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REMARK ON A PROPERTY OF THE INTUITIONISTIC FUZZY INTERPRETATION TRIANGLE

Krassimir T. Atanassov
CLBME - Bulg. Academy of Sci., P.O.Box 12, Sofia-1113, Bulgaria
e-mail: krat@bgcict.acad.bg

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There are two problems related to the Intuitionistic Fuzzy Sets (IFSs; see [1]) which are related to the following
Problem: *To find a continued bijective transformation that transform the unit square $ABCD$ (from Fig. 1) to the IFS interpretation triangle ABD (from the same figure).*

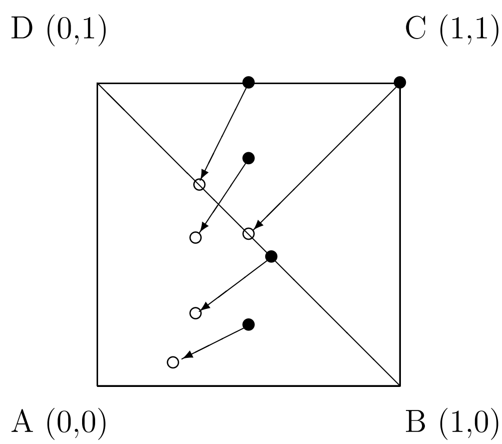

Fig. 1.

The solution of this question gives the possibility:
- to prove that each bi-lattice (see e.g., [2]) can be interpreted by an IFS;
- to prove that each intuitionistic L-fuzzy set (see [1]) can be interpreted by an IFS;
- to construct new algorithm for modifying incorrect expert estimations (see [1,3]).

Theorem 1. Transformation

$$F(x,y) = \begin{cases} \langle \frac{x^2}{x+y}, \frac{xy}{x+y} \rangle & \text{if } x, y \in [0,1] \text{ and } x \geq y \\ \\ \langle \frac{xy}{x+y}, \frac{y^2}{x+y} \rangle & \text{if } x, y \in [0,1] \text{ and } x \leq y \end{cases} \qquad (1)$$

satisfies the conditions of the Problem.
Proof. Firstly, we shall prove that F is a bijective transformation.

[1] This is the second day of the Fifth International Conference on Intuitionistic Fuzzy Sets in Sofia

We shall discuss the first case of (1), but the second case is similar.

Let points with coordinates $\langle x, y\rangle$ and $\langle u, v\rangle$ such that $x, y, u, v \in [0, 1]$ do not lie on a line containig point A with coordinates $\langle 0, 0\rangle$. Obviously, points $F(x, y)$ and $F(u, v)$ do not lie on a line containing point A. If $\langle x, y\rangle$ and $\langle u, v\rangle$ lie on a line containing point A, then

$$\frac{x}{y} = \frac{u}{v} = k > 0,$$

$y \neq v$ and then

$$\frac{x^2}{x+y} = \frac{k^2 y}{1+k} \neq \frac{k^2 v}{1+k} = \frac{u^2}{u+v}.$$

Let

$$\begin{cases} \dfrac{x^2}{x+y} = \dfrac{u^2}{u+v} \\ \\ \dfrac{xy}{x+y} = \dfrac{uv}{u+v}. \end{cases}$$

Let us assume that $x < u$. Of course, if $x = u$, then $y = v$. Therefore, $u = x + a$ and $v = y + b$ for $a > 0$ and $b \in [-1, 1]$. Therefore

$$\begin{cases} \dfrac{x^2}{x+y} = \dfrac{x^2 + 2ax + a^2}{x+y+a+b} \\ \\ \dfrac{xy}{x+y} = \dfrac{(x+a)(y+b)}{x+y+a+b}. \end{cases}$$

Therefore

$$\begin{cases} a(x^2 + x + 2xy + y) - bx^2 = 0 \\ \\ bx(x+a) + ay(y+b) = 0 \end{cases} \tag{2}$$

From the first equality of (2) is seen that $a, b \geq 0$ simultaneously, or $a, b \leq 0$ simultaneously. If the first case is valid, then from the second equality of (2) it follows that $a \geq 0$ and $b \leq 0$ or $a \leq 0$ and $b \geq 0$, that is possible only for $a = b = 0$. The second case, having in mind our assumption, we see that is being reduced to $a = b = 0$, i.e., $x = u$ and $y = v$.

