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Solving intuitionistic fuzzy differential equations with linear differential operator by Adomian decomposition method

Suvankar Biswas*, Sanhita Banerjee and Tapan Kumar Roy

Department of Mathematics
Indian Institute of Engineering Science and Technology
Shibpur, Howrah-711103, West Bengal, India
e-mail: suvo180591@gmail.com
* *Corresponding author*

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Abstract: In this paper we have taken the intuitionistic fuzzy differential equation with linear differential operator. Adomian decomposition method (ADM) has been used to find the approximate solution. We have given two numerical examples and by comparing the numerical results obtain from ADM with the exact solution, we have studied their accuracy.
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1 Introduction

Fuzzy and intuitionistic fuzzy differential equations are very useful to model dynamical systems whose uncertainty is characterized by a non-random process [9]. Fuzzy differential equations have a large area of application in civil engineering [37], biology [38–40], physics [41], medical science [42, 43] and so many other fields of science [44, 45]. So the existence and uniqueness of the fuzzy differential equation is an area of great interest and this has been studied in [10, 13, 14, 19, 21, 23]. Various kind of fuzzy differential equation and their application have been studied by many researchers. Bede et al. interpret first order linear fuzzy differential equations by using the strongly generalized differentiability concept [11]. Chalco-Cano, Roman-Flores study the class of first order fuzzy differential equations where the dynamics is given by a continuous fuzzy mapping which is obtain via Zadeh's extension principle [12]. Solutions of first order fuzzy differential equations have been also studied in [15, 17, 18, 20, 22, 24–26]. An extension of differential transformation method using the concept of generalized H-differentiability has been studied by Allahviranloo et al. in [16]. But most of them have studied only first or second order fuzzy differential equation.

The concept of fuzzy set theory has been extended into intuitionistic fuzzy set (IFS) theory by Atanassov [27–29]. There are huge applications of IFS in different fields of science like medical diagnosis [31], microelectronic fault analysis [32], pattern recognition [33], decision-making problems [34, 35], drug selection [36] and etc. The studies on improvement of IFS theory, together with intuitionistic fuzzy geometry, intuitionistic fuzzy logic, intuitionistic fuzzy topology, an intuitionistic fuzzy approach to artificial intelligence, and intuitionistic fuzzy generalized nets have been given in [30].

There are only few work have been done on intuitionistic fuzzy differential equation (IFDE) [46–49] although like fuzzy differential equations there are a large area of possible applications of IFDE in civil engineering, biology, physics, medical science and so many other fields of science. V. Nirmala and S. C. Pandian have studied Intuitionistic fuzzy differential equation with initial condition by using Euler method [46]. The existence and uniqueness of a solution of the intuitionistic fuzzy differential equation using the method of successive approximation has been discussed by R. Ettoussi et al. [47]. The existence and uniqueness theorem of a solution to the nonlocal intuitionistic fuzzy differential equation using the concept of intuitionistic fuzzy semigroup and the contraction mapping principle has been given by S. Melliani [48]. A system of differential equation of first order with initial value as triangular intuitionistic fuzzy number has been solved in [49].

The Adomian Decomposion Method (ADM) was first introduced by Adomian in 1980 [1]. ADM is very powerful tool to solve algebraic, differential, integral and integro-differential equations involving non-linear functional [2–5]. A second-order fuzzy differential equation has been solved by using Adomian method under strongly generalized differentiability in [8]. Using ADM hybrid fuzzy differential equations have been solved in [7]. Numerical approximation of fuzzy first-order initial value problem by using ADM is presented in [6]. Our study is the first attempt to solve intuitionistic fuzzy differential equation with linear differential operator of any order. In this paper, we develop numerical method for intuitionistic fuzzy differential equations with linear differential operator by an application of the ADM. The structure of this paper is organized as follows: Section 2 contains some basic definitions of intuitionistic fuzzy sets and intuitionistic fuzzy number. Section 3 contains solution procedure of intuitionistic fuzzy differential equations with linear differential operator by ADM. In section 4, the proposed method is illustrated by two numerical examples and we compare ADM solution with exact solution to check the accuracy of the method. Finally the conclusion and future research is given in section 5.

2 Preliminaries

Definition 2.1. (see [1]). If X is a collection of objects denoted by x then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs denoted and defined by: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$, were $\mu_{\tilde{A}}(x)$ is called membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$ which maps X to [0,1].

Definition 2.2. [1]. α-cut of a fuzzy $\tilde{A}$ set is a crisp set A_α and defined by A_α or $\tilde{A}\ [\alpha] = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}$, where $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$.

Definition 2.3. A fuzzy set $\tilde{A}$ is said to be convex fuzzy set if A_α is a convex set for all $\alpha \in (0,1]$.

Definition 2.4. A fuzzy set $\tilde{A}$ is said to be normal fuzzy set if there exist an element $(a, 1) \in \tilde{A}$.

