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A simplification of the Neutrosophic Sets. Neutrosophic Logic and Intuitionistic Fuzzy Sets

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Abstract. This article explores some properties of the Neutrosophic logic and proposes a general simplification of the Neutrosophic Sets into a subclass of theirs, comprising of elements of R^3. The simplified variant is shown to preserve the properties of the Neutrosophic logic. The possibility to generalize Intuitionistic Fuzzy Sets with Neutrosophic Sets is also investigated.

Keywords: Neutrosophic Sets, Neutrosophic Logic, Intuitionistic Fuzzy Sets, Intuitionistic Fuzzy Logic

1. Introduction

In his work [1] F. Smarandache has provided a variety of real-life examples for possible applications of his neutrosophic sets. However, it seems like the information stored about each neutrosophic element is logically just too much and the Neutrosophic logic doesn't create significant benefit as a modeling tool. We will define an automorphism of neutrosophic sets into a kind of simplified neutrosophic sets that will preserve the operations of the neutrosophic logic, including the subset relation.

2. A transformation of non-standard intervals into standard intervals.

Let

$$R^* = \left\{ x \mid \forall n \in N^+ (x < \frac{1}{n}) \right\}$$

be the set of all infinitely small numbers. Then

$$a^+ = \left\{ a + x \mid x \in R^* \right\} \text{ and } {}^-a = \left\{ a - x \mid x \in R^* \right\}$$

are non-standard elements. Let $L \subseteq]^-0,1^+[$ be a *subset* of the non-standard unit interval. The following definition introduces a transformation $|.|_{st} :]^-0,1^+[\to [0,1]$.

Definition.

$$|a^+|_{st} = a \in R \,,\ |{}^-a|_{st} = a \in R$$

for all non standard elements a^+ and ${}^-a$, and

$$\forall a \in R (|a|_{st} = a) \,.$$

We will extend $|.|_{st}$ for subsets of $]^-0,1^+[$.

Definition. Let $L \subseteq]^{-}0,1^{+}[$. Then

$$\left|L\right|_{st} = \left\{\left|x\right|_{st} \mid x \in L\right\}.$$

Now $\left|.\right|_{st} : \left(]^{-}0,1^{+}[\right)^{\omega} \to \left([0,1]\right)^{\omega}$.

3. Reducing Neutrosophic Sets to subsets of R^3 .

The elements of a neutrosophic set are comprised of three subsets of the nonstandard unit interval $]^{-}0,1^{+}[$. A neutrosophic set has the form

$$A = \left\{x\left(T, I, F\right) \mid T, I, F \subseteq]^{-}0,1^{+}[\right\}.$$

We will define a subclass of neutrosophic sets – simplified neutrosophic sets.

Definition. The neutrosophic set A is called a simplified neutrosophic set if

$$A = \left\{x(\{i_T\}, \{i_I\}, \{i_F\}) \mid i_T, i_I, i_F \in [0,1]\right\}.$$

A neutrosophic set is simplified if its elements are comprised of singleton subsets of the real unit interval. Therefore each simplified neutrosophic set can be described by the means of three real numbers.

We will define an automorphism κ of neutrosophic sets into simplified neutrosophic sets.

Definition. Let $L \subseteq]^{-}0,1^{+}[$. Then the operation

$$[L] = \left\{\inf\left|L\right|_{st}\right\} \subseteq [0,1]$$

transforms a subset of the non-standard unit interval into a singleton subset of the real unit interval.

Definition. Let A be a neutrosophic set.

$$\kappa(A) = \left\{x([T],[I],[F]) \mid x(T, I, F) \in A\right\}$$

is a simplification of the neutrosophic set A.

