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ON THE FIRST TYPE OF INTUITIONISTIC FUZZY GENERALIZED NETS

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Abstract: Generalized Nets (GNs) and intuitionistic fuzzy logic are briefly reminded. Three versions of the First type of an intuitionistic fuzzy GN are defined and some of their properties are discussed.

1 Introduction

Generalized Nets (GNs) are extensions of Petri nets and Petri net modifications and extensions. They were defined in 1982 (see [3]).

Intuitionistic Fuzzy Sets (IFSs), defined in 1983, are extensions of fuzzy sets (see [5]). They have two degrees - degree of membership (μ) and degree of non-membership (ν) such that their sum can be smaller that 1, i.e., a third degree - of uncertainty ($\pi = 1-\mu-\nu$) - can be defined, too. A variety of operations, relations and operators (from modal, topological and others types) are defined over the IFSs.

GNs have so far over 20 extensions. The first one, proposed in 1985 (see [1]), was called Intuitionistic Fuzzy GN (IFGN). The transition condition predicates of these nets are estimated in intuitionistic fuzzy sense. Later, this extension was called IFGN of type 1, because IFGN of a second type was defined. Two other extensions are described in [6].

2 Short introduction to Generalized Nets (GNs)

Since 1983 more than 600 papers being related to the concept of the GNs have been published. A part of them is included in the bibliography [7].

GNs are defined as extensions of the ordinary Petri nets and their modifications, but in a way that is principaly different from the ways of defining the other types of Petri nets. The additional components in the GN-definition provide more and greater modelling possibilities and determine the place of the GNs among the individual types of Petri nets, similar to the place of the Turing machine among the finite automata.

The first basic difference between GNs and the ordinary Petri nets is the "place – transition" relation [8]. Here, the transitions are objects of a more complex nature. A transition may contain m input and n output places where $m, n \geq 1$.

Formally, every transition is described by a seven-tuple (Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

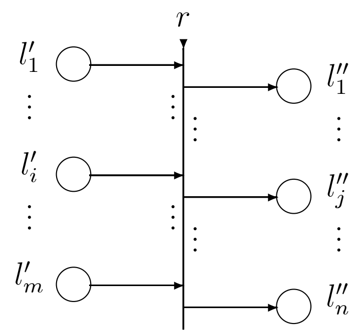

Fig. 1

(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Fig. 1 these are $L' = \{l'_1, l'_2, \ldots, l'_m\}$ and $L'' = \{l''_1, l''_2, \ldots, l''_n\}$;

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will pass (or *transfer*) from the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [2]):

$$r = \begin{array}{c|c} & l''_1 \;\ldots\; l''_j \;\ldots\; l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \;\; - \;\; \text{predicate}\ ) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array} \quad ;$$

$r_{i,j}$ is the predicate that corresponds to the i-th input and j-th output places. When its truth value is "*true*", a token from i-th input place transfers to j-th output place; otherwise, this is not possible;

(e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \geq 0 - \text{ natural number }) \\ \vdots & (1 \leq i \leq m, 1 \leq j \leq n) \\ l'_m & \end{array} ;$$

(f) $\square$ is an object of a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places, and $\square$ is an expression built up from variables and the Boolean connectives $\wedge$ and $\vee$ whose semantics is defined as follows:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,
$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – there must be at least one token in all places $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "*true*", the transition can become active, otherwise it cannot.