Definition 2.5. If a fuzzy set is convex, normalized and its membership function, defined in ℝ, is piecewise continuous then it is called as fuzzy number.

A triangular fuzzy number $\tilde{A}$ is denoted by $(a_1,\ a_2,\ a_3)$ and it is a fuzzy set $\{(x, \mu_{\tilde{A}}(x))\}$ where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, a_2 \leq x \leq a_3 \\ 0, \quad \text{otherwise} \end{cases}. \ \tilde{A}$$

is called positive triangular fuzzy number if $a_1 > 0$ and negative triangular fuzzy number if $a_3 < 0$.

Definition 2.6. [18]. Let E be the set of all upper semicontinuous normal convex fuzzy numbers with bounded α-cut intervals. It means if $\tilde{v} \in E$ then the α-cutset is a closed bounded interval which is denoted by $v_{\ \alpha} = [v_1, v_2]$. For arbitrary $u_{\ \alpha} = [u_1, u_2]$,$v_{\ \alpha} = [v_1, v_2]$ and $k \geq 0$, addition $(u_{\ \alpha} + v_{\ \alpha})$ and multiplication by k are defined as $(u + v)_1(\alpha) = u_1(\alpha) + v_1(\alpha)$, $(u + v)_2(\alpha) = u_2(\alpha) + v_2(\alpha)$, $(ku)_1(\alpha) = ku_1(\alpha)$, $(ku)_2(\alpha) = ku_2(\alpha)$.

Since each $y \in R$ can be regarded as a fuzzy number $\tilde{y}$ defined by

$$\mu_{\tilde{y}}(x) = \begin{cases} 1 \ \ if \ \ x = y \\ 0 \ \ if \ x \neq y \end{cases},$$

the Hausdorff distance between fuzzy numbers given by $D : E \times E \rightarrow R_+ \cup \{0\}$,

$$D(\tilde{u}, \tilde{v}) = \sup_{\alpha \in [0,1]} \operatorname{Max}\{|u_1(\alpha) - v_1(\alpha)|, |u_2(\alpha) - v_2(\alpha)|\}.$$

It is easy to see that D is a metric in E and has the following properties (see [19])

(i) $D(\tilde{u} \oplus \tilde{w}, \tilde{v} \oplus \tilde{w}) = D(\tilde{u}, \tilde{v}), \forall\ \tilde{u}, \tilde{v}, \tilde{w} \in E$,

(ii) $D(k \odot \tilde{u}, k \odot \tilde{v}) = |k| D(\tilde{u}, \tilde{v}), \forall k \in R,\ \tilde{u}, \tilde{v} \in E$,

(iii) $D(\tilde{u} \oplus \tilde{v}, \tilde{w} \oplus \tilde{e}) \leq D(\tilde{u}, \tilde{w}) + D(\tilde{v}, \tilde{e}), \forall\ \tilde{u}, \tilde{v}, \tilde{w}, \tilde{e} \in E$,

(iv) (D, E) is a complete metric space.

Definition 2.7. (see [15]). Let $f{:}R \rightarrow E$ be a fuzzy valued function. If for arbitrary fixed $t_0 \in R$ and $\epsilon > 0$, a $\delta > 0$ such that $|t - t_0| < \delta \Longrightarrow \mathrm{D}(f(t), f(t_0)) < \epsilon$, f is said to be continuous.

Definition 2.8. Let $U = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. An intuitionistic fuzzy set $\tilde{A}^i$ in a given universal set U is an object having the form

$$\tilde{A}^i = \{\langle x_i, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle : x_i \in U\},$$

where the functions

$$\mu_{\tilde{A}^i}: U \to [0,1]; \text{ i.e. }, x_i \in U \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$$

and

$$\upsilon_{\tilde{A}^i}: U \to [0,1]; \text{ i.e. }, x_i \in U \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$$

define the degree of membership and the degree of non-membership of an element $x_i \in U$, such that they satisfy the following conditions:

$$0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1, \forall x_i \in U$$

which is known as intuitionistic fuzzy condition. The degree of acceptance $\mu_{\tilde{A}^i}(x_i)$and of non-acceptance $\upsilon_{\tilde{A}^i}(x_i)$ can be arbitrary.

Definition 2.9. A set of (α, β)-cut, generated by IFS $\tilde{A}^i$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\tilde{A}^i{}_{\alpha,\beta} = \begin{cases} \left(x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x)\right); & x \in U \\ \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta; & \alpha, \beta \in [0,1] \end{cases},$$

where (α, β)-cut, denoted by $\tilde{A}^i{}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\tilde{A}^i$ at least to the degree α and which does belong to $\tilde{A}^i$ at most to the degree β.

