Notes on Intuitionistic Fuzzy Sets
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Common fixed point theorems for weakly commuting of type (J) in generalized intuitionistic fuzzy metric spaces

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This paper is dedicated to our advisors.

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Abstract: This paper focuses on the existence of a fixed point in generalized intuitionistic fuzzy metric spaces. The presentation of generalized intuitionistic fuzzy metric spaces in n-tuple encourages us to define different mappings in the symmetric generalized intuitionistic fuzzy metric spaces. Notation for pair of mappings (f, g) on generalized intuitionistic fuzzy metric spaces called weakly commuting of type (J) and R-weakly commuting of type (J) is introduced. We prove common fixed point theorems in generalized intuitionistic fuzzy metric spaces employing the effectiveness of property (E.A.) and CLRg property.

Keywords: Common fixed point, V-fuzzy metric spaces, Generalized intuitionistic fuzzy metric spaces.

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1 Introduction

The notion of fuzzy sets was initially proposed and investigated by Zadeh [10] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [7] defined the notion of an intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [4]. In 2002, Aamri and Moutawakil [1] introduced the idea of property (E.A.). In 2011, Sintunavarat and Kumam [9] opinion of CLRg property the requirement of completeness of underlying subspace. Abbas et al. [2] established the notion of A-metric spaces, a generalization of S-metric space [8]. Vishal Gupta and Ashima Kanwar [5] introduced the V-fuzzy metric space. In 2016, Jeyaraman and Malligadevi [6] introduce generalized intuitionistic fuzzy metric spaces and discuss their properties. We prove common fixed point theorems for weakly commuting of type (J) in symmetric generalized intuitionistic fuzzy metric spaces.

2 Preliminaries

Definition 2.1. Let X be a nonempty set. A function $A : X^n \to [0, +\infty)$ is called an A-metric on X if for any $x_i, a \in X, i = 1, 2, \dots, n$, the following conditions hold:

(A-1) $A(x_1, x_2, x_3, \dots, x_n) \geq 0$,

(A-2) $A(x_1, x_2, x_3, \dots, x_n) = 0$, if and only if $x_1 = x_2 = x_3 = \cdots = x_n$,

(A-3) $$\begin{aligned} A(x_1, x_2, x_3, \dots, x_n) \leq\ & A(x_1, x_1, x_1, \dots, (x_1)_{n-1}, a) \\ & + A(x_2, x_2, x_2, \dots, (x_2)_{n-1}, a) \\ & + \cdots + A(x_n, x_n, x_n, \dots, (x_n)_{n-1}, a). \end{aligned}$$

The pair (X, A) is called an A-metric space.

Example
Let $X = R$. Define the function $A : X^n \to$ [0, +∞) by

$$A(x_1, x_2, x_3, \dots, x_n) = \sum_{i=1}^{n} \sum_{i<j} |x_i - y_j|.$$

Then (X, A) is called the usual A-metric space.

Definition 2.2. Let X be a nonempty set. A triple $(X, V, *)$ is said to be a V-fuzzy metric space (denoted by VF-space), where $*$ is a continuous t-norm, V is a fuzzy set on $X^n \times (0, \infty)$ satisfying the following conditions: for all $t, s > 0$,

(VF-1) $V(x, x, x, \dots, x, y, t) > 0$ for all $x, y \in X$ with $x \neq y$, (VF-2) $V(x_1, x_1, x_1, \ldots, x_1, x_2, t) \geq V(x_1, x_2, x_3, \ldots, x_n, t)$ for all $x_1, x_2, \ldots, x_n \in X$ with $x_2 \neq x_3 \neq \cdots \neq x_n$,

(VF-3) $V(x_1, x_2, x_3, \ldots, x_n, t) = 1$ if and only if $x_1 = x_2 = x_3 = \ldots = x_n$,

(VF-4) $V(x_1, x_2, x_3, \ldots, x_n, t) = V(p(x_1, x_2, x_3, \ldots, x_n), t)$, where p is a permutation function,

(VF-5) $V(x_1, x_2, x_3, \ldots, x_n, t+s) \geq V(x_1, x_2, x_3, \ldots, x_{n-1}, l, t) * V(l, l, l, \ldots, l, x_n, s)$,

(VF-6) $\lim_{t\to\infty} V(x_1, x_2, x_3, \ldots, x_n, t) = 1$,

(VF-7) $V(x_1, x_2, x_3, \ldots, x_n, .) : (0, \infty) \to [0, 1]$ is continuous.

3 Generalized intuitionistic fuzzy metric spaces

Definition 3.1. Let X be a nonempty set. A 5-tuple $(X, V, W, *, \diamond)$ is said to be a generalized intuitionistic fuzzy metric space, where $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and V, W are fuzzy sets on $X^n \times (0, \infty)$ satisfying the following conditions: for every $x_1, x_2, x_3, \ldots, x_n, l \in X, t, s > 0$,

(i) $V(x_1, x_2, x_3, \ldots, x_n, t) + W(x_1, x_2, x_3, \ldots, x_n, t) \leq 1$,

(ii) $V(x, x, x, \ldots, x, y, t) > 0$ for all $x, y \in X$ with $x \neq y$,

(iii) $V(x_1, x_1, x_1, \ldots, x_1, x_2, t) \geq V(x_1, x_2, x_3, \ldots, x_n, t)$ for all $x_1, x_2, \ldots, x_n \in X$ with $x_2 \neq x_3 \neq \cdots \neq x_n$,

(iv) $V(x_1, x_2, x_3, \ldots, x_n, t) = 1$ if and only if $x_1 = x_2 = x_3 = \ldots = x_n$,

(v) $V(x_1, x_2, x_3, \ldots, x_n, t) = V(p(x_1, x_2, x_3, \ldots, x_n), t)$, where p is a permutation function,