The ordered four-tuple

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^o, t^* \rangle, \langle X, \Phi, b \rangle \rangle$$

is called a *Generalized Net* (GN) if:

(a) A is a set of transitions;

(b) π_A is a function giving the priorities of the transitions, i.e., $\pi_A : A \to N$, where $N = \{0, 1, 2, \ldots\} \cup \{\infty\}$;

(c) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to N$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in N, n \geq 1$ and $1 \leq k \leq n$ (obviously, L is the set of all GN-places);

(d) c is a function giving the capacities of the places, i.e., $c : L \to N$;

(e) f is a function that calculates the truth values of the predicates of the transition's conditions (for the GN described here let the function f have the value "$false$" or "$true$", i.e., a value from the set $\{0, 1\}$;

(f) θ_1 is a function giving the next time-moment when a given transition Z can be activated, i.e., $\theta_1(t) = t'$, where $pr_3 Z = t, t' \in [T, T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when the transition terminates its functioning;

(g) θ_2 is a function giving the duration of the active state of a given transition Z, i. e., $\theta_2(t) = t'$, where $pr_4 Z = t \in [T, T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts its functioning;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider it as a set of the form

$$K = \bigcup_{l \in Q^I} K_l \ ,$$

where K_l is the set of tokens that enter the net from place l, and Q^I is the set of all input places of the net;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \to N$;

(j) θ_K is a function giving the time-moment when a given token can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the time-moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale;

(l) t^o is an elementary time-step, related to the fixed (global) time-scale;

(m) t^* is the duration of the GN functioning;

(n) X is the set of all initial characteristics the tokens can receive on entering the net;

(o) Φ is a characteristic function that assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition.

(p) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \to N$.

A given GN may lack some of the above components. In these cases, any missing component will be omitted. The GNs of this kind form a special class of GNs called *"reduced GNs"*.

The definition of a GN is more complex than the definition of a Petri net. Thus the algorithms of the tokens' transfer in the GNs are also more complex. On the other hand, as the GNs are more general, the algorithms for movement of tokens in the GN are more general than those of Petri nets. In a Petri net implementation, parallelism is reduced to a sequential firing of its transitions and the order of their activation in the general case is probabilistic or dependent on the transitions' priorities, if such exist. The GN's algorithms provide a means for a more detailed modelling of the described process. The algorithms for the token's transfer take into account the priorities of the places, transitions and tokens, i. e., they are more precise.

Operations and relations are defined as over the transitions, as well as over the GNs in general.

The operations, defined over the GNs - "union", "intersection", "composition" and "iteration" (see [3]) do not exist anywhere else in the Petri net theory. They can be transferred to virtually all other types of Petri nets (obviously with some modifications concerning the structure of the corresponding nets). These operations are useful for constructing GN models of real processes.

In [3] different properties of the operations over transitions and GNs are formulated and proved. Certain relations over transitions and GNs are also introduced there.

Now, the operator aspect has an important place in the theory of GNs. Six types of operators are defined in its framework. Every operator assigns to a given GN a new GN with some desired properties. The comprised groups of operators are:
• global ($G-$) operators,
• local ($P-$) operators,
• hierarchical ($H-$) operators,
• reducing ($R-$) operators,
• extending ($O-$) operators,
• dynamic ($D-$) operators.

The *global operators* transform, according to a definite procedure, a whole given net or all its components of a given type.

The second type of operators are local operators. They transform single components of some of the transitions of a given GN.

The third type of operators are the *hierarchical operators.*They are of five different types and fall into two groups according to their way of action:
• expanding a given GN (H_1, H_3, H_5 and H_6),
• shrinking a given GN (H_2, H_4 and H_5),
The H_5 operator can be expanding as well as shrinking, depending on its form. According to their object of action the operators fall again into three groups:
• acting upon or giving as a result a place (H_1 and H_2),
• acting upon or giving as a result a transition (H_3 H_4 and H_5),
• acting upon a token (H_6).

The next (fourth) group of operators defined over the GNs produce a new, reduced GN from a given net. They would allow the construction of elements of the classes of reduced GNs. To find the place of a given Petri net modification among the classes of reduced GNs, it must be compared to some reduced GN obtained by an operator of this type. These operators are called *reducing operators.*

Operators from the fifth group extend a given GN. These operators are called *extending operators.* The extending operators are associated with every one of the GN extensions.

