Notes on Intuitionistic Fuzzy Sets
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InterCriteria Analysis: Ideas and problems

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Abstract: The basic concepts of the InterCriteria Analysis are given. Some possible forms of the scales for estimation of the degrees of consonance or dissonance between criteria, and their independence, are discussed. Some important open problems for the area of the InterCriteria Analysis are formulated.

Keywords: Consonance, Dissonance, Independence, InterCriteria Analysis.
AMS Classification: 03E72.

1 Introduction

The InterCriteria Analysis, which idea and first steps of research began in the end of 2013, presented in front of the 12th International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets, 2013, Warsaw, and published in [3] is finding increasing number of application areas. This has led to the necessity to specify a series of its components. So far, every researcher has independently determined the scale according to which every pair of criteria are in consonance or in dissonance, as well as the precision of digits after the decimal point of the real numbers staying for evaluations of the objects against the criteria.

In the present paper, we propose several rules for defining the ways of estimating the degrees of similarity and dissimilarity, with respect to the type of data, as well as scales for determining the presence or absence of consonance or dissonance between the pairs of criteria.

2 Short notes on ICA

Here we will briefly repeat the theoretical framework of the proposed approach, firstly proposed in [3], by slightly improving the notation from [2, 3]. The approach employs an index matrix M of m rows $\{O_1, \ldots, O_m\}$ and n columns $\{C_1, \ldots, C_n\}$, where for every p, q $(1 \le p \le m, 1 \le q \le n)$, O_p is an evaluated object, C_q is an evaluation criterion, and $e_{O_p C_q}$ is the evaluation of the p-th object against the q-th criterion, defined as a real number or another object that is comparable according to relation R with all the rest elements of the index matrix M.

$$M = \begin{array}{c|ccccccc} & C_1 & \cdots & C_k & \cdots & C_l & \cdots & C_n \\ \hline O_1 & e_{O_1,C_1} & \cdots & e_{O_1,C_k} & \cdots & e_{O_1,C_l} & \cdots & e_{O_1,C_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ O_i & e_{O_i,C_1} & \cdots & e_{O_i,C_k} & \cdots & e_{O_i,C_l} & \cdots & e_{O_i,C_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ O_j & e_{O_j,C_1} & \cdots & e_{O_j,C_k} & \cdots & e_{O_j,C_l} & \cdots & e_{O_j,C_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ O_m & e_{O_m,C_1} & \cdots & e_{O_m,C_j} & \cdots & e_{O_m,C_l} & \cdots & e_{O_m,C_n} \end{array},$$

From the above requirement for comparability follows the relation $R(e_{O_iC_k}, e_{O_jC_k})$ for each i, j, k. The relation R has dual relation $\overline{R}$, which is true in the cases when relation R is false, and vice versa.

For the needs of our decision making method, pairwise comparisons between every two different criteria are made along all evaluated objects. During the comparison, it is maintained one counter of the number of times when the relation R holds, and another counter for the dual relation.

Let $S^{\mu}_{k,l}$ be the number of cases in which the relations $R(e_{O_iC_k}, e_{O_jC_k})$ and $R(e_{O_i C_l}, e_{O_jC_l})$ are simultaneously satisfied. Let also $S^{\nu}_{k,l}$ be the number of cases in which the relations $R(e_{O_iC_k}, e_{O_jC_k})$ and its dual $\overline{R}(e_{O_iC_l}, e_{O_jC_l})$ are simultaneously satisfied. As the total number of pairwise comparisons between the object is $m(m-1)/2$, it is seen that there hold the inequalities:

$$0 \le S^{\mu}_{k,l} + S^{\nu}_{k,l} \le \frac{m(m-1)}{2}.$$

For every k, l, such that $1 \le k \le l \le m$, and for $m \ge 2$ the following two numbers are defined:

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{m(m-1)},\ \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{m(m-1)}.$$

Obviously, both $\mu_{C_k,C_l}, \nu_{C_k,C_l}$ are numbers in the [0, 1]-interval, and their sum is also a number in this interval. What is complement to their sum to 1 is the number π_{C_k,C_l}, which corresponds to the degree of uncertainty.

