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INTUITIONISTIC FUZZY INTERPRETATION OF FRAME-STRUCTURES [1]

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ABSTRACT: A intuitionistic fuzzy interpretation of the expert knowledge-based estimations of the validity of frames is discussed.

KEYWORDS: Estimation, Expert knowledge, Intuitionistic fuzzy set

Introduction

Assume we are given a set of objects and a finite number of criteria. We are interested in finding out the degree to which each object fulfills every criterion. To do this, we collect the opinions of a group of experts about the objects; also, each expert has her own rating with respect to each criterion.

This work presents a way to build an estimation of every object by each criterion, given the experts' ratings and opinions.

Procedure

Let us denote the patterns (objects) by $P_1, P_2, ..., P_p$, the criteria by $C_1, C_2, ..., C_c$, the experts by $E_1, E_2, ..., E_e$, where p, c, e are natural numbers. Let $i-$th expert have a rating of $\langle \rho_{i,j}, \sigma_{i,j} \rangle$ with respect to the $j-$th criterion and let the estimation of $i-$th expert for $k-$th object be $\langle \varepsilon_{i,k}, \eta_{i,k} \rangle$.

Everywhere below $1 \leq i \leq e, 1 \leq j \leq c, 1 \leq k \leq p$, $\rho_{i,j}, \sigma_{i,j}, \varepsilon_{i,k}, \eta_{i,k} \in [0,1]$, and $0 \leq \rho_{i,j} + \sigma_{i,j} \leq 1$, $0 \leq \varepsilon_{i,k} + \eta_{i,k} \leq 1$, i.e., $\langle \rho_{i,j}, \sigma_{i,j} \rangle$, and $\langle \varepsilon_{i,k}, \eta_{i,k} \rangle$ are intuitionistic fuzzy couples in the sense of [1,2].

Now we can construct the Index Matrices (IM, see [3]):

	C_1	...	C_j	...	C_c
E_1					
⋮					
E_i			$\langle \rho_{i,j}, \sigma_{i,j} \rangle$		
⋮			$(1 \leq i \leq e, 1 \leq j \leq c)$		
E_e					

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	P_1	...	P_k	...	P_p
E_1					
⋮					
E_i			$\langle \varepsilon_{i,k}, \eta_{i,k} \rangle$		
⋮	$(1 \leq i \leq e, 1 \leq j \leq c)$				
E_e					

The first one corresponds to the expert's ratings and the second one – to the expert's estimations.

We can modify the first IM to the transposed form:

	E_1	...	E_i	...	E_e
C_1					
⋮					
C_j			$\langle \rho_{j,i}, \sigma_{j,i} \rangle$		
⋮	$(1 \leq j \leq c, 1 \leq i \leq e)$				
C_c					

and construct the IM

	P_1	...	P_k	...	P_p
C_1					
⋮					
C_j			$\langle \alpha_{j,k}, \beta_{j,k} \rangle$		
⋮	$(1 \leq j \leq c, 1 \leq k \leq p)$				
C_c					

$$= \begin{array}{|c|c|}\hline & E_1 \ \ldots \ E_i \ \ldots \ E_e \\ \hline C_1 & \\ \vdots & \\ C_j & \langle \rho_{j,i}, \sigma_{j,i} \rangle \\ \vdots & (1 \leq j \leq c, 1 \leq i \leq e) \\ C_c & \\ \hline \end{array} + \begin{array}{|c|c|}\hline & P_1 \ \ldots \ P_k \ \ldots \ P_p \\ \hline E_1 & \\ \vdots & \\ E_i & \langle \varepsilon_{i,k}, \eta_{i,k} \rangle \\ \vdots & (1 \leq i \leq e, 1 \leq j \leq c) \\ E_e & \\ \hline \end{array}$$

where

$$\alpha_{j,k} = \frac{\sum\limits_{i=1}^{e} 2\rho_{j,i}\varepsilon_{i,k} + 1 - \sigma_{j,i}\eta_{i,k}}{3e} \tag{1}$$

and

$$\beta_{j,k} = \frac{\sum\limits_{i=1}^{e} 2\sigma_{j,i}.\eta_{i,k} + 1 - \rho_{j,i}.\varepsilon_{i,k}}{3e}. \tag{2}$$

We call $\langle \alpha_{j,k}, \beta_{j,k} \rangle$ the intuitionistic fuzzy estimation of the j-th object with respect to the k-th criterion.