We will prove that κ preserves the following relations and logical operations over neutrosophic sets, defined by F. Smarandache in [1] and [2]:

1. Subset. A is a subset of B when for each $x\left(T_1, I_1, F_1\right)$ from A there is some $y(T_2, I_2, F_2)$ from B such that $\inf(T_1) \le \inf(T_2)$, $\sup(T_1) \le \sup(T_2)$, $\inf(F_1) \ge \inf(F_2)$, and $\sup(F_1) \ge \sup(F_2)$
2. Negation. $\neg A = \left\{x\left(1-T, 1-I, 1-F\right) \mid x(T, I, F) \in A\right\}$ where $1 - L = \left\{1 - x \mid x \in L\right\}$ for $L \subseteq]^{-}0,1^{+}[$.
3. Conjunction $A \wedge B = \left\{z(T_1 \otimes T_2, I_1 \otimes I_2, F_1 \otimes F_2) \mid x(T_1, I_1, F_1) \in A, y(T_2, I_2, F_2) \in B\right\}$ where $L_1 \otimes L_2 = \left\{x.y \mid x \in L_1, y \in L_2\right\}$ for $L_1, L_2 \subseteq]^{-}0,1^{+}[$.
4. Implication $A \to B =$
$\left\{z\left((1-T_1) + (T_1 \otimes T_2), (1-I_1) + (I_1 \otimes I_2), (1-F_1) + (F_1 \otimes F_2)\right) \mid x(T_1, I_1, F_1) \in A, y(T_2, I_2, F_2) \in B\right\}$
Where $L_1 + L_2 = \left\{x + y \mid x \in L_1, y \in L_2\right\}$

Note. When it comes to neutrosophic sets A and B we will denote $A \subseteq B$ meaning the relation in point one, defined by F. Smarandache.

Lemma 1. Let A and B are neutrosophic sets and $A \subseteq B$. Then $\kappa(A) \subseteq \kappa(B)$.

Let $x(T_1, I_1, F_1) \in A$ and $y(T_2, I_2, F_2) \in B$ such that $\inf(T_1) \le \inf(T_2)$, $\sup(T_1) \le \sup(T_2)$, $\inf(F_1) \ge \inf(F_2)$, $\sup(F_1) \ge \sup(F_2)$. Then $x([T_1],[I_1],[F_1]) \in \kappa(A)$ and $y([T_2],[I_2],[F_2]) \in \kappa(B)$. Now since $\sup[T_l] = \inf[T_1] = \inf|T_1|_{st}$, $\sup[T_2] = \inf[T_2] = \inf|T_2|_{st}$, $\sup[F_l] = \inf[F_1] = \inf|F_1|_{st}$ and $\sup[F_2] = \inf[F_2] = \inf|F_2|_{st}$ it is trivial that $\kappa(A) \subseteq \kappa(B)$.

Lemma 2. Let A is a neutrosophic set. The $\kappa(\neg A) = \neg\kappa(A)$.

$\neg A = \{x(1-T, 1-I, 1-F) \mid x(T,I,F) \in A\}$.
$\neg\kappa(A) = \{x(1-[T], 1-[I], 1-[F]) \mid x(T,I,F) \in A\}$.
$\kappa(\neg A) = \{x[([1-T],[1-I],[1-F]) \mid x(1-T,1-I,1-F) \in \neg A\}$.
Now since $[1-L] = 1-[L]$ we have proven the lemma.

Lemma 3. Let A and B are neutrosophic sets. Then $\kappa(A \wedge B) = \kappa(A) \wedge \kappa(B)$.

$A \wedge B = \{z(T_1 \otimes T_2, I_1 \otimes I_2, F_1 \otimes F_2) \mid x(T_1, I_1, F_1) \in A, y(T_2, I_2, F_2) \in B\}$.
$\kappa(A \wedge B) = \{z([T_1 \otimes T_2],[I_1 \otimes I_2],[F_1 \otimes F_2]) \mid x(T_1, I_1, F_1) \in A, y(T_2, I_2, F_2) \in B\}$.
$\kappa(A) = \{x([T],[I],[F]) \mid x(T,I,F) \in A\}$.
$\kappa(B) = \{y([T],[I],[F]) \mid y(T,I,F) \in B\}$.
$\kappa(A) \wedge \kappa(B) = \{z([T_1] \otimes [T_2],[I_1] \otimes [I_2],[F_1] \otimes [F_2]) \mid x(T_1, I_1, F_1) \in A, y(T_2, I_2, F_2) \in B\}$.
Now the fact that $[L_1 \otimes L_2] = [L_1] \otimes [L_2]$ for each $L_1, L_2 \subseteq]^-0, 1^+[$ proves the lemma.