The pair, constructed from these two numbers, plays the role of the intuitionistic fuzzy evaluation of the relations that can be established between any two criteria C_k and C_l. In this way the index matrix M that relates evaluated objects with evaluating criteria can be transformed to another index matrix M^* that gives the relations among the criteria:

$$M^* = \begin{array}{c|ccc} & C_1 & \cdots & C_n \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \cdots & \langle \mu_{C_1,C_n}, \nu_{C_1,C_n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_n & \langle \mu_{C_n,C_1}, \nu_{C_n,C_1} \rangle & \cdots & \langle \mu_{C_n,C_n}, \nu_{C_n,C_n} \rangle \end{array}.$$

From practical considerations, it has been more flexible to work with two index matrices M^μ and M^ν, rather than with the index matrix M^* of IF pairs.

The final step of the algorithm is to determine the degrees of correlation between the criteria, depending on the user's choice of μ and ν. We call these correlations between the criteria: 'positive consonance', 'negative consonance' or 'dissonance'. Let $\alpha, \beta \in [0; 1]$ be the threshold values, against which we compare the values of μ_{C_k,C_l} and ν_{C_k,C_l}. We call that criteria C_k and C_l are in:

- (α, β)-positive consonance, if $\mu_{C_k,C_l} > \alpha$ and $\nu_{C_k,C_l} < \beta$;
- (α, β)-negative consonance, if $\mu_{C_k,C_l} < \beta$ and $\nu_{C_k,C_l} > \alpha$;
- (α, β)-dissonance, otherwise.

In a completely identical way, it is possible (though not always meaningful) to build a matrix giving the correlations between the objects. The only difference is that the input index matrix M has to be transposed, and the resultant matrix, say, M^{**} is with dimensions $m \times m$.

It is noteworthy that in previous publications on ICA some small misprints have been detected (indices, misplaced variables, etc), which are given here in correct form.

3 Rules for determining the degrees of consonance and dissonance

Having analysed various types of data, medical, chemical, economic, ecological, the authors reached the conclusion that it is necessary to specify the algorithms for determining the values of the degrees of consonance and dissonance. In the applications so far, the comparison of relations leads to incrementing the degrees of membership, non-membership or uncertainty according to the following rule (Table 1).

$>, >$	$>, =$	$>, <$	$=, >$	$=, =$	$=, <$	$<, >$	$<, =$	$<, <$
$\mu+$	$\pi+$	$\nu+$	$\pi+$	$\pi+$	$\pi+$	$\nu+$	$\pi+$	$\mu+$

Table 1

In the course of the investigation of various applications, we have discovered that the rule in Table 1 is just one of the possible interpretations of the comparisons of the relations. In this paper, we propose three new rules for determining the degrees of consonance and dissonance between pairs of criteria.

When ICA is applied to a set of objects, evaluated against a set of criteria using real numbers, then the rule of determining the degrees of consonance and dissonance has different variants, with the difference occurring only in the case of comparing the relations '= ,='. If we have no certainty that two equal values in the input matrix correspond to equal rounded real numbers, it is better to increment the degree of uncertainty. For instance, comparing the two values 2.1 and 2.1, obtained as a result of rounding to the first decimal place, the sign between them should be equality, while if they were rounded to the second decimal place, one of the values could be rounded to 2.06, while the other one could receive any of the values 2.06, 2.07, …, 2.13, 2.14. Therefore, the probability of coincidence is obviously much smaller. We will note that if two numbers are not identical, then regardless of the precision of rounding, they will remain to be different. Thus, we can define the following rule of determining the degrees of consonance and dissonance, see Table 2.