Definition 2.10. An intuitionistic fuzzy number $\tilde{A}^i$ is

i. An intuitionistic fuzzy subset on the real line

ii. Normal i.e. there exists at least one $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\upsilon_{\tilde{A}^i}(x_0) = 0$)

iii. Convex for the membership function $\mu_{\tilde{A}^i}$ i.e.

$$\mu_{\tilde{A}^i}\left(\lambda x_1 + (1 - \lambda x_2)\right) \ge \min\{\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

iv. Concave for the non-membership function $\upsilon_{\tilde{A}^i}$ i.e.

$$\upsilon_{\tilde{A}^i}\left(\lambda x_1 + (1 - \lambda x_2)\right) \le \max\{\upsilon_{\tilde{A}^i}(x_1), \upsilon_{\tilde{A}^i}(x_2)\}; \forall\, x_1, x_2 \in \mathbb{R}\ , \lambda \in [0,1]$$

3 Intuitionistic fuzzy differential equation with linear differential operator

The intuitionistic fuzzy differential equation with linear differential operator is as follows:

$$L\tilde{u}^i(t) + R\tilde{u}^i(t) + N\left(t, \tilde{u}^i(t)\right) = \tilde{g}^i(t) \tag{1}$$

where $\tilde{g}^i(t)$ is an intuitionistic fuzzy function of t, L is the highest order linear differential operator, R is the remaining part of the linear differential operator and N may be linear or nonlinear function of t and $\tilde{u}^i(t)$. Here, in general, we take N as a nonlinear function of t and $\tilde{u}^i(t)$ such that

$$N_{\alpha_l}\left(t, u_{\alpha_l}, u_{\alpha_r}\right) = \sum_{i=1}^{m} E_{1i}(t, u_{\alpha_l}) E_{2i}(t, u_{\alpha_r}) \tag{2}$$

$$N_{\alpha_r}\big(t,u_{\alpha_l},u_{\alpha_r}\big)=\sum_{j=1}^{l}F_{1j}(t,u_{\alpha_l})F_{2j}(t,u_{\alpha_r}) \quad (3)$$

$$N_{\beta_l}\big(t,u_{\beta_l},u_{\beta_r}\big)=\sum_{i=1}^{m}G_{1i}(t,u_{\beta_l})G_{2i}(t,u_{\beta_r}) \quad (4)$$

$$N_{\beta_r}\big(t,u_{\beta_l},u_{\beta_r}\big)=\sum_{j=1}^{l}H_{1j}(t,u_{\beta_l})H_{2j}(t,u_{\beta_r}) \quad (5)$$

where

$$u_{\alpha,\beta}(t)=\langle u_\alpha(t),u_\beta(t)\rangle=\langle[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)],[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]\rangle,$$

$$N_{\alpha,\beta}(t)=N\big(t,\langle[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)],[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]\rangle\big)$$

$$=\langle[N_{\alpha_l}\big(t,u_{\alpha_l},u_{\alpha_r}\big),N_{\alpha_r}\big(t,u_{\alpha_l},u_{\alpha_r}\big)],[N_{\beta_l}\big(t,u_{\beta_l},u_{\beta_r}\big),N_{\beta_r}\big(t,u_{\beta_l},u_{\beta_r}\big)]\rangle,$$

$E_{1i}\big(t,u_{\alpha_l}\big)$, $F_{1j}\big(t,u_{\alpha_l}\big)$ are functions of t and $u_{\alpha_l}(t,\alpha)$ and $E_{2i}\big(t,u_{\alpha_r}\big)$, $F_{2j}\big(t,u_{\alpha_r}\big)$ are functions of t and $u_{\alpha_r}(t,\alpha)$, i= $1,2,\dots,m$, $j=1,2,\dots,l$.

Again, $G_{1i}\big(t,u_{\beta_l}\big)$, $H_{1j}\big(t,u_{\beta_l}\big)$ are functions of t and $u_{\beta_l}(t,\beta)$ and $G_{2i}\big(t,u_{\beta_r}\big)$, $H_{2j}\big(t,u_{\beta_r}\big)$ are functions of t and $u_{\beta_r}(t,\beta)$, i= $1,2,\dots,m$, $j=1,2,\dots,l$.

$$g_{\alpha,\beta}(t)=\langle g_\alpha(t),g_\beta(t)\rangle=\langle[g_{\alpha_l}(t,\alpha),g_{\alpha_r}(t,\alpha)],[g_{\beta_l}(t,\beta),g_{\beta_r}(t,\beta)]\rangle.$$

Now, from (1) we get

$$L\langle[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)],[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]\rangle+R\langle[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)],[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]\rangle$$
$$+N\big(t,\langle[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)],[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]\rangle\big)$$
$$=\langle[g_{\alpha_l}(t,\alpha),g_{\alpha_r}(t,\alpha)],[g_{\beta_l}(t,\beta),g_{\beta_r}(t,\beta)]\rangle$$

Therefore,

$$L[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)]+R[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)]+N\big(t,[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)]\big)$$
$$=[g_{\alpha_l}(t,\alpha),g_{\alpha_r}(t,\alpha)]$$

and

$$L[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]+R[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]+N\big(t,[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]\big)$$
$$=[g_{\beta_l}(t,\beta),g_{\beta_r}(t,\beta)]$$

Let, $[P_{\alpha_l}\big(u_{\alpha_l},u_{\alpha_r}\big),P_{\alpha_r}\big(u_{\alpha_l},u_{\alpha_r}\big)]=R[u_{\alpha_l}(t,\alpha),u_{\alpha_r}(t,\alpha)]$ and $[P_{\beta_l}\big(u_{\beta_l},u_{\beta_r}\big),P_{\beta_r}\big(u_{\beta_l},u_{\beta_r}\big)]=R[u_{\beta_l}(t,\beta),u_{\beta_r}(t,\beta)]$.