(vi) $V(x_1, x_2, x_3, \ldots, x_n, t+s) \geq V(x_1, x_2, x_3, \ldots, x_{n-1}, l, t) * V(l, l, l, \ldots, l, x_n, s)$,

(vii) $V(x_1, x_2, x_3, \ldots, x_n, .) : (0, \infty) \to [0, 1]$ is continuous,

(viii) V is a non-decreasing function on R^+, $\lim \to \infty V(x_1, x_2, x_3, \ldots, x_n, t) = 1$ and $\lim_{t\to 0} V(x_1, x_2, x_3, \ldots, x_n, t) = 0$, for all $x_1, x_2, x_3, \ldots, x_n \in X, t > 0$,

(ix) $W(x, x, x, \ldots, x, y, t) < 1$ for all $x, y \in X$ with $x \neq y$,

(x) $W(x_1, x_1, x_1, \ldots, x_1, x_2, t) \leq W(x_1, x_2, x_3, \ldots, x_n, t)$ for all $x_1, x_2, \ldots, x_n \in X$ with $x_2 \neq x_3 \neq \cdots \neq x_n$,

(xi) $W(x_1, x_2, x_3, \ldots, x_n, t) = 0$ if and only if $x_1 = x_2 = x_3 = \ldots = x_n$,

(xii) $W(x_1, x_2, x_3, \ldots, x_n, t) = W(p(x_1, x_2, x_3, \ldots, x_n), t)$, where p is a permutation function,

(xiii) $W(x_1, x_2, x_3, \ldots, x_n, t+s) \leq W(x_1, x_2, x_3, \ldots, x_{n-1}, l, t) \diamond W(l, l, l, \ldots, l, x_n, s)$,

(xiv) $W(x_1, x_2, x_3, \ldots, x_n, .) : (0, \infty) \to [0, 1]$ is continuous,

(xv) W is a non-increasing function on R^+, $\lim_{n\to\infty} W(x_1, x_2, x_3, \ldots, x_n, t) = 0$ and $\lim_{t\to 0} W(x_1, x_2, x_3, \ldots, x_n, t) = 1$, for all $x_1, x_2, x_3, \ldots, x_n \in X, t > 0$,

In this case, the pair (V, W) is called a generalized intuitionistic fuzzy metric spaces.

Example
Let (X, A) be an A-metric space. For all $x_1, x_2, x_3, \ldots, x_n \in X$ and every $t > 0$, consider (V, W) to be fuzzy sets on $X^n \times (0, \infty)$ defined by

$$V(x_1, x_2, x_3, \ldots, x_n, t) = \frac{t}{t + A(x_1, x_2, x_3, \ldots, x_n)}$$

and

$$W(x_1, x_2, x_3, \ldots, x_n, t) = \frac{A(x_1, x_2, x_3, \ldots, x_n)}{t + A(x_1, x_2, x_3, \ldots, x_n)}.$$

Denote $a * b = ab$ and $a\Diamond b = \min\{a + b\}$. Then, $(X, V, W, *, \Diamond)$ is a generalized intuitionistic fuzzy metric space.

Lemma 3.2. *Let $(X, V, W, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. Then $V(x_1, x_2, \ldots, x_n, t)$ is non-decreasing and $W(x_1, x_2, \ldots, x_n, t)$ in non-increasing with respect to t.*

Lemma 3.3. *Let $(X, V, W, *, \diamond)$ be a generalized intuitionistic fuzzy metric space such that $V(x_1, x_2, \ldots, x_n, kt) \geq V(x_1, x_2, \ldots, x_n, t)$ and $W(x_1, x_2, \ldots, x_n, kt) \leq W(x_1, x_2, \ldots, x_n, t)$ with $k \in (0, 1)$. Then, $x_1 = x_2 = \cdots = x_n$.*

Definition 3.4. Let $(X, V, W, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. A sequence $\{x_r\}$ is said to converge to a point $x \in X$ if $V(x_r, x_r, x_r, \ldots, x_r, x, t) \to 1$ and $W(x_r, x_r, x_r, \ldots, x_r, x, t) \to 0$ as $r \to \infty$ for all $t > 0$, that is, for each $\epsilon > 0$, there exists $n \in N$ such that for all $r \geq n$, we have $V(x_r, x_r, x_r, \ldots, x_r, x, t) > 1 - \epsilon$ and $W(x_r, x_r, x_r, \ldots, x_r, x, t) < \epsilon$, we write $\lim_{r\to\infty} x_r = x$.

Definition 3.5. Let $(X, V, W, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. A sequence x_r is said to be a Cauchy sequence if $V(x_r, x_r, x_r, \ldots, x_r, x_q, t) \to 1$ and $W(x_r, x_r, x_r, \ldots, x_r, x_q, t) \to 0$ as $r, q \to \infty$ for all $t > 0$, that is, for each $\epsilon > 0$, there exists $n_0 \in N$ such that for all $r, q \geq n_0$, we have $V(x_r, x_r, x_r, \ldots, x_r, x_q, t) > 1 - \epsilon$ and $W(x_r, x_r, x_r, \ldots, x_r, x_q, t) < \epsilon$.

Definition 3.6. The generalized intuitionistic fuzzy metric space $(X, V, W, *, \diamond)$ is said to be complete if every Cauchy sequence in X is convergent.

Remark. Define $\Phi = \phi : R^+ \to R^+$, where $R^+ = [0, \infty)$ and each $\phi \in \Phi$ satisfying the following conditions:

(ϕ-1) ϕ is strictly increasing,

(ϕ-2) ϕ is upper semicontinuous from the right,

(ϕ-3) $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for all $t > 0$.

Lemma 3.7. *Let $(X, V, W, *, \diamond)$ be a generalized intuitionistic fuzzy metric space. If there exists $\phi \in \Phi$, such that*

$$V(x_1, x_2, x_3, \ldots, x_n, \phi(t)) \geq V(x_1, x_2, x_3, \ldots, x_n, t)$$

and

$$W(x_1, x_2, x_3, \ldots, x_n, \phi(t)) \leq W(x_1, x_2, x_3, \ldots, x_n, t)$$

for all $t > 0$, then $x_1 = x_2 = x_3 = \cdots = x_n$.