>, >	>, =	>, <	=, >	=, =	=, <	<, >	< ,=	<, <
$\mu+$	$\pi+$	$\nu+$	$\pi+$	½ $\mu+$ ½ $\nu+$	$\pi+$	$\nu+$	$\pi+$	$\mu+$

Table 2

In the case '=, =' the degree of membership is incremented in half of the cases, and in half – the degree of non-membership. The so described scale guarantees equal treatment of the positive and negative consonances, for which it is really reasonable to expect that symmetric behaviour will be exhibited.

When the equivalence of two pairs of values of objects against a pair of criteria is predetermined from the specificity of the particular problem, then, from the user's perspective, it may be suitable to use the rule given in Table 3.

>, >	>, =	>, <	=, >	=, =	=, <	<, >	< ,=	<, <
$\mu+$	$\pi+$	$\nu+$	$\pi+$	$\mu+$	$\pi+$	$\nu+$	$\pi+$	$\mu+$

Table 3

Completely different should be the rule, if we are working with integers, strings, and other finite objects. It will be appropriate to have the form from Table 4.

>, >	>, =	>, <	=, >	=, =	=, <	<, >	< ,=	<, <
$\mu+$	$\nu+$	$\nu+$	$\nu+$	$\mu+$	$\nu+$	$\nu+$	$\nu+$	$\mu+$

Table 4

In this case, if we dispose of all data in the index matrix, we will obtain a fuzzy, and not an intuitionistic fuzzy evaluation. The IF nature of the evaluation will be exhibited in the cases of lack of sufficient data.

When we have to work with data, for which we know that are either correct (+), incorrect (–) or missing (~), then it is reasonable to use the rule from Table 5.

+ , +	+ , –	+ , ~	– , +	– , –	– , ~	~ , +	~ , –	~ , ~
$\mu+$	$\nu+$	$\pi+$	$\nu+$	$\nu+$	$\pi+$	$\pi+$	$\pi+$	$\pi+$

Table 5

Such a rule would require the development of a new algorithm for evaluation of the degrees of membership and non-membership, which would displace the one described in Section 2. The rule can find real application, for instance, in studying systems of logical axioms.

4 Scales for determining of consonance or dissonance between pairs of criteria

Here a scale for determining the presence or absence of consonance or dissonance between pairs of criteria, which takes into consideration the intervals in normal distribution $N(0, 1)$ defined as a function of the standard deviation, is proposed. The probability for such a normally distributed random variable to be in the interval $[-t, t]$ is calculated with the distribution function $\Phi(t)$, known as Laplace function [6, 7].

The respective values, connected with the consonance/dissonance scale may have the following form from Figure 1, where 0% – 5% is Strong Negative Consonance (SNC), 5% – 15% is Negative Consonance (NC), 15% – 25% is Weak Negative Consonance (WNC), 25% – 33%is Weak Dissonance (WD), 33% – 43% is Dissonance (D), 43% – 57% is Strong Dissonance (SD), 57% – 67% is Dissonance (D), 67% – 75% is Weak Dissonance (WD), 75% – 85% is Weak Positive Consonance (WPC), 85% – 95% is Positive Consonance (PC), 95% – 100% is Strong Positive Consonance (SPC).

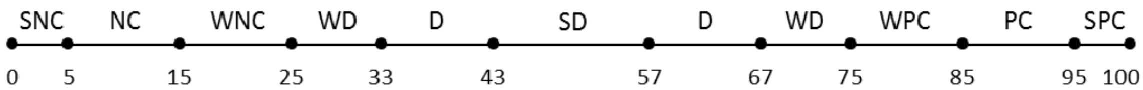

Figure 1.

In the Strong Dissonance interval half of the values will be above 50%, while the other half of them will be less than 50%. In that case, no statistical dependence between the corresponding criteria exists. This leads to a supposition of independent criteria.

Similar scale, though very rough and simplified is the one from Figure 2.

Figure 2.

