$

Theorem 4: For every two IFSs A and B and for every natural number $n \geq 1$:

(a) $\sqrt[n]{A+B} \supset \sqrt[n]{A} + \sqrt[n]{B}$,

(b) $\sqrt[n]{A.B} \subset \sqrt[n]{A}.\sqrt[n]{B}$.

The simplest modal operators defined over IFSs (see, e.g., [1]) are:

$$\Box A = \{\langle x, \mu_A(x), 1-\mu_A(x)\rangle | x \in E\};$$
$$\Diamond A = \{\langle x, 1-\nu_A(x), \nu_A(x)\rangle | x \in E\}.$$

They are analogous of the modal logic operators *"necessity"* and *"possibility"*. For them it is valid

Theorem 5: For every IFS A and for every natural number $n \geq 1$:

(a) $\Box \sqrt[n]{A} = \sqrt[n]{\Box A}$,

(b) $\Diamond \sqrt[n]{A} = \sqrt[n]{\Diamond A}$.

In IFSs theory some level operators are defined. Two of them are:

$$P_{\alpha,\beta}(A) = \{\langle x, \max(\alpha, \mu_A(x)), \min(\beta, \nu_A(x))\rangle | x \in E\},$$

$$Q_{\alpha,\beta}(A) = \{\langle x, \min(\alpha, \mu_A(x)), \max(\beta, \nu_A(x))\rangle | x \in E\},$$

where $\alpha + \beta \leq 1$. For them it is valid

Theorem 6: For every IFS A, for every natural number $n \geq 1$ and for every $\alpha, \beta \in [0,1]$, so that $\alpha + \beta \leq 1$:

(a) $P_{\alpha,\beta}(\sqrt[n]{A}) = \sqrt[n]{P_{\alpha^n, 1-(1-\beta)^n} A}$,

(b) $Q_{\alpha,\beta}(\sqrt[n]{A}) = \sqrt[n]{Q_{\alpha^n, 1-(1-\beta)^n} A}$.

References

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