

ON SOME TYPES OF INTUITIONISTIC FUZZY NEGATIONS

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Abstract: Nine intuitionistic fuzzy negations are introduced and some of their basic properties are discussed.

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1 Introduction

A great number of fuzzy implications have been discussed in the book [1] by Georg Klir and Bo Yuan. In [2] analogous of these implications for the case of intuitionistic fuzzy logics are given (for intuitionistic fuzzy sets and logics see, e.g., [3]).

Let x be a fixed variable. Its truth-value is represented by the ordered couple $V(x) = \langle a, b \rangle$ so that $a, b, a+b \in [0, 1]$, where a and b are respectively degrees of validity and of non-validity of x .

For the needs of the research below we shall define the notion of Intuitionistic Fuzzy Tautology (IFT, see [3]) by:

$$x \text{ is an IFT if and only if } a \geq b.$$

We shall use functions sg and \overline{sg} :

$$sg(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

$$\overline{sg}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

The implications from [2] are given in Table 1.

TABLE 1: List of intuitionistic fuzzy implications

Name	Form of implication
Zadeh	$\langle \max(b, \min(a, c)), \min(a, d) \rangle$
Gaines-Rescher	$\langle 1 - sg(a - c), d.sg(a - c) \rangle$
Gödel	$\langle 1 - (1 - c).sg(a - c), d.sg(a - c) \rangle$
Kleene-Dienes	$\langle \max(b, c), \min(a, d) \rangle$
Lukasiewicz	$\langle \min(1, b + c), \max(0, a + d - 1) \rangle$
Reichenbach	$\langle b + ac, ad \rangle$
Willmott	$\langle \min(\max(b, c), \max(a, b), \max(c, d)), \max(\min(a, d), \min(a, b), \min(c, d)) \rangle$
Wu	$\langle 1 - (1 - \min(b, c)).sg(a - c), \max(a, d).sg(a - c).sg(d - b) \rangle$
Klir and Yuan 1	$\langle b + a^2c, ab + a^2d \rangle$
Klir and Yuan 2	$\langle c.\overline{sg}(1 - a) + sg(1 - a).(\overline{sg}(1 - c) + b.sg(1 - c)), d.\overline{sg}(1 - a) + a.sg(1 - a).sg(1 - c) \rangle$

Now, using as a basis equality

$$\neg x = x \rightarrow 0$$

or

$$\neg\langle a, b \rangle = \langle a, b \rangle \rightarrow \langle 0, 1 \rangle$$

we shall construct negations, corresponding to each of the above implications. These negations are introduced in Table 2.

TABLE 2: List of intuitionistic fuzzy negations

Name	Form of negation
Zadeh	$\langle b, a \rangle$
Gaines-Rescher	$\langle 1 - sg(a), sg(a) \rangle$
Gödel	$\langle 1 - sg(a), sg(a) \rangle$
Kleene-Dienes	$\langle b, a \rangle$
Lukasiewicz	$\langle b, a \rangle$
Reichenbach	$\langle b, a \rangle$
Willmott	$\langle b, a \rangle$
Wu	$\langle 1 - sg(a), sg(a).sg(1 - b) \rangle$
Klir and Yuan 2	$\langle sg(1 - a).b, \overline{sg}(1 - a) + a.sg(1 - a) \rangle$

For them the following three properties are checked:

Propertiy P1: $A \rightarrow \neg\neg A$,

Propertiy P2: $\neg\neg A \rightarrow A$,

Propertiy P3: $\neg\neg\neg A = \neg A$.

Since negarion is related to implication, we shall name the introduced negations after the names of the authors of the respective implications.

We shall omit the negation that corresponds to the first Klir and Yuan's implication, because this negation is very complex and unusable.

Theorem 1. Each of the negations from Table 2 satisfies Property 1.

Theorem 2. Only negations of Zadeh, Kleene-Dienes, Lukasiewicz, Reichenbach and Willmott from Table 2 satisfy Property 2.

Theorem 3. Each of the negations from Table 2 satisfies Property 3.

For example, we shall prove the validity of Property 3 for the last negation in the Table 2.

$$\begin{aligned}
\neg\neg\neg\langle a, b \rangle &= \neg\neg\langle sg(1-a).b, \overline{sg}(1-a) + a.sg(1-a) \rangle \\
&= \neg\langle sg(1-sg(1-a).b).(\overline{sg}(1-a) + a.sg(1-a)), \\
&\quad \overline{sg}(1-sg(1-a).b) + sg(1-a).b.sg(1-sg(1-a).b) \rangle \\
&= \langle sg(1-sg(1-sg(1-a).b).(\overline{sg}(1-a) + a.sg(1-a))), \\
&\quad .(\overline{sg}(1-sg(1-a).b) + sg(1-a).b.sg(1-sg(1-a).b)), \\
&\quad \overline{sg}(1-sg(1-sg(1-a).b).(\overline{sg}(1-a) + a.sg(1-a))) \\
&+ sg(1-sg(1-a).b).(\overline{sg}(1-a) + a.sg(1-a)).sg(1-sg(1-sg(1-a).b).(\overline{sg}(1-a) + a.sg(1-a)))) \rangle
\end{aligned}$$

Let $a = 1$. Then $b = 0$ and

$$\neg\neg\neg\langle a, b \rangle == \langle 0, 1 \rangle = \langle sg(1-a).b, \overline{sg}(1-a) + a.sg(1-a) \rangle = \neg\langle a, b \rangle.$$

Let $a < 1$. Then $b \geq 0$ and

$$\neg\neg\neg\langle a, b \rangle == \langle b.sg(1-b), \overline{sg}(1-sg(1-b).a) + sg(1-b).a.sg(1-sg(1-b).a) \rangle$$

If $b = 1$, then $a = 0$ and

$$\neg\neg\neg\langle a, b \rangle = \langle 1, 0 \rangle = \langle sg(1-a).b, \overline{sg}(1-a) + a.sg(1-a) \rangle = \neg\langle a, b \rangle.$$

If $b < 1$, then

$$\neg\neg\neg\langle a, b \rangle == \langle b, a \rangle = \langle sg(1-a).b, \overline{sg}(1-a) + a.sg(1-a) \rangle.$$

References

- [1] Klir G. and Bo Yuan, *Fuzzy Sets and Fuzzy Logic*. Prentice Hall, New Jersey, 1995.
- [2] Atanassov K., Intuitionistic fuzzy implications and Modus Ponens. *NIFS*, Vol. 11, 2005, No. 1, 1-4.
- [3] Atanassov, K. *Intuitionistic Fuzzy Sets*. Springer Physica-Verlag, Heidelberg, 1999.