To convince ourselves that the definition is correct, observe that $\alpha_{j,k} \geq 0$, $\beta_{j,k} \geq 0$ and

$$\alpha_{j,k} + \beta_{j,k} = \sum_{i=1}^{e} \frac{2 + \rho_{j,i}\varepsilon_{i,k} + \sigma_{j,i}\eta_{i,k}}{3e} \leq \sum_{i=1}^{e} \frac{2 + \rho_{j,i} + \sigma_{j,i}}{3e} \leq 1, \tag{3}$$

therefore $\langle \alpha_{j,k}, \beta_{j,k} \rangle$ form an intuitionistic fuzzy couple.

Frames and indexed matrices

In fact, the IM

$$S = \begin{array}{c|c} & P_1 \ \ldots \ P_k \ \ldots \ P_p \\ \hline C_1 & \\ \vdots & \\ C_j & \langle \alpha_{j,k}, \beta_{j,k} \rangle \\ \vdots & (1 \leq j \leq c, 1 \leq k \leq p) \\ C_c & \end{array}$$

is a more abstract representation of a frame structure.

If we have a single object P_1, the IM S degenerates to

$$S' = \begin{array}{c|c} & P \\ \hline C_1 & \\ \vdots & \\ C_j & \langle \alpha_j, \beta_j \rangle \\ \vdots & (1 \leq j \leq c) \\ C_c & \end{array}$$

which can be directly related to a frame having c slots $C_1, \ldots, C_c$ and values $\langle \alpha_j, \beta_j \rangle$ for $(1 \leq j \leq c)$, all of which are intuitionistic fuzzy couples.

Given an IM of the kind of S', we define new criteria related to P. For an object P_i, let

$$C_j^P(P_i) = \text{“the object } P \text{ fulfills criterion } C_j \text{ better than the object } P_i\text{”}$$

Going now back to the set of p objects $P_1, \ldots, P_p$, we can construct the IM

$$S^P = \begin{array}{c|c} & P_1 \ \ldots \ P_k \ \ldots \ P_p \\ \hline C_1^P & \\ \vdots & \\ C_j^P & \langle \alpha_{j,k}^P, \beta_{j,k}^P \rangle \\ \vdots & (1 \leq j \leq c, 1 \leq k \leq p) \\ C_c^P & \end{array}$$

Now we can apply the procedure described in the previous section in such a way that a target IM of the above kind is obtained for a given P.

Thus, our procedure can be interpreted as construction of an intuitionistic fuzzy estimation of the validity of a frame structure.

Conclusion

The results are a continuation of the work published in [4,5]. The approach proposed here can be useful, in particular, in application areas where much depends on the expert's experience, and where experts' opinions differ significantly, such as expert estimations of art work.

As it is seen, this time we focus on estimating the validity of objects with respect to given criteria, using expert's opinions. In a subsequent work we plan to study how this can be used further to recalculate experts' ratings, depending on the plausibility of their initial opinions.

References

[1] Atanassov K., Intuitionistic fuzzy sets, Fuzzy sets and Systems Vol. 20 (1986), No. 1, 87-96.

[2] Atanassov K., Intuitionistic Fuzzy Sets, Springer-Verlag, Heidelberg, 1999.

[3] Atanassov K., Generalized index matrices, Comptes rendus de l'Academie Bulgare des Sciences, vol.40, 1987, No.11, 15-18.

[4] Atanassov K., Intuitionistic fuzzy sets and expert estimations, BUSEFAL, Vol. 55, 1993, 67-71.

[5] Atanassov K., Intuitionistic fuzzy sets and expert estimations. II, BUSEFAL, Vol. 59, 1994, 64-69.