Hence,

$$Lu_{\alpha_l}(t,\alpha)+P_{\alpha_l}\big(u_{\alpha_l},u_{\alpha_r}\big)+N_{\alpha_l}\big(t,u_{\alpha_l},u_{\alpha_r}\big)=g_{\alpha_l}(t,\alpha) \quad (6)$$

$$Lu_{\alpha_r}(t,\alpha)+P_{\alpha_r}\big(u_{\alpha_l},u_{\alpha_r}\big)+N_{\alpha_r}\big(t,u_{\alpha_l},u_{\alpha_r}\big)=g_{\alpha_r}(t,\alpha) \quad (7)$$

$$Lu_{\beta_l}(t,\beta)+P_{\beta_l}\big(u_{\beta_l},u_{\beta_r}\big)+N\left(t,u_{\beta_l}(t,\beta)\right)=g_{\beta_l}(t,\beta) \quad (8)$$

$$Lu_{\beta_r}(t,\beta)+P_{\beta_r}\big(u_{\beta_l},u_{\beta_r}\big)+N\left(t,u_{\beta_r}(t,\beta)\right)=g_{\beta_r}(t,\beta) \quad (9)$$

Applying the inverse operator L^{-1} of L on both sides of (6), (7), (8) and (9) we get

$$u_{\alpha_l}(t,\alpha) = L^{-1}g_{\alpha_l}(t,\alpha) + \varphi_{\alpha_l}(t) - L^{-1}P_{\alpha_l}\big(u_{\alpha_l}, u_{\alpha_r}\big) - L^{-1}N_{\alpha_l}\big(t, u_{\alpha_l}, u_{\alpha_r}\big)$$

$$u_{\alpha_r}(t,\alpha) = L^{-1}g_{\alpha_r}(t,\alpha) + \varphi_{\alpha_r}(t) - L^{-1}P_{\alpha_r}\big(u_{\alpha_l}, u_{\alpha_r}\big) - L^{-1}N_{\alpha_r}\big(t, u_{\alpha_l}, u_{\alpha_r}\big)$$

$$u_{\beta_l}(t,\beta) = L^{-1}g_{\beta_l}(t,\beta) + \varphi_{\beta_l}(t) - L^{-1}P_{\beta_l}\big(u_{\beta_l}, u_{\beta_r}\big) - L^{-1}N_{\beta_l}\big(t, u_{\beta_l}, u_{\beta_r}\big)$$

$$u_{\beta_r}(t,\beta) = L^{-1}g_{\beta_r}(t,\beta) + \varphi_{\beta_r}(t) - L^{-1}P_{\beta_r}\big(u_{\beta_l}, u_{\beta_r}\big) - L^{-1}N_{\beta_r}\big(t, u_{\beta_l}, u_{\beta_r}\big)$$

where $\varphi_i(t)$ satisfies $L\varphi_i(t) = 0$, $i = \alpha_l, \alpha_r, \beta_l, \beta_r$ those are normally found by the initial conditions.

The Adomian decomposition method assume an infinite series solution for the unknown functions $u_{\alpha_l}(t,\alpha), u_{\alpha_r}(t,\alpha), u_{\beta_l}(t,\beta)$ and $u_{\beta_r}(t,\beta)$ given by

$$u_{\alpha_l}(t,\alpha) = \textstyle\sum_{n=0}^{\infty} u_{\alpha_l n}(t,\alpha) \tag{10}$$

$$u_{\alpha_r}(t,\alpha) = \textstyle\sum_{n=0}^{\infty} u_{\alpha_r n}(t,\alpha) \tag{11}$$

$$u_{\beta_l}(t,\beta) = \textstyle\sum_{n=0}^{\infty} u_{\beta_l n}(t,\beta) \quad 12)$$

$$u_{\beta_r}(t,\beta) = \textstyle\sum_{n=0}^{\infty} u_{\beta_r n}(t,\beta) \tag{13}$$