Finally, the operators from the last - sixth - group are related to the ways the GN functions, so that they are called *dynamic operators.* These are the following:
• operators $D(1,i)$ that determine the procedure of evaluating the transition condition predicates ($1 \leq i \leq 18$);
• operators governing token splitting: one that allows ($D(2,1)$) and one that prohibits splitting ($D(2,2)$), respectively; and operators governing the union of tokens having a

common predecessor: an allowing one ($D(2,4)$) and a prohibiting one ($D(2,3)$);
• operators that determine the strategies of the tokens transfer: one by one at a time vs. all in groups (the operator $D(3,2)$; the operator $D(3,1)$ does not allow this);
• operators related to the ways of evaluating the transition condition predicates: predicate checking ($D(4,1)$); changing the predicates by probability functions with corresponding forms ($D(4,2)$); expert estimations of predicate values ($D(4,3)$); predicates depending on solutions of optimization problems (e.g., transportation problem) ($D(4,4)$).

The operators of different types, as well as the others that can be defined, have a major theoretical and practical value. On the one hand, they help us study the properties and the behaviour of GNs. On the other hand, they facilitate the modelling of many real processes. The basic properties of the operators are discussed in [3, 4].

3 The basics of Intuitionistic Fuzzy Logic (IFL)

To each proposition (in the classical sense) we can assign its truth value: truth – denoted by 1, or falsity – 0. In the case of fuzzy logic this truth value is a real number in the interval $[0,1]$ and may be called "truth degree" of a particular proposition. Here we add one more value – "falsity degree" – which will be in the interval $[0,1]$ as well. Thus two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition p with the following constraint to hold (see [5]):

$$\mu(p) + \nu(p) \leq 1.$$

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

For the needs of the discussion below we shall define the notion of *intuitionistic fuzzy tautology* (IFT) through:

"A is an IFT" if and only if, if $V(A) = \langle a, b \rangle$, then $a \geq b$.

4 Short remarks on the extensions of the different types of IFGNs

First, we shall note that the GNs have more than 20 conservative extensions. An ordinary GN can represent the functioning and the results of work of each of these extensions.

The first type of GN-extensions, related to the IFSs, is **Intuitionistic fuzzy GN of type 1 (IFGN1)**. Its transition condition predicates are evaluated over the set $[0,1]^2$

with a degree of truth (μ) and a degree of falsity (ν) for which $\mu + \nu \leq 1$ (see [3]). If $r_{i,j}$ is the predicate, corresponding to the condition for a token transfer from i-th input to j-th output place, then this transfer is possible, if

$$\mu(r_{i,j}) > 0 \;\text{ or }\; \nu(r_{i,j}) < 1.$$

Now, we shall introduce another rule for token transfer. It corresponds to the idea for the IFT and has the form: if $r_{i,j}$ is the predicate, corresponding to the condition for a token transfer from i-th input to j-th output place, then the transfer is possible, if

$$\mu(r_{i,j}) \geq \nu(r_{i,j}).$$

Third modification is the following. Let the two constants $\alpha, \beta \in [0, 1]$ be given, so that $\alpha \geq \beta$. The rule has the form: if $r_{i,j}$ is the predicate, corresponding to the condition for a token transfer from i-th input to j-th output place, then the transfer is possible, if

$$\mu(r_{i,j}) \geq \alpha \;\text{ and }\; \nu(r_{i,j}) \leq \beta.$$

It is obvious that if a token can transfer following the third rule (for given α, β satisfying the above conditions, then the token will transfer from the same input and to the same output places, following the second rule, because from

$$\mu(r_{i,j}) \geq \alpha \geq \beta \geq \nu(r_{i,j})$$

it follows that $\mu(r_{i,j}) \geq \nu(r_{i,j})$, i.e., $\langle \mu(r_{i,j}), \nu(r_{i,j}) \rangle$ is an IFT.