Proof. Since

$$\begin{aligned} V(x_1, x_2, x_3, \ldots, x_n, \phi(t)) \geq V(x_1, x_2, x_3, \ldots, x_n, t) \text{ and} \\ W(x_1, x_2, x_3, \ldots, x_n, \phi(t)) \leq W(x_1, x_2, x_3, \ldots, x_n, t) \end{aligned} \tag{3.1}$$

Also $\phi(t) < t$, by using Lemma 3.2 we have

$$\begin{aligned} V(x_1, x_2, x_3, \ldots, x_n, \phi(t)) \leq V(x_1, x_2, x_3, \ldots, x_n, t) \text{ and} \\ W(x_1, x_2, x_3, \ldots, x_n, \phi(t)) \geq W(x_1, x_2, x_3, \ldots, x_n, t) \end{aligned} \tag{3.2}$$

From (3.1), (3.2) and the definition of generalized intuitionistic fuzzy metric space, we get $x_1 = x_2 = x_3 = \cdots = x_n$. □

Remark. Let $x_1 = w, x_2 = x_3 = \cdots = x_{n-1} = u$, in (vi) and (xiii) we have

$$V(w, u, u, \ldots, u, t + s) \geq V(w, v, v, \ldots, v, t) * V(v, u, u, \ldots, u, s)$$

and

$$W(w, u, u, \ldots, u, t + s) \leq W(w, v, v, \ldots, v, t) \diamond W(v, u, u, \ldots, u, s),$$

which implies that

$$V(u, u, u, \ldots, w, s + t) \geq V(v, v, v, \ldots, v, w, t) * V(u, u, u, \ldots, u, v, s)$$

and

$$W(u, u, u, \ldots, w, s + t) \leq W(v, v, v, \ldots, v, w, t) \diamond W(u, u, u, \ldots, u, v, s)$$

for all $u, v, w \in X$ and $s, t > 0$.

A generalized intuitionistic fuzzy metric space is said to be symmetric if:

$$V(x, x, x, \ldots, x, y, t) = V(x, y, y, \ldots, y, t)$$

and

$$W(x, x, x, \ldots, x, y, t) = W(x, y, y, \ldots, y, t)$$

for all $x, y \in X$ and for each $t > 0$.

Lemma 3.8. *Let $(X, V, W, *, \diamond)$ be a generalized intuitionistic fuzzy metric space, if we define $E_\lambda : X \times X \times X \times \cdots \times X \to [0, \infty)$ by*

$$\begin{aligned} E_\lambda(x_1, x_2, x_3, \ldots, x_n) = \inf\{t > 0, V(x_1, x_2, x_3, \ldots, x_n, t) > 1 - \lambda \textit{ and} \\ W(x_1, x_2, x_3, \ldots, x_n, t) < \lambda\} \end{aligned} \tag{3.3}$$

for all $\lambda \in (0, 1]$ and $x_1, x_2, x_3, \ldots, x_n \in X$, then we have

(a) *for each* $\lambda \in (0,1]$, *there exists* $\mu \in (0,1]$ *such that*

$$E_\lambda(x_1, x_2, x_3, \dots, x_n) \le \sum_{i=1}^{n-1} E_\mu(x_i, x_i, x_i, \dots, x_{i+1})$$

for all $x_1, x_2, x_3, \dots, x_n \in X$

(b) *the sequence* $\{x_n\}_{n\in N}$ *in* X *is convergent if and only if*

$$E_\lambda(x_n, x_n, x_n, \dots, x_n, x) \to 0$$

as $n \to \infty$ *for all* $\lambda \in (0,1]$.

Proof. (a) For any $\lambda \in (0,1]$, let $\mu \in (0,1]$ and $\mu < \lambda$. Therefore, by the triangular inequality (vi) and (xiii) and the previous Remark

$$\begin{aligned}
&V(x_1, x_1, \dots, x_n, \sum_{i=1}^{n-1} E_\mu(x_i, x_i, \dots, x_{i+1}) + (n-1)\delta) \\
&\ge V(x_1, x_1, \dots, x_2, E_\mu(x_1, x_1, \dots, x_2) + \delta) * V(x_2, x_2, \dots, x_3, E_\mu(x_2, x_2, \dots, x_3) + \delta) * \\
&\quad \cdots * V(x_{n-1}, x_{n-1}, \dots, x_n, E_\mu(x_{n-1}, x_{n-1}, \dots, x_n) + \delta) \\
&\ge \min\{(1-\mu), (1-\mu), \dots, (1-\mu)\} \ge 1 - \lambda
\end{aligned}$$

and

$$\begin{aligned}
&W(x_1, x_1, \dots, x_n, \sum_{i=1}^{n-1} E_\mu(x_i, x_i, \dots, x_{i+1}) + (n-1)\delta) \\
&\le W(x_1, x_1, \dots, x_2, E_\mu(x_1, x_1, \dots, x_2) + \delta) \diamond W(x_2, x_2, \dots, x_3, E_\mu(x_2, x_2, \dots, x_3) + \delta) \diamond \\
&\quad \cdots \diamond W(x_{n-1}, x_{n-1}, \dots, x_n, E_\mu(x_{n-1}, x_{n-1}, \dots, x_n) + \delta) \\
&\le \max\{\mu, \mu, \dots, \mu\} \le \lambda,
\end{aligned}$$

which gives, using (3.3),

$$\begin{aligned}
E_\lambda(x_1, x_1, \dots, x_n) \le{}& E_\mu(x_1, x_1, \dots, x_2) + E_\mu(x_2, x_2, \dots, x_3) + \cdots + E_\mu(x_{n-1}, x_{n-1}, \dots, x_n) \\
&+ (n-1)\delta.
\end{aligned}$$