The nonlinear functions $E_{1i}\big(t,u_{\alpha_l}\big)$, $E_{2i}\big(t,u_{\alpha_r}\big)$, $F_{1j}\big(t,u_{\alpha_l}\big), F_{2j}\big(t,u_{\alpha_r}\big), G_{\ 1i}\big(t,u_{\beta_l}\big)$, $G_{2i}\big(t,u_{\beta_r}\big), H_{1j}\big(t,u_{\beta_l}\big)$, $H_{2i}\big(t,u_{\beta_r}\big)$ into an infinite series of polynomials given by

$$E_{1i}\big(t,u_{\alpha_l}\big) = \textstyle\sum_{n=0}^{\infty} A_{1in}$$

$$E_{2i}\big(t,u_{\alpha_r}\big) = \textstyle\sum_{n=0}^{\infty} A_{2in}$$

$$F_{1j}\big(t,u_{\alpha_l}\big) = \textstyle\sum_{n=0}^{\infty} B_{1jn}$$

$$F_{2j}\big(t,u_{\alpha_r}\big) = \textstyle\sum_{n=0}^{\infty} B_{2jn}$$

$$G_{1i}\big(t,u_{\beta_l}\big) = \textstyle\sum_{n=0}^{\infty} C_{1in}$$

$$G_{2i}\big(t,u_{\beta_r}\big) = \textstyle\sum_{n=0}^{\infty} C_{2in}$$

$$H_{1j}\big(t,u_{\beta_l}\big) = \textstyle\sum_{n=0}^{\infty} D_{1in}$$

$$H_{2i}\big(t,u_{\beta_r}\big) = \textstyle\sum_{n=0}^{\infty} D_{2in}$$

where A_{1in} , A_{2in} , B_{1jn} , B_{2jn}, C_{1in} , C_{2in} , D_{1jn} , D_{2jn} are the so-called Adomian polynomial defined by

$$A_{1in} = \frac{1}{n!}\Big[\frac{d^n}{d\lambda^n} E_{1i}(t, \textstyle\sum_{k=0}^{n} \lambda^k u_{\alpha_l k}(t,\alpha))\Big]_{\lambda=0},$$

$$A_{2in} = \frac{1}{n!}\Big[\frac{d^n}{d\lambda^n} E_{2i}(t, \textstyle\sum_{k=0}^{n} \lambda^k u_{\alpha_r k}(t,\alpha))\Big]_{\lambda=0},$$

$$B_{1jn} = \frac{1}{n!}\Big[\frac{d^n}{d\lambda^n} F_{1j}(t, \textstyle\sum_{k=0}^{n} \lambda^k u_{\alpha_l k}(t,\alpha))\Big]_{\lambda=0},$$

$$B_{2jn} = \frac{1}{n!}\Big[\frac{d^n}{d\lambda^n} F_{2j}(t, \textstyle\sum_{k=0}^{n} \lambda^k u_{\alpha_r k}(t,\alpha))\Big]_{\lambda=0},$$

$$C_{1in} = \frac{1}{n!}\Big[\frac{d^n}{d\lambda^n} G_{1i}(t, \textstyle\sum_{k=0}^{n} \lambda^k u_{\beta_l k}(t,\beta))\Big]_{\lambda=0},$$

$$C_{2in} = \frac{1}{n!}[\frac{d^n}{d\lambda^n} G_{2i}(t, \sum_{k=0}^{n} \lambda^k u_{\beta_r k}(t,\beta))]_{\lambda=0},$$

$$D_{1jn} = \frac{1}{n!}[\frac{d^n}{d\lambda^n} H_{1j}(t, \sum_{k=0}^{n} \lambda^k u_{\beta_l n}(t,\beta))]_{\lambda=0},$$

$$D_{2jn} = \frac{1}{n!}[\frac{d^n}{d\lambda^n} H_{2j}(t, \sum_{k=0}^{n} \lambda^k u_{\beta_r n}(t,\beta))]_{\lambda=0}$$

$$i = 1,2, \dots, m, \quad j = 1,2, \dots, l, n \geq 0.$$

We can see that A_{1i0} and B_{1j0} depend only on $u_{\alpha_l 0}$, A_{1i1} and B_{1j1} depend only on $u_{\alpha_l 0}$ and $u_{\alpha_l 1}$, and so on. Similarly, A_{2i0} and B_{2j0} depend only on $u_{\alpha_r 0}$, A_{2i1} and B_{2j1} depend only on $u_{\alpha_r 0}$ and $u_{\alpha_r 1}$, and so on, where $i = 1,2, \dots, m, \; j = 1,2, \dots, l$.