On the other hand, if $\langle \mu(r_{i,j}), \nu(r_{i,j}) \rangle$ is an IFT, then

$$\mu(r_{i,j}) \geq \nu(r_{i,j}) \geq 0,$$

and

$$1 \geq 1 - \mu(r_{i,j}) \geq \nu(r_{i,j}).$$

Of course, if the third rule can be realized, then the first one can be realized, too.

In [3] some logical operator are defined over the class of all GNs. Here we shall modify them in the following form. Let place l that is input place of transition Z be fixed. We can note this fact by $l \in pr_1 Z$ and the predicate between places l and l' - by $r_{l,l'}$. We define the sets

$$\pi_1(l) = \{l' \;/\; (l' \in pr_2 Z) \;\&\; (\mu(r_{l,l'}) > 0 \;\&\; \nu(r_{l,l'}) < 1)\}.$$

$$\pi_2(l) = \{l' \;/\; (l' \in pr_2 Z) \;\&\; (\mu(r_{l,l'}) \geq \nu(r_{l,l'}))\}.$$

When the real numbers $\alpha, \beta \in [0, 1]$, satisfying the above conditions, are given and the third rule is use, then we define the set

$$\pi_3(l) \equiv \pi_{3,\alpha,\beta}(l) = \{l' \;/\; (l' \in pr_2 Z) \;\&\; (\mu(r_{l,l'}) \geq \alpha \;\&\; \nu(r_{l,l'}) \leq \beta)\}.$$

Let L_E be the set of all places of GN E, Q_E^I and Q_E^O be the input and output places of E.

The following assertion is valid.

Theorem 1. For every $l \in L_E - Q_E^O$ and for every $\alpha, \beta \in [0, 1]$:

$$\pi_2(l) \subset \pi_1(l), \tag{1}$$

$$\pi_3(l) \subset \pi_2(l). \tag{2}$$

Proof. Let $m \in \pi_2(l)$. Therefore, $l \in pr_1 Z$, $m \in pr_2 Z$ and for

$$f(r_{l,m}) = \langle \mu(r_{l,m}), \nu(r_{l,m}) \rangle$$

is valid inequality

$$\mu(r_{l,m}) \geq \nu(r_{l,m}).$$

If $\nu(r_{l,m}) = 0$, then $\mu(r_{l,m}) \geq 0$ and $\nu(r_{l,m}) < 1$, i.e., the first rule is valid, too. Hence $m \in \pi_1(l)$.

If

$$0 < \nu(r_{l,m}) \leq \mu(r_{l,m}) < 1,$$

then again we obtain that $m \in \pi_1(l)$. Therefore, inclusion (1) is valid.

Inclusion (2) is proved by analogy.

Let us define for $l \in Q_E^I$ and for i-th rule for $i = 1, 2$:

$$\Pi_i(l) = \{l' \ / \ (l' \in Q_E^O) \ \& \ (\exists l_1, l_2, ..., l_s \in L_E)(l_1 = l \& l_s = l'$$

$$\&(\forall j : 2 \leq j \leq s)(l_j \in \pi_i(l_{j-1})))\}.$$

When the real numbers $\alpha, \beta \in [0, 1]$ are given and the third rule is use, then we define the set

$$\Pi_3(l) \equiv \Pi_{3,\alpha,\beta}(l) = \{l' \ / \ (l' \in Q_E^O) \ \& \ (\exists l_1, l_2, ..., l_s \in L_E)(l_1 = l \& l_s = l'$$

$$\&(\forall j : 2 \leq j \leq s)(l_j \in \pi_{3,\alpha,\beta}(l_{j-1})))\}.$$

Similarly to above, we can prove the following

Theorem 2. For every $l \in Q_E^I$ and for every $\alpha, \beta \in [0, 1]$:

$$\Pi_2(l) \subset \Pi_1(l),$$

$$\Pi_3(l) \subset \Pi_2(l).$$

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