Other scales with different levels of detailization are also possible and worth studying for possible applications in future. The presented scales are designed to correspond to the confidence intervals

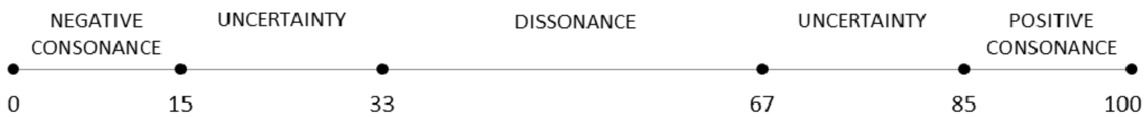

Figure 3.

For each of these three, and other, scales, as given in Figures 1–3, geometric interpretation in the IF triangle is possible. For the simplest case (Figure 2) this geometric interpretation will have the form from Figure 4.

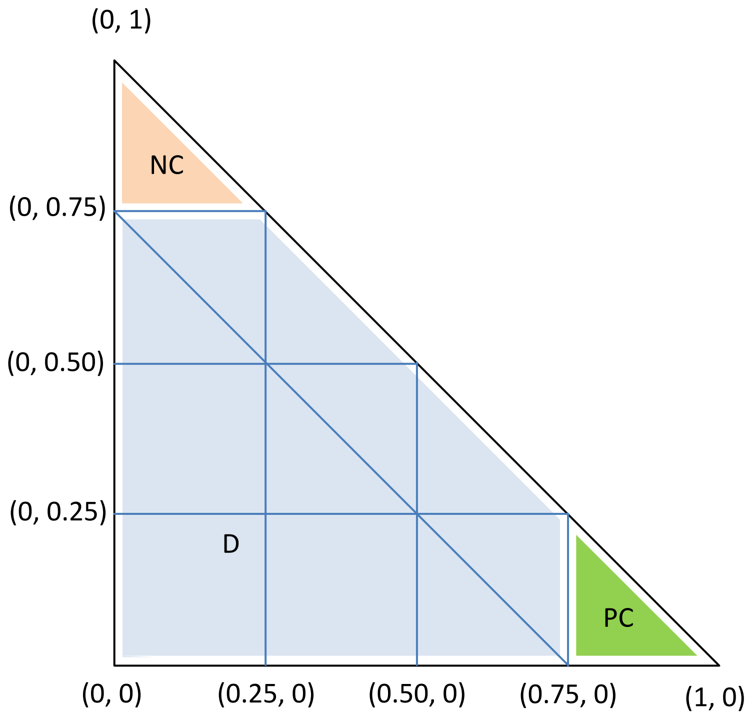

Figure 4.

The triangular zone *NC* from Figure 4 corresponds to the place where the pairs of criteria which exhibit negative consonance would be located. Formally, this area can be expressed as:

$$NC = \{\langle \mu, \nu \rangle \mid \mu \in [0, 0.25] \ \& \ \nu \in [0.75, 1] \ \& \ \mu + \nu \leq 1\}.$$

The triangular zone *PC* corresponds to the place where the pairs of criteria which exhibit positive consonance would be located. Formally, this area can be expressed as:

$$PC = \{\langle \mu, \nu \rangle \mid \mu \in [0.75. 1] \ \& \ \nu \in [0, 0.25] \ \& \ \mu + \nu \leq 1\}.$$

The pentagonal zone *D* corresponds to the place where the pairs of criteria which are in dissonance would be located. Formally, this area can be expressed as:

$$D = \{\langle \mu, \nu \rangle \mid \mu \in [0, 0.75] \ \& \ \nu \in [0, 0.75] \ \& \ \mu + \nu \leq 1\}.$$

Geometrical interpretations of the other, more sophisticated, scales, as well as other scale variants can be designed by analogy in future, if necessary.

This scale is consistent with the statistical tables (Cramer). For instance, the values 25% and 75% match the first and thirds quartiles of the distribution, while the values 67% and 95% – match the intervals for σ and 2σ, respectively. It is necessary in this relation to specify the maximal value of the degree of indeterminacy. Here we propose that it does not exceed 0.1, which is twice the admissible statistical error [6, 7].