Using the Adomian decomposition method we set the recurrence relation as follows:

$$u_{\alpha_l 0} = L^{-1} g_{\alpha_l}(t,\alpha) + \varphi_{\alpha_l}(t) \tag{14}$$

$$u_{\alpha_r 0} = L^{-1} g_{\alpha_r}(t,\alpha) + \varphi_{\alpha_r}(t) \tag{15}$$

$$u_{\alpha_l 1} = -L^{-1} P_{\alpha_l}\left(u_{\alpha_l 0}, u_{\alpha_r 0}\right) - L^{-1} \sum_{i=1}^{m} A_{1i0} A_{2i0} \tag{16}$$

$$u_{\alpha_r 1} = -L^{-1} P_{\alpha_r}\left(u_{\alpha_l 0}, u_{\alpha_r 0}\right) - L^{-1} \sum_{j=1}^{l} B_{1j0} B_{2j0} \tag{17}$$

$$u_{\alpha_l 2} = -L^{-1} P_{\alpha_l}\left(u_{\alpha_l 1}, u_{\alpha_r 1}\right) - L^{-1} \sum_{i=1}^{m} A_{1i1} A_{2i1} \tag{18}$$

$$u_{\alpha_r 2} = -L^{-1} P_{\alpha_r}\left(u_{\alpha_l 1}, u_{\alpha_r 1}\right) - L^{-1} \sum_{j=1}^{l} B_{1j1} B_{2j1} \tag{19}$$

$$\dots$$

$$u_{\alpha_l k+1} = -L^{-1} P_{\alpha_l}\left(u_{\alpha_l k}, u_{\alpha_r k}\right) - L^{-1} \sum_{i=1}^{m} A_{1ik} A_{2ik} \tag{20}$$

$$u_{\alpha_r k+1} = -L^{-1} P_{\alpha_r}\left(u_{\alpha_l k}, u_{\alpha_r k}\right) - L^{-1} \sum_{j=1}^{l} B_{1jk} B_{2jk} \;, k \geq 0 \tag{21}$$

Again we can also see that C_{1i0} and D_{1j0} depend only on $u_{\beta_l 0}$, C_{1i1} and D_{1j1} depend only on $u_{\beta_l 0}$ and $u_{\beta_l 1}$, and so on. Similarly, C_{2i0} and D_{2j0} depend only on $u_{\beta_r 0}$, C_{2i1} and D_{2j1} depend only on $u_{\beta_r 0}$ and $u_{\beta_r 1}$, and so on. Where $i = 1,2, \dots, m, \; j = 1,2, \dots, l$.

Using the Adomian decomposition method we set the recurrence relation as follows:

$$u_{\beta_l 0} = L^{-1} g_{\beta_l}(t,\beta) + \varphi_{\beta_l}(t) \tag{22}$$

$$u_{\beta_r 0} = L^{-1} g_{\beta_r}(t,\beta) + \varphi_{\beta_r}(t) \tag{23}$$

$$u_{\beta_l 1} = -L^{-1} P_{\beta_l}\left(u_{\beta_l 0}, u_{\beta_r 0}\right) - L^{-1} \sum_{i=1}^{m} C_{1i0} C_{2i0} \tag{24}$$

$$u_{\beta_r 1} = -L^{-1} P_{\beta_r}\left(u_{\beta_l 0}, u_{\beta_r 0}\right) - L^{-1} \sum_{j=1}^{l} D_{1j0} D_{2j0} \tag{25}$$

$$u_{\beta_l 2} = -L^{-1} P_{\beta_l}\left(u_{\beta_l 1}, u_{\beta_r 1}\right) - L^{-1} \sum_{i=1}^{m} C_{1i1} C_{2i1} \tag{26}$$

$$u_{\beta_r 2} = -L^{-1} P_{\beta_r}\left(u_{\beta_l 1}, u_{\beta_r 1}\right) - L^{-1} \sum_{j=1}^{l} D_{1j1} D_{2j1} \tag{27}$$

$$\dots$$

$$u_{\beta_l k+1} = -L^{-1} P_{\beta_l}\left(u_{\beta_l k}, u_{\beta_r k}\right) - L^{-1} \sum_{i=1}^{m} C_{1ik} C_{2ik} \tag{28}$$

$$u_{\beta_r k+1} = -L^{-1} P_{\beta_r}\left(u_{\beta_l k}, u_{\beta_r k}\right) - L^{-1} \sum_{j=1}^{l} D_{1jk} D_{2jk} \;, k \geq 0 \tag{29}$$

Then, we define the nth term approximation to the solution

$u_{\alpha,\beta}(t) = \langle [u_{\alpha_l}(t,\alpha), u_{\alpha_r}(t,\alpha)], [u_{\beta_l}(t,\beta), u_{\beta_r}(t,\beta)] \rangle$ by $\psi_{n\alpha,\beta}(t) = \langle [\psi_{n\alpha_l}(t,\alpha), \psi_{n\alpha_r}(t,\alpha)], [\psi_{n\beta_l}(t,\beta), \psi_{n\beta_r}(t,\beta)] \rangle$ where, $\psi_{n\alpha_l}(t,\alpha) = \sum_{i=0}^{n-1} u_{\alpha_l i}(t,\alpha)$, $\psi_{n\alpha_r}(t,\alpha) = \sum_{i=0}^{n-1} u_{\alpha_r i}(t,\alpha)$, $\psi_{n\beta_l}(t,\beta) = \sum_{i=0}^{n-1} u_{\beta_l i}(t,\beta)$ and $\psi_{n\beta_r}(t,\beta) = \sum_{i=0}^{n-1} u_{\beta_r i}(t,\beta)$.