Since $\delta > 0$ is arbitrary, we have

$$E_\lambda(x_1, x_1, \dots, x_n) \le E_\mu(x_1, x_1, \dots, x_2) + E_\mu(x_2, x_2, \dots, x_3) + \cdots + E_\mu(x_{n-1}, x_{n-1}, \dots, x_n).$$

(b) Since V and W are continuous in its $(n+1)$-st argument by (3.3), we have $V(x_n, x_n, \dots, x_n, x, \eta) > 1 - \lambda$ and $W(x_n, x_n, \dots, x_n, x, \eta) < \lambda$ for all $\eta > 0$. □

Lemma 3.9. *Let* $(X, V, W, *, \diamond)$ *be a generalized intuitionistic fuzzy metric space and* $\{y_n\}$ *be a sequence in* X. *If there exists* $\phi \in \Phi$, *such that*

$$\begin{aligned}
&V(y_n, y_n, y_n, \dots, y_n, y_{n+1}, \phi(t)) \\
&\qquad \ge V(y_{n-1}, y_{n-1}, y_{n-1}, \dots, y_{n-1}, y_n, t) * V(y_n, y_n, y_n, \dots, y_n, y_{n+1}, t) \\
&\quad \text{and} \\
&W(y_n, y_n, y_n, \dots, y_n, y_{n+1}, \phi(t)) \\
&\qquad \le W(y_{n-1}, y_{n-1}, y_{n-1}, \dots, y_{n-1}, y_n, t) \diamond W(y_n, y_n, y_n, \dots, y_n, y_{n+1}, t)
\end{aligned} \tag{3.4}$$

for all $t > 0$ and $n = 1, 2, \ldots$, then $\{y_n\}$ is a Cauchy sequence in X.

Proof. Let $\{E_\lambda(x_1, x_2, x_3, \ldots, x_n)\}_{\lambda\in(0,1]}$ be defined by (3.3). For each $\lambda \in (0, 1]$ and $n \in N$, put

$$a_n = E_\lambda(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n). \tag{3.5}$$

We will prove that

$$a_{n+1} \leq \phi(a_n) \text{ for all } n \in N. \tag{3.6}$$

Since ϕ is upper semicontinuous from right, for a given $\epsilon > 0$ and each a_n, there exists $p_n > a_n$ such that $\phi(p_n) < \phi(a_n) + \epsilon$.

From (3.3), it follows from $p_n > a_n = E_\lambda(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n)$ that

$$V(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n, p_n) > 1 - \lambda \text{ and } W(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n, p_n) < \lambda$$

for all $n \in N$. Thus, by (3.4), (3.6) and Lemma 3.2 we get

$$\begin{aligned} &V(y_n, y_n, y_n, \ldots, y_{n+1}, \phi(\max\{p_n, p_{n+1}\})) \\ &\quad \geq V(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_n, \max\{p_n, p_{n+1}\}) * V(y_n, y_n, y_n, \ldots, y_{n+1}, \max\{p_n, p_{n+1}\}) \\ &\quad \geq V(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_n, p_n) * V(y_n, y_n, y_n, \ldots, y_{n+1}, p_{n+1}) > 1 - \lambda \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} &W(y_n, y_n, y_n, \ldots, y_{n+1}, \phi(\max\{p_n, p_{n+1}\})) \\ &\quad \leq W(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_n, \max\{p_n, p_{n+1}\}) \diamond W(y_n, y_n, y_n, \ldots, y_{n+1}, \max\{p_n, p_{n+1}\}) \\ &\quad \leq W(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_n, p_n) \diamond W(y_n, y_n, y_n, \ldots, y_{n+1}, p_{n+1}) < \lambda. \end{aligned} \tag{3.8}$$

Again, by (3.3), we get
$E_\lambda(y_n, y_n, y_n, \ldots, y_{n+1}) \leq \phi(\max\{p_n, p_{n+1}\}) = \max\{\phi(p_n), \phi(p_{n+1})\}$
therefore,

$$E_\lambda(y_n, y_n, y_n, \ldots, y_{n+1}) \leq \max\{\phi(a_n), \phi(a_{n+1})\} + \epsilon \tag{3.9}$$

Since, ϵ is arbitrary, we get

$$a_{n+1} = E_\lambda(y_n, y_n, y_n, \ldots, y_{n+1}) \leq \max\{\phi(a_n), \phi(a_{n+1})\}. \tag{3.10}$$

Suppose, $\max\{\phi(a_n), \phi(a_{n+1})\} = \phi(a_{n+1})$. We have $a_{n+1} \leq \phi(a_{n+1}) < a_{n+1}$, this is a contradiction. Hence, (3.9) implies that $a_{n+1} \leq \phi(a_n)$.

By repeated application of (3.6), we get

$$\begin{aligned} E_\lambda(y_n, y_n, y_n, \ldots, y_n, y_{n+1}) &\leq \phi(E_\lambda(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n)) \\ &\leq \cdots \leq \phi^n(E_\lambda(y_0, y_0, y_0, \ldots, y_0, y_1) \text{ for all } n \in N. \end{aligned} \tag{3.11}$$

By Lemma 3.8, for each $\lambda \in$(0,1], there exists $\mu \in$ (0,1] such that

$$E_\lambda(y_n, y_n, y_n, \ldots, y_n, y_m) \leq \sum_{i=n}^{m-1} E_\mu(y_i, y_i, y_i, \ldots, y_i, y_{i+1}), \tag{3.12}$$

for all $m, n \in N$ with $m > n.$

Since $\phi \in \Phi$, by condition (ϕ - 3), we have

$$\sum_{n=0}^{\infty} \phi^n(E_\mu(y_0, y_0, y_0, \dots, y_0, y_1)) < +\infty. \tag{3.13}$$

