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Four equalities connected with intuitionistic fuzzy sets

Tsvetan V. Vasilev

Faculty of Mathematics and Computer Science, Sofia University,
5 James Baucher Blvd., 1164 Sofia, Bulgaria,
tsvetanv@fmi.uni-sofia.bg

Abstract: We prove 4 new equalities connected with intuitionistic fuzzy sets, which reveal new contacts between the basic operations: $+$ $\boldsymbol{.}$ $@$. \$ $\cup$ $\cap$.
Key words: intuitionistic fuzzy sets, equality

Here we shall prove the following
Theorem: For every two IFSs A, B the following equalities hold:

$$((A + B) \cap (A \,.\, B)) \;@\; ((A + B) \cup (A \,.\, B)) = A@B \qquad (1)$$

$$((A \,\$\, B) \,.\, (A \,\$\, B)) \;@\; ((A \,\$\, B) + (A \,\$\, B)) = A \,\$\, B \qquad (2)$$

$$\overline{(A + B)} \;@\; (A \,.\, B) = \overline{(\,\overline{A \,.\, B}\,) \;@\; (A + B)} \qquad (3)$$

$$\left[\, \overline{(A \,.\, B)} \;@\; (A \,.\, B) \,\right] @ \left[\, (\,\overline{A + B}\,) \;@\; (A + B) \,\right] = \overline{(A \,@\, B)} \;@\; (A \,@\, B) \qquad (4)$$

Proof: First, we have to note the following elementary statement, which is necessary for the proof: for every two real numbers a and b, it follows that

$$\max(a,b) + \min(a,b) = a + b \qquad (*)$$

1) Proof of equality (1)

This equality is very similar to the following equality, which is proved in [1]:

$$((A \cap B) + (A \cup B)) \;@\; ((A \cap B) \,.\, (A \cup B)) = A@B$$

Let us prove equality (1):

$$((A + B) \cap (A \,.\, B)) \;@\; ((A + B) \cup (A \,.\, B))$$

$$= (\{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x),\ \nu_A(x).\nu_B(x)\rangle \mid x \in E\}$$
$$\cap \{\langle x, \mu_A(x).\mu_B(x),\ \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle \mid x \in E\})$$
$$@ (\{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x),\ \nu_A(x).\nu_B(x)\rangle \mid x \in E\}$$
$$\cup \{\langle x, \mu_A(x).\mu_B(x),\ \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle \mid x \in E\})$$

$$= (\{\langle x, \min(\mu_A(x).\mu_B(x),\ \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)),$$
$$\max(\nu_A(x).\nu_B(x),\ \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x))\rangle \mid x \in E\})$$
$$@ (\{\langle x, \max(\mu_A(x).\mu_B(x),\ \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)),$$
$$\min(\nu_A(x).\nu_B(x),\ \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x))\rangle \mid x \in E\})$$

$$\overset{\text{from } (*)}{=} \left\{ \left\langle x, \frac{(\mu_A(x).\mu_B(x)) + (\mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x))}{2}, \frac{(\nu_A(x).\nu_B(x)) + (\nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x))}{2} \right\rangle \mid x \in E \right\}$$

$$= \left\{ \left\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2} \right\rangle \mid x \in E \right\}$$

$= A @ B$

2) Proof of equality (2)

$((A \$ B) . (A \$ B)) @ ((A \$ B) + (A \$ B))$

$$= \left(\left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\} . \left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\} \right)$$

$$@ \left(\left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\} + \left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\} \right)$$

$$= \left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)} . \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)} + \sqrt{\nu_A(x).\nu_B(x)} - \sqrt{\nu_A(x).\nu_B(x)}.\sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\}$$

$$@ \left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)} + \sqrt{\mu_A(x).\mu_B(x)} - \sqrt{\mu_A(x).\mu_B(x)} . \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)}.\sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\}$$

$$= \left\{ \left\langle x, \mu_A(x).\mu_B(x), 2.\sqrt{\nu_A(x).\nu_B(x)} - \nu_A(x).\nu_B(x) \right\rangle \mid x \in E \right\}$$

$$@ \left\{ \left\langle x, 2.\sqrt{\mu_A(x).\mu_B(x)} - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x) \right\rangle \mid x \in E \right\}$$

$$= \left\{ \left\langle x, \frac{(\mu_A(x).\mu_B(x)) + (2.\sqrt{\mu_A(x).\mu_B(x)} - \mu_A(x).\mu_B(x))}{2}, \frac{(\nu_A(x).\nu_B(x)) + (2.\sqrt{\nu_A(x).\nu_B(x)} - \nu_A(x).\nu_B(x))}{2} \right\rangle \mid x \in E \right\}$$

$$= \left\{ \left\langle x, \sqrt{\mu_A(x).\mu_B(x)}, \sqrt{\nu_A(x).\nu_B(x)} \right\rangle \mid x \in E \right\}$$