5 Independence between criteria

The intuitionistic fuzzy interpretation of the results from application of the InterCriteria Analysis, as described in the previous Section 4, generated a series of new questions of theoretical nature. From the triangle on Figure 4, we see the possibility to add in it one more area, the triangular zone *I* of the IF triangle, see Figure 5. It is speculated that the intercriteria correlation points which fall into this area correspond to pairs of criteria, which are independent of each other. This speculation needs to be verified in the various case studies of application of the ICA approach to particular problems and areas of research.

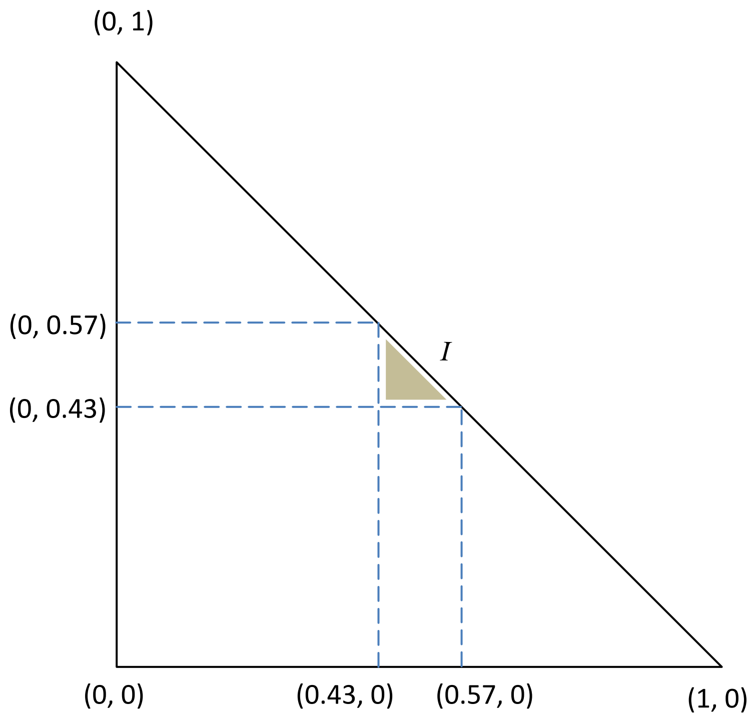

Figure 5.

The motivation behind our interest especially in the triangle *I*, is explained with the fact that in about half of the cases the pair of criteria satisfy relation one of the relations, R, and in the other half – the opposite relation, i.e. there is no demonstrated trend of satisfiability of any of the two relations. This matches the idea of strong dissonance (SD) in the most detailed scale presented above, the one in Figure 1.

Another important approach that needs to be clarified is related to the degree of uncertainty π. From the statistical point of view, it is appropriate to choose for π degrees of 0.05 or 0.10, while the triangle *I* in Figure 5, hints of the possibility to set the value of π to about 0.14. Thus the triangles NC and PC from Figure 4, hint of the possibility to set the value of π to about 0.25.

6 Open problems

The application of ICA gives rise to a number of important problems that need to be approached in near future. Among these are the following.

1. To develop algorithms, realizing the above discussed rules for defining the ways of estimating the degrees of similarity and dissimilarity.
2. To clarify the applicability of ICA towards various kinds of estimations (with integers, with real numbers, logical ones, etc.).

3. To specify the minimal number of criteria involved, for the algorithms to function correctly.
4. To specify the minimal number of evaluated objects, for the algorithms to function correctly.
5. To determine which of the scales proposed so far (or a completely new one) would be appropriate to use, depending on the specifics of the application.
6. To specify the degree of uncertainty, due to missing data, above which it becomes meaningless to apply the ICA method.
7. To develop algorithms for clustering of the resultant points in the IF interpretation triangle.
8. To develop algorithms for processing of three-dimensional index matrices, and operations over these, which contain not only evaluated objects and evaluation criteria, but also the factor of time. In this case, we could seek relations between pairs of criteria, or pairs of objects, which are subject to changes in time.

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