So, for a given $\epsilon > 0$, there exists $n_0 \in N$ such that

$$\sum_{i=n_0}^{\infty} \phi^i(E_\mu(y_0, y_0, y_0, \dots, y_0, y_1)) < \epsilon.$$

Thus, it follows from (3.11) that

$$E_\lambda(y_n, y_n, y_n, \dots, y_n, y_m) \leq \sum_{i=n}^{m-1} \phi^i(E_\mu(y_0, y_0, y_0, \dots, y_0, y_1)) < \epsilon$$

for all $n \geq n_0$, which implies that $V(y_n, y_n, y_n, \dots, y_n, \epsilon) > 1 - \lambda$ and $W(y_n, y_n, y_n, \dots, y_n, \epsilon) < \lambda$ for all $m.n \in N$ with $m > n \geq n_0$. Therefore, $\{y_n\}$ is a Cauchy sequence in X. □

Definition 3.10. A pair of self mappings (f, g) of a generalized intuitionistic fuzzy metric space $(X, V, W, *, \diamond)$ is said to be weakly commuting of type (J) if $V(fgx, gfx, fgx, \dots, ffx, t) \geq V(fx, gx, fx, \dots, fx, t)$ and $W(fgx, gfx, fgx, \dots, ffx, t) \leq W(fx, gx, fx, \dots, fx, t)$ for all $x \in X$ and $t > 0$.

Definition 3.11. A pair of self mappings (f, g) of a generalized intuitionistic fuzzy metric space $(X, V, W, *, \diamond)$ is said to be R-weakly commuting of type (J) if there exists some positive real number R such that $V(fgx, gfx, fgx, gfx, \dots, ffx, t) \geq V(fx, gx, fx, gx, \dots, fx, \frac{t}{R})$ and $W(fgx, gfx, fgx, gfx, \dots, ffx, t) \leq W(fx, gx, fx, gx, \dots, fx, \frac{t}{R})$, for all $x \in X$ and $t > 0$.

Remark. If we interchange f and g in above definitions, then the pair of self-mappings (f, g) of the generalized intuitionistic fuzzy metric space $(X, V, W, *, \diamond)$ is said to be weakly commuting of type (J-1) and R-weakly commuting of type (J-1), respectively.

Lemma 3.12. *If f and g are weakly commuting of type (J) or R-weakly commuting of type (J), then f and g are weakly compatible.*

Proof. Let x be a coincidence point of f and g, i.e., $f(x) = g(x)$, then if the pair (f, g) is weakly commuting of type (J), we have $V(fgx, gfx, \dots, fgx, t) = V(fgx, gfx, \dots, ffx, t) \geq V(fx, gx, \dots, fx, t) = 1$ and $W(fgx, gfx, \dots, fgx, t) = W(fgx, gfx, \dots, ffx, t) \leq W(fx, gx, \dots, fx, t) = 0.$

It follows that $fgx = gfx$. Hence f and g commute at their coincidence point. Similary, if pair (f, g) is R-weakly commuting of type (J), we have $V(fgx, gfx, \dots, fgx, t) = V(fgx, gfx, \dots, ffx, t) \geq V(fx, gx, \dots, fx, \frac{t}{R}) = 1$ and $W(fgx, gfx, \dots, fgx, t) = W(fgx, gfx, \dots, ffx, t) \leq W(fx, gx, \dots, fx, \frac{t}{R}) = 0$. Thus $fgx = gfx$, then the pair (f, g) is weakly compatible.

The converse of the lemma needs not be true. □

Definition 3.13. A pair of self mappings (f, g) on X is said to satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for all $z \in X$.

Definition 3.14. A pair of self mappings (f, g) on X is said to satisfy the CLRg property if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gz$ for all $z \in X$.

4 Main results

In this section, we prove fixed point theorems for symmetric generalized intuitionistic fuzzy metric spaces.

Theorem 4.1. *Let $(X, V, W, *, \diamond)$ be a symmetric generalized intuitionistic fuzzy metric space and let mappings $f, g : X \to X$ satisfy the following conditions:*

(4.1.1) f and g are weakly commuting of type (J),

(4.1.2) $f(X) \subseteq g(X)$,

(4.1.3) $g(X)$ is a V and W complete subspace of X,

(4.1.4) there exists $\phi \in \Phi$ such that $x_i \in X$ and $i = 1, 2, \dots, n, t > 0$

$$V(fx_1, fx_2, \dots, fx_n, \phi(t)) \geq V(gx_1, gx_1, \dots, fx_1, t) * V(gx_2, gx_2, \dots, fx_2, t) * \cdots * V(gx_n, gx_n, \dots, fx_n, t)$$

and

$$W(fx_1, fx_2, \dots, fx_n, \phi(t)) \leq W(gx_1, gx_1, \dots, fx_1, t) \diamond W(gx_2, gx_2, \dots, fx_2, t) \diamond \cdots \diamond W(gx_n, gx_n, \dots, fx_n, t).$$

Then, f and g have a unique common fixed point.

Proof. Let $z_1 \in X$ such that $fz_0 = gz_1$ and $z_2 \in X$ such that $fz_1 = gz_2$, and then by induction we can define a sequence $\{y_n\} \in X$ as follows

$$y_n = fz_n = gz_{n+1}, \quad n \in N.$$

To prove that, $\{y_n\}$ is a Cauchy sequence in X.