Lemma 4. $[(1-L_1) + (L_1 \otimes L_2)] = (1-[L_1]) + ([L_1] \otimes [L_2])$.

We have already noticed that $[1-L] = 1-[L]$ and $[L_1 \otimes L_2] = [L_1] \otimes [L_2]$. Now
$[L_1] + [L_2] = \{\inf|L_1|_{st}\} + \{\inf|L_2|_{st}\} = \{\inf|L_1|_{st} + \inf|L_2|_{st}\} = \{\inf|L_1 + L_2|_{st}\} = [L_1 + L_2]$.

Having lemma 3 proven we may conclude that $\kappa(A \to B) = \kappa(A) \to \kappa(B)$. If we continue our investigation in the same fashion we may prove that all the neutrosophic logical operations defined in [1] and [2] are preserved by the simplification automorphism κ.

4. Neutrosophic Logic, Intuitionistic Fuzzy Logic and Modal Logic

Smarandache [1,2] suggests that the Neutrosophic logic may be applied as a generalization of the Intuitionistic Fuzzy logic [3]. However, according to the neutrosophic operations we have $\neg\neg A = A$ and since $\neg\neg A \ne A$ is just the assumption that has brought intuitionism to life, the neutrosophic logic could not be a generalization of any Intuitionistic logic.

Smarandache has also adopted Leibniz's "worlds" in his work, but it seems to be more like a game of words. The neutrosophic logic is not capable of maintaining modal operators, since there is no normalization rule for the components T, I and F. The Intuitionistic Fuzzy sets, on the other hand, have the chance to become a consistent model of the modal logic, adopting all the necessary properties [4].

5. Neutrosophic Logic and Simplified Neutrosophic Logic

Apparently there isn't a clear definition of the truth value of the neutrosophic formulas. But in every meaningful logical system if A and B are sets (formulas) such that $A \subseteq B$ then B ′ A, i.e. when B is true then A is true.

Theorem 1. Either A H $\kappa(A)$ (*i.e. A is true if and only if* $\kappa(A)$ *is true*) **or** the neutrosophic logic is contradictory.

Now whatever the definition of the truth value is, since $\kappa(A) \subseteq A$ we have A′ $\kappa(A)$. Let $\kappa(A)$ be true and assume that A is not true. Then since the Neutrosophic logic is not an intuitionistic one, $\neg A$ should be true leading to the conclusion that $\kappa(\neg A) = \neg\kappa(A)$ is true. We found that $\kappa(A) \wedge \neg\kappa(A)$ is true which means that the simplified neutrosophic logic is contradictory. But since the simplified neutrosophic logic is only a subclass of the neutrosophic logic, then the neutrosophic logic is contradictory. If it is not, the assumption that A is not true is incorrect.

While there is no investigation about whether the neutrosophic logic is contradictory or not, we should be satisfied by the proposition of Theorem 1. If we trust in the consistency of the Neutrosophic logic, then we find it to be equivalent to a logic based on three real numbers.

6. Conclusion

Such a great generalization as the neutrosophic sets seems to be just too great. Moreover, the suggestion that the neutrosophic logic in its generality is capable of absorbing the properties of a great variety of known logical systems also seems to be too strong.

An interesting direction for future work on Neutrosophic logic could be a prove that it is consistent. The properties of a R^3 logic may also turn to be very interesting and should be explored to find the place of the Neutrosophic logic in the wide family of logical systems that are recently known.

References

[1] Smarandache, F., Definition of neutrosophic logic – a generalization of intuitionistic fuzzy logic. Proc. of the Third Conf. of the European Society for Fuzzy Logic and Technology EUSFLAT' 2003, Zittau, 10-12 Sept. 2003, 141-146.
[2] Smarandache, F., Neutrosophic set – a generalization of intuitionistic fuzzy set (in press).
[3] Atanassov, K., T. Trifonov, Towards combining two constructions of intuitionistic fuzzy sets, see the present issue, 1-11.
[4] Atanassov, K., Intuitionistic Fuzzy Sets. Springer-Verlag, Heidelberg, 1999.