Hence, $\lim_{n\to\infty} \psi_{n\alpha_l}(t,\alpha) = u_{\alpha_l}(t,\alpha)$and $\lim_{n\to\infty} \psi_{n\alpha_r}(t,\alpha) = u_{\alpha_r}(t,\alpha)$, $\lim_{n\to\infty} \psi_{n\beta_l}(t,\beta) = u_{\beta_l}(t,\beta)$ and $\lim_{n\to\infty} \psi_{n\beta_r}(t,\beta) = u_{\beta_r}(t,\beta)$.

4 Numerical examples

Example 4.1

Let us consider the fuzzy differential equation of the following form

$$\frac{d^2\tilde{u}^i}{dt^2} + \frac{d\tilde{u}^i}{dt} = e^t \tag{30}$$

with initial conditions

$$u_{\alpha,\beta}(0) = \langle [1+\alpha, 3-\alpha], [2-2\beta, 2+2\beta] \rangle \; (31)$$

$$u'_{\alpha,\beta}(0) = \langle [\alpha, 2-\alpha], [1-2\beta, 1+\beta] \rangle \tag{32}$$

The exact solution, given by classical solution method, is

$$u_{\alpha_l}(t,\alpha) = cosht + 2\alpha - \alpha e^{-t}$$

$$u_{\alpha_r}(t,\alpha) = sinht + (\alpha-1)e^{-t} + 4 - 2\alpha$$

$$u_{\beta_l}(t,\beta) = cosht + (2\beta-1)e^{-t} + (2-4\beta)$$

$$u_{\beta_r}(t,\beta) = sinht - \beta e^{-t} + (2+3\beta)$$

Now we will use ADM to find the approximate solution. The equation becomes

$$\frac{d^2u_{\alpha_l}}{dt^2} = e^t - \frac{du_{\alpha_l}}{dt}$$

$$\frac{d^2u_{\alpha_r}}{dt^2} = e^t - \frac{du_{\alpha_r}}{dt}$$

with the initial conditions

$$u_{\alpha_l}(0,\alpha) = 1+\alpha \;, \; u_{\alpha_r}(0,\alpha) = 3-\alpha$$

$$u'_{\alpha_l}(0,\alpha) = \alpha \;, \qquad u'_{\alpha_r}(0,\alpha) = 2-\alpha$$

and

$$\frac{d^2u_{\beta_l}}{dt^2} = e^t - \frac{du_{\beta_l}}{dt}$$

$$\frac{d^2u_{\beta_r}}{dt^2} = e^t - \frac{du_{\beta_r}}{dt}$$

with the initial conditions

$$u_{\beta_l}(0,\beta) = 2 - 2\beta \ , \ u_{\beta_r}(0,\beta) = 2 + 2\beta$$

$$u'_{\beta_l}(0,\beta) = 1 - 2\beta \ , \qquad u'_{\beta_r}(0,\beta) = 1 + \beta$$

Here, $L \equiv \frac{d^2}{dt^2}$ and by operating the two sides of the above equation with the inverse operator (namely $L^{-1}(*) \equiv \iint (*)dtdt$) and using the initial conditions, we get

$$u_{\alpha_l}(t,\alpha) = e^t + 2\alpha t + \alpha - \int_0^t u_{\alpha_l}(x,\alpha)dx$$

$$u_{\alpha_r}(t,\alpha) = e^t + (4 - 2\alpha)t + (2 - \alpha) - \int_0^t u_{\alpha_r}(x,\alpha)dx$$

$$u_{\beta_l}(t,\beta) = e^t + (2-4\beta)t + (1-2\beta) - \int_0^t u_{\beta_l}(x,\beta)dx$$

$$u_{\beta_r}(t,\beta) = e^t + (2+3\beta)t + (1+2\beta) - \int_0^t u_{\beta_r}(x,\beta)dx$$

Now applying the ADM we get

$$u_{\alpha_l 0}(t,\alpha) = e^t + 2\alpha t + \alpha$$

$$u_{\alpha_r 0}(t,\alpha) = e^t + (4 - 2\alpha)t + (2 - \alpha)$$

$$u_{\alpha_l k+1}(t,\alpha) = -\int_0^t u_{\alpha_l k}(x,\alpha)dx \ , \ k \geq 0.$$

$$u_{\alpha_r k+1}(t,\alpha) = -\int_0^t u_{\alpha_r k}(x,\alpha)dx \ , \ k \geq 0.$$

$$u_{\beta_l 0}(t,\beta) = e^t + (2-4\beta)t + (1-2\beta)$$

$$u_{\beta_r 0}(t,\beta) = e^t + (2+3\beta)t + (1+2\beta)$$

$$u_{\beta_l k+1}(t,\beta) = -\int_0^t u_{\beta_l k}(x,\beta)dx \ , \ k \geq 0.$$

$$u_{\beta_r k+1}(t,\beta) = -\int_0^t u_{\beta_r k}(x,\beta)dx \ , \ k \geq 0.$$