$$\begin{aligned} V(y_n, y_n, \dots, y_n, y_{n+1}, \phi(t)) &= V(fz_n, fz_n, \dots, fz_n, fz_{n+1}, \phi(t)) \\ &\geq V(gz_n, gz_n, \dots, gz_n, fz_n, t) * V(gz_n, gz_n, \dots, gz_n, fz_n, t) \\ &\quad * \cdots * V(gz_{n+1}, gz_{n+1}, \dots, gz_{n+1}, fz_{n+1}, t) \\ &\geq V(gz_n, gz_n, \dots, gz_n, gz_{n+1}, t) * V(gz_n, gz_n, \dots, gz_n, gz_{n+1}, t) \\ &\quad * \cdots * V(gz_{n+1}, gz_{n+1}, \dots, gz_{n+1}, gz_{n+2}, t) \\ &\geq V(y_{n-1}, y_{n-1}, \dots, y_{n-1}, y_n, t) * V(y_{n-1}, y_{n-1}, \dots, y_{n-1}, y_n, t) \\ &\quad * \cdots * V(y_n, y_n, \dots, y_n, y_{n+1}, t) \end{aligned}$$

and

$$
\begin{aligned}
W(y_n, y_n, \ldots, y_n, y_{n+1}, \phi(t)) &= W(fz_n, fz_n, \ldots, fz_n, fz_{n+1}, \phi(t)) \\
&\le W(gz_n, gz_n, \ldots, gz_n, fz_n, t) \ \diamond W(gz_n, gz_n, \ldots, gz_n, fz_n, t) \\
&\quad \diamond \cdots \diamond W(gz_{n+1}, gz_{n+1}, \ldots, gz_{n+1}, fz_{n+1}, t) \\
&\le W(gz_n, gz_n, \ldots, gz_n, gz_{n+1}, t) \ \diamond W(gz_n, gz_n, \ldots, gz_n, gz_{n+1}, t) \\
&\quad \diamond \cdots \diamond W(gz_{n+1}, gz_{n+1}, \ldots, gz_{n+1}, gz_{n+2}, t) \\
&\le W(y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n, t) \ \diamond W(y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n, t) \\
&\quad \diamond \cdots \diamond W(y_n, y_n, \ldots, y_n, y_{n+1}, t).
\end{aligned}
$$

By Lemma 3.9, the sequence $\{y_n\}$ is a Cauchy sequence, since $y_n = gz_{n+1}$, $\{gz_{n+1}\}$ is a Cauchy sequence in $g(X)$.
By hypothesis $(4.1.3)$, $g(X)$ is complete, then there exists $u \in g(X)$ such that

$$\lim_{n\to\infty} gz_n = u = \lim_{n\to\infty} fz_n.$$

Now $u \in g(X)$, so there exists $p \in X$ such that $u = gp$.
Therefore,

$$\lim_{n\to\infty} gz_n = gp = \lim_{n\to\infty} fz_n.$$

To prove $fp = gp$.

$$
\begin{aligned}
&V(fp, fp, \ldots, fp, fz_n, \phi(t)) \\
&\qquad \ge V(gp, gp, \ldots, gp, fp, t) \ * V(gp, gp, \ldots, gp, fp, t) \ * \\
&\qquad\qquad \cdots * V(gp, gp, \ldots, gp, fp, t) \ * V(gz_n, gz_n, \ldots, gz_n, fz_n, t)
\end{aligned}
$$

and

$$
\begin{aligned}
&W(fp, fp, \ldots, fp, fz_n, \phi(t)) \\
&\qquad \le W(gp, gp, \ldots, gp, fp, t) \ \diamond W(gp, gp, \ldots, gp, fp, t) \ \diamond \\
&\qquad\qquad \cdots \diamond W(gp, gp, \ldots, gp, fp, t) \ \diamond W(gz_n, gz_n, \ldots, gz_n, fz_n, t).
\end{aligned}
$$

Taking limit as $n \to \infty$,

$$
\begin{aligned}
&V(fp, fp, \ldots, fp, gp, \phi(t)) \\
&\qquad \ge V(gp, gp, \ldots, gp, fp, t) \ * V(gp, gp, \ldots, gp, fp, t) \ * \\
&\qquad\qquad \cdots * V(gp, gp, \ldots, gp, fp, t) \ * V(gp, gp, \ldots, gp, gp, t)
\end{aligned}
$$

and

$$
\begin{aligned}
&W(fp, fp, \ldots, fp, gp, \phi(t)) \\
&\qquad \le W(gp, gp, \ldots, gp, fp, t) \ \diamond W(gp, gp, \ldots, gp, fp, t) \ \diamond \\
&\qquad\qquad \cdots \diamond W(gp, gp, \ldots, gp, fp, t) \ \diamond W(gp, gp, \ldots, gp, gp, t),
\end{aligned}
$$

which implies

$$V(fp, fp, \dots, fp, gp, \phi(t)) \geq V(gp, gp, \dots, gp, fp, t)$$

and

$$W(fp, fp, \dots, fp, gp, \phi(t)) \leq W(gp, gp, \dots, gp, fp, t).$$

Since the generalized intuitionistic fuzzy metric space is symmetric, we have

$$V(fp, fp, \dots, gp, \phi(t)) \geq V(gp, gp, \dots, gp, fp, t) = V(fp, fp, \dots, fp, gp, t)$$

and

$$W(fp, fp, \dots, gp, \phi(t)) \leq W(gp, gp, \dots, gp, fp, t) = W(fp, fp, \dots, fp, gp, t),$$

which implies $fp = gp$ (by Lemma 3.7).

Since the pair (f, g) is weakly commuting of type (J), then

$$V(fgp, gfp, fgp, \dots, ffp, \phi(t)) \geq V(fp, gp, fp, gp \dots, fp, t) = 1$$

and

$$W(fgp, gfp, fgp, \dots, ffp, \phi(t)) \leq W(fp, gp, fp, gp \dots, fp, t) = 0,$$

which implies

$$ffp = fgp = gfp = ggp.$$

Hence,

$$fu = fgp = gfp = gu.$$

Finally, to show that $u = gp$ is a common fixed point of f and g.