Therefore, F is a bijective transformation. It can be easily seen that F is a continued transformation and that

$$\begin{cases} \lim\limits_{x\to 0,\ y\to 0} \dfrac{x^2}{x+y} = 0 \\ \\ \lim\limits_{x\to 0,\ y\to 0} \dfrac{xy}{x+y} = 0 \end{cases}$$

For every $u, v \in (0, 1]$, such that $u + v \leq 1$:

$$F^{-1}(u, v) = \begin{cases} \langle u+v, \frac{v}{u}(u+v)\rangle & \text{if } u \geq v \\ \\ \langle \frac{u}{v}(u+v), u+v\rangle & \text{if } u \leq v \end{cases}$$

Corolloaries For every $x, y \in [0,1]$: $F(1,y) = \langle \frac{1}{1+y}, \frac{y}{1+y} \rangle$, $F(x,1) = \langle \frac{x}{x+1}, \frac{1}{x+1} \rangle$, $F(1,1) = \langle \frac{1}{2}, \frac{1}{2} \rangle$, $F(x, 1-x) = \langle x^2, x - x^2 \rangle$, $F(\frac{1}{2}, \frac{1}{2}) = \langle \frac{1}{4}, \frac{1}{4} \rangle$, $F(0,0) = \langle 0, 0 \rangle$.

Analogically it can be proved

Theorem 2. Transformation

$$G(x,y) = \begin{cases} \langle x - \frac{y}{2}, \frac{y}{2} \rangle & \text{if } x, y \in [0,1] \text{ and } x \geq y \\ \langle \frac{x}{2}, y - \frac{x}{2} \rangle & \text{if } x, y \in [0,1] \text{ and } x \leq y \end{cases}$$

satisfies the conditions of the Problem (see Fig. 2).

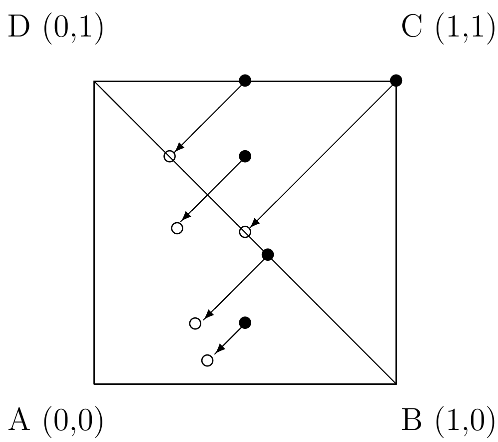

Fig. 2.

For every $u, v \in (0,1]$ such that $u + v \leq 1$:

$$G^{-1}(u,v) = \begin{cases} \langle u + v, 2v \rangle & \text{if } u \geq v \\ \langle 2u, u + v \rangle & \text{if } u \leq v \end{cases}$$

Corolloaries For every $x, y \in [0,1]$: $G(1,y) = \langle 1 - \frac{y}{2}, \frac{y}{2} \rangle$; $G(x,1) = \langle \frac{x}{2}, 1 - \frac{x}{2} \rangle$; $G(1,1) = \langle \frac{1}{2}, \frac{1}{2} \rangle$; $G(x, 1-x) = \langle \frac{3x-1}{2}, \frac{1-x}{2} \rangle$, if $x \geq \frac{1}{2}$; $G(x, 1-x) = \langle \frac{x}{2}, 1 - \frac{3x}{2} \rangle$, if $x \leq \frac{1}{2}$; $G(\frac{1}{2}, \frac{1}{2}) = \langle \frac{1}{4}, \frac{1}{4} \rangle$, $G(0,0) = \langle 0, 0 \rangle$.

References:

[1] Atanassov K., Intuitionistic Fuzzy Sets, Springer Physica-Verlag, Berlin, 1999.

[2] Gargov G., Knowledge, uncertainty and ignorance in logic: bilattices and beyond, Journal of Applied Non-Classical Logics, Vol. 9, 1999, No. 2-3, 195-283.

[3] Atanassov K., Intuitionistic fuzzy sets and expert estimations. II, BUSEFAL, Vol. 59, 1994, 64-69.