$= A \$ B$

3) Proof of equality (3)

Let us work out the left part of equality (3):

$$(\overline{A + B}) \,@\, (A \,.\, B)$$

$$= \{ \langle x,\ \nu_A(x).\nu_B(x)\ ,\ \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x) \rangle \mid x \in E \}$$

$$@\ \{ \langle x,\ \mu_A(x).\mu_B(x)\ ,\ \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x) \rangle \mid x \in E \}$$

$$= \{ \langle\ x,\ \frac{\mu_A(x).\mu_B(x) + \nu_A(x).\nu_B(x)}{2},$$

$$\frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)-\mu_A(x).\mu_B(x)-\nu_A(x).\nu_B(x)}{2} \rangle \mid x \in E \}$$

Let us work out the right part of equality (3):

$$\overline{(\ \overline{A \,.\, B}\ ) \,@\, (A + B)}$$

$$= \overline{\{ \langle x,\ \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x),\ \mu_A(x).\mu_B(x) \rangle \mid x \in E \}}$$

$$\overline{@\ \{ \langle x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x),\ \nu_A(x).\nu_B(x) \rangle \mid x \in E \}}$$

$$= \overline{\{\langle\ x,\ \frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)-\mu_A(x).\mu_B(x)-\nu_A(x).\nu_B(x)}{2},}$$

$$\overline{\frac{\mu_A(x).\mu_B(x)\ +\ \nu_A(x).\nu_B(x)}{2}\ \rangle \mid x \in E \}}$$

$$= \{\langle\ x,\ \frac{\mu_A(x).\mu_B(x)\ +\ \nu_A(x).\nu_B(x)}{2},$$

$$\frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)-\mu_A(x).\mu_B(x)-\nu_A(x).\nu_B(x)}{2},\ \rangle \mid x \in E \}$$

After estimation we get that left part and right part of (3) are equivalent. Hence, equality(3) is proved.

4) Proof of equality (4)

Let us work out the left part of equality (4):

$$\left[\ (\overline{A \,.\, B}) \,@\, (A \,.\, B)\ \right] @ \left[\ (\ \overline{A\ +\ B}\ ) \,@\, (A\ +\ B)\ \right]$$

$$= \Big[\ \{ \langle x,\ \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x),\ \mu_A(x).\mu_B(x) \rangle \mid x \in E \}$$

$$@\ \{ \langle x,\ \mu_A(x).\mu_B(x)\ ,\ \nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x) \rangle \mid x \in E \}\ \Big]$$

$$@\Big[\ \{ \langle x,\ \nu_A(x).\nu_B(x)\ ,\ \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x) \rangle \mid x \in E \}$$

$$@\ \{ \langle x, \mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x),\ \nu_A(x).\nu_B(x) \rangle \mid x \in E \}\ \Big]$$

$$= \left(\left\{ \left\langle x, \frac{\nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x)+\mu_A(x).\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)-\nu_A(x).\nu_B(x)+\mu_A(x).\mu_B(x)}{2} \right\rangle \mid x \in E \right\} \right)$$

$$@\left(\left\{ \left\langle x, \frac{\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x)+\nu_A(x).\nu_B(x)}{2}, \frac{\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x)+\nu_A(x).\nu_B(x)}{2} \right\rangle \mid x \in E \right\} \right)$$

$$= \left\{ \left\langle x, \frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)}{4}, \frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)}{4} \right\rangle \mid x \in E \right\}$$

Let us work out the right part of equality (4):

$$(\overline{A @ B}) @ (A @ B)$$

$$= \left(\left\{ \left\langle x, \frac{\nu_A(x)+\nu_B(x)}{2}, \frac{\mu_A(x)+\mu_B(x)}{2} \right\rangle \mid x \in E \right\} \right)$$

$$@\left(\left\{ \left\langle x, \frac{\mu_A(x)+\mu_B(x)}{2}, \frac{\nu_A(x)+\nu_B(x)}{2} \right\rangle \mid x \in E \right\} \right)$$

$$= \left\{ \left\langle x, \frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)}{4}, \frac{\mu_A(x)+\mu_B(x)+\nu_A(x)+\nu_B(x)}{4} \right\rangle \mid x \in E \right\}$$

After estimation we get that left part and right part of (4) are equivalent. Hence, equality(4) is proved. ■

Reference

[1] Atanassov, K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Heidelberg, 1999