Suppose, $fu \neq u$, then

$$\begin{aligned} V(fu, fp, fp, \dots, fp, \phi(t)) \geq & V(gu, gu, \dots, fu, t) * V(gp, gp, \dots, fp, t) \\ & * \cdots * V(gp, gp, \dots, fp, t) \\ \geq & V(fu, fu, \dots, fu, t) * V(fp, fp, \dots, fp, t) \\ & * \cdots * V(fp, fp, \dots, fp, t) geq 1 * 1 * \cdots * 1 = 1 \end{aligned}$$

and

$$\begin{aligned} W(fu, fp, fp, \dots, fp, \phi(t)) \leq & W(gu, gu, \dots, fu, t) \diamond W(gp, gp, \dots, fp, t) \\ & \diamond \cdots \diamond W(gp, gp, \dots, fp, t) \\ \leq & V(fu, fu, \dots, fu, t) \diamond W(fp, fp, \dots, fp, t) \\ & \diamond \cdots \diamond W(fp, fp, \dots, fp, t) \leq 0 \diamond 0 \diamond \cdots \diamond 0 = 0, \end{aligned}$$

which is a contradiction.

Hence,

$$fu = gu = u.$$

To prove uniqueness, suppose u and v are such that $u \neq v, fv = gv = v$ and $fu = gu = u$, then again using condition $(4.1.4)$, we have

$$\begin{aligned} V(u, v, \dots, v, \phi(t)) &= V(fu, fv, \dots, fv, \phi(t)) \\ &\geq V(gu, gu, \dots, fu, t) * V(gv, gv, \dots, fv, t) * \dots * V(gv, gv, \dots, fv, t) \\ &= 1 * 1 * \dots * 1 = 1 \end{aligned}$$

and

$$\begin{aligned} W(u, v, \dots, v, \phi(t)) &= W(fu, fv, \dots, fv, \phi(t)) \\ &\leq W(gu, gu, \dots, fu, t) \diamond W(gv, gv, \dots, fv, t) \diamond \dots \diamond W(gv, gv, \dots, fv, t) \\ &= 0 \diamond 0 \diamond \dots \diamond 0 = 0. \end{aligned}$$

Hence, $V(u, v, \dots, v, \phi(t)) \geq 1$ and $W(u, v, \dots, v, \phi(t)) \leq 0$ which gives a contradiction.

Hence, $u = v$. Therefore, u is a unique common fixed point of f and g. $\square$

Example

Let $X = [0, 1]$ be a symmetric generalized intuitionistic fuzzy metric space. Let $\phi(t) = \frac{t}{2}$ and define $f, g : X \to X$ by $f(x) = \frac{x}{6}, \quad g(x) = \frac{x}{2}(x + 1), x \in X$. We see that $x = 0$ is the only coincidence point and f and g are weakly compatible.

Let $x_n = \frac{1}{n}$ be a sequence such that

$$V(fp, fp, \dots, fx_n, \phi(t)) \geq V(fp, fp, \dots, gp, t)$$

and

$$W(fp, fp, \dots, fx_n, \phi(t)) \leq W(fp, fp, \dots, gp, t),$$

where p is a coincidence point. Then the pair (f, g) is weakly commuting of type (J). Further f and g have a unique common fixed point of f and g.

Theorem 4.2. *Let $(X, V, W, *, \diamond)$ be a symmetric generalized intuitionistic fuzzy metric space and mappings $f, g : X \to X$ are weakly commuting of type (J) satisfying the following conditions:*

(4.2.1) f and g satisfy the property (E.A),

(4.2.2) $g(X)$ is a closed subspace of X,

(4.2.3) there exists $\phi \in \Phi$ such that $x_i \in X$ and $i = 1, 2, \dots, n, t > 0$

$$\begin{aligned} V(fx_1, fx_2, \dots, fx_n, \phi(t)) \geq\ & V(gx_1, gx_1, \dots, fx_1, t) * V(gx_2, gx_2, \dots, fx_2, t) \\ & * \dots * V(gx_n, gx_n, \dots, fx_n, t) \end{aligned}$$

and

$$\begin{aligned} W(fx_1, fx_2, \dots, fx_n, \phi(t)) \leq\ & W(gx_1, gx_1, \dots, fx_1, t) \diamond W(gx_2, gx_2, \dots, fx_2, t) \\ & \diamond \dots \diamond W(gx_n, gx_n, \dots, fx_n, t). \end{aligned}$$

Then f and g have unique common fixed point.

Proof. Since, the mappings f and g satisfy the property (E.A), then there exists a sequence $\{z_n\}$ in X satisfying $\lim_{n\to\infty} gz_n = u = \lim_{n\to\infty} fz_n$ for some $u \in X$.
Since $g(X)$ is a closed subspace of X and $\lim_{n\to\infty} gz_n = u$, then there exists $p \in X$ such that $gp = u$.

Also, $\lim\limits_{n\to\infty} gz_n = gp = \lim\limits_{n\to\infty} fz_n$.

To prove $fp = gp$.

$$\begin{aligned} &V(fp, fp, \ldots, fp, fz_n, \phi(t)) \\ &\qquad \geq V(gp, gp, \ldots, gp, fp, t) * V(gp, gp, \ldots, gp, fp, t) * \\ &\qquad\qquad \cdots * V(gp, gp, \ldots, gp, fp, t) * V(gz_n, gz_n, \ldots, gz_n, fz_n, t) \end{aligned}$$

and

$$\begin{aligned} &W(fp, fp, \ldots, fp, fz_n, \phi(t)) \\ &\qquad \leq W(gp, gp, \ldots, gp, fp, t) \diamond W(gp, gp, \ldots, gp, fp, t) \diamond \\ &\qquad\qquad \cdots \diamond W(gp, gp, \ldots, gp, fp, t) \diamond W(gz_n, gz_n, \ldots, gz_n, fz_n, t). \end{aligned}$$

Taking limit as $n \to \infty$,

$$\begin{aligned} &V(fp, fp, \ldots, fp, gp, \phi(t)) \\ &\qquad \geq V(gp, gp, \ldots, gp, fp, t) * V(gp, gp, \ldots, gp, fp, t) * \\ &\qquad\qquad \cdots * V(gp, gp, \ldots, gp, fp, t) * V(gp, gp, \ldots, gp, gp, t) \end{aligned}$$

and

$$\begin{aligned} &W(fp, fp, \ldots, fp, gp, \phi(t)) \\ &\qquad \leq W(gp, gp, \ldots, gp, fp, t) \diamond W(gp, gp, \ldots, gp, fp, t) \diamond \\ &\qquad\qquad \cdots \diamond W(gp, gp, \ldots, gp, fp, t) \diamond W(gp, gp, \ldots, gp, gp, t), \end{aligned}$$

which implies,

$$V(fp, fp, \ldots, fp, gp, \phi(t)) \geq V(gp, gp, \ldots, gp, fp, t)$$

and

$$W(fp, fp, \ldots, fp, gp, \phi(t)) \leq W(gp, gp, \ldots, gp, fp, t).$$

Since the generalized intuitionistic fuzzy metric space is symmetric, we have

$$V(fp, fp, \ldots, gp, \phi(t)) \geq V(gp, gp, \ldots, gp, fp, t) = V(fp, fp, \ldots, fp, gp, t)$$

and

$$W(fp, fp, \ldots, gp, \phi(t)) \leq W(gp, gp, \ldots, gp, fp, t) = W(fp, fp, \ldots, fp, gp, t),$$

which implies $fp = gp$ (by Lemma 3.7).

Since the pair (f, g) is weakly commuting of type (J), then

$$V(fgp, gfp, fgp, \ldots, ffp, \phi(t)) \geq V(fp, gp, fp, gp \ldots, fp, t) = 1$$

and

$$W(fgp, gfp, fgp, \ldots, ffp, \phi(t)) \leq W(fp, gp, fp, gp \ldots, fp, t) = 0,$$

which implies

$$ffp = fgp = gfp = ggp.$$

Hence,

$$fu = fgp = gfp = gu.$$

Finally, to show that $u = gp$ is a common fixed point of f and g.

Suppose, $fu \neq u$, then

$$\begin{aligned}
&V(fu,fp, fp, \ldots, fp, \phi(t)) \\
&\quad \geq V(gu, gu, \ldots, fu, t) \, * V(gp, gp, \ldots, fp, t) \, * \cdots * V(gp, gp, \ldots, fp, t) \\
&\quad \geq V(fu, fu, \ldots, fu, t) \, * V(fp, fp, \ldots, fp, t) \, * \cdots * V(fp, fp, \ldots, fp, t) \\
&\quad \geq 1 \, * 1 \, * \cdots * 1 = 1
\end{aligned}$$

and

$$\begin{aligned}
&W(fu,fp, fp, \ldots, fp, \phi(t)) \\
&\quad \leq W(gu, gu, \ldots, fu, t) \, \diamond W(gp, gp, \ldots, fp, t) \, \diamond \cdots \diamond W(gp, gp, \ldots, fp, t) \\
&\quad \leq V(fu, fu, \ldots, fu, t) \, \diamond W(fp, fp, \ldots, fp, t) \, \diamond \cdots \diamond W(fp, fp, \ldots, fp, t) \\
&\quad \leq 0 \, \diamond 0 \, \diamond \cdots \diamond 0 = 0.
\end{aligned}$$

which is a contradiction.

Hence,

$$fu = gu = u.$$

To prove uniqueness, suppose u and v are such that $u \neq v$, $fv = gv = v$ and $fu = gu = u$, then again using condition $(4.2.4)$, we have

$$\begin{aligned}
V(u, v, \ldots, v, \phi(t)) &= V(fu, fv, \ldots, fv, \phi(t)) \\
&\geq V(gu, gu, \ldots, fu, t) \, * V(gv, gv, \ldots, fv, t) \, * \cdots * V(gv, gv, \ldots, fv, t) \\
&= 1 * 1 * \cdots * 1 = 1
\end{aligned}$$

and

$$\begin{aligned}
W(u, v, \ldots, v, \phi(t)) &= W(fu, fv, \ldots, fv, \phi(t)) \\
&\leq W(gu, gu, \ldots, fu, t) \diamond W(gv, gv, \ldots, fv, t) \diamond \cdots \diamond W(gv, gv, \ldots, fv, t) \\
&= 0 \diamond 0 \diamond \cdots \diamond 0 = 0.
\end{aligned}$$

Hence. $V(u, v, \ldots, v, \phi(t)) \geq 1$ and $W(u, v, \ldots, v, \phi(t)) \leq 0$, which gives a contradiction.

Hence, $u = v$. Therefore, u is the unique common fixed point of f and g. □

Theorem 4.3. *Let $(X, V, W, *, \diamond)$ be a symmetric generalized intuitionistic fuzzy metric space and the mappings $f, g : X \to X$ are weakly commuting of type (J) satisfying the following conditions:*

(4.3.1) f and g satisfy the CLRg property,

(4.3.2) $g(X)$ is a closed subspace of X,

(4.3.3) there exists $\phi \in \Phi$ such that $x_i \in X$ and $i = 1, 2, \ldots, n, t > 0$

$$V(fx_1, fx_2, \ldots, fx_n, \phi(t)) \geq V(gx_1, gx_1, \ldots, fx_1, t) * V(gx_2, gx_2, \ldots, fx_2, t) * \cdots * V(gx_n, gx_n, \ldots, fx_n, t) \quad and$$
$$W(fx_1, fx_2, \ldots, fx_n, \phi(t)) \leq W(gx_1, gx_1, \ldots, fx_1, t) \diamond W(gx_2, gx_2, \ldots, fx_2, t) \diamond \cdots \diamond W(gx_n, gx_n, \ldots, fx_n, t).$$

Then, f and g have a unique common fixed point.

Proof. The proof follows on the same lines of Theorem 4.1 and by definition of CLRg property. □

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