On substituting and solving the above equation, we obtain the approximate solution after four iterations as

$$u_{\alpha_l 0}(t,\alpha) + u_{\alpha_l 1}(t,\alpha) + u_{\alpha_l 2}(t,\alpha) + u_{\alpha_l 3}(t,\alpha)$$

$$= 1 + \alpha + \alpha t + \frac{(1-\alpha)}{2}t^2 + \frac{\alpha}{2}t^3 + \frac{\alpha}{12}t^4$$

$$u_{\alpha_r 0}(t,\alpha) + u_{\alpha_r 1}(t,\alpha) + u_{\alpha_r 2}(t,\alpha) + u_{\alpha_r 3}(t,\alpha)$$

$$= (3 - \alpha) + (2 - \alpha)t + \frac{(\alpha-1)}{2}t^2 + \frac{(2-\alpha)}{2}t^3 + \frac{(2-\alpha)}{12}t^4$$

$$u_{\beta_l 0}(t,\beta) + u_{\beta_l 1}(t,\beta) + u_{\beta_l 2}(t,\beta) + u_{\beta_l 3}(t,\beta)$$

$$= -(2 - 4\beta)\frac{t^4}{4!} + (1 - 2\beta)\frac{t^3}{3!} + 2\beta\frac{t^2}{2} + (1 - 2\beta)t + (2 - 2\beta)$$

$$u_{\beta_r 0}(t,\beta) + u_{\beta_r 1}(t,\beta) + u_{\beta_r 2}(t,\beta) + u_{\beta_r 3}(t,\beta)$$

$$= -(2 + 3\beta)\frac{t^4}{4!} + (1 + \beta)\frac{t^3}{3!} - \beta\frac{t^2}{2} + (1 + \beta)t + (2 + 2\beta)$$

The comparison between the exact and approximate solutions at $t = 1$ for some $\alpha \in [0,1]$ has been shown at Figures 1 and 2. We have calculated all data by using MATLAB.

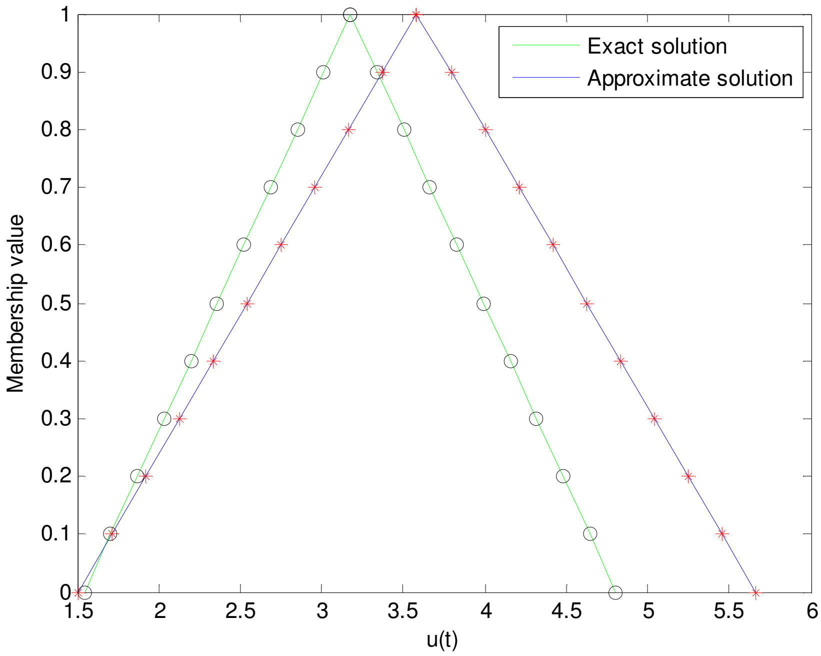

Figure 1. Exact solution and approximate solution for the membership function at $t = 1$ for Example 1.

Example 4.2

Let us consider the fuzzy differential equation of the following form $\frac{d^2\tilde{u}^i}{dt^2} - 2\frac{d\tilde{u}^i}{dt} + \tilde{u}^i = \tilde{a}^i$ where $\tilde{a}^i = \langle(-1, 0, 1), (-1, 0, 2)\rangle$ i.e. $a_{\alpha,\beta} = \langle[\alpha - 1, 1 - \alpha][-\beta, 2\beta]\rangle$ with initial conditions

$$u_{\alpha,\beta}(0) = \langle[1 + \alpha, 3 - \alpha][2 - 2\beta, 2 + 2\beta]\rangle$$

$$u'_{\alpha,\beta}(0) = \langle[\alpha, 2 - \alpha][1 - 2\beta, 1 + \beta]\rangle$$

The exact solution, given by classical solution method, is

$$u_{\alpha_l}(t, \alpha) = (2 - t)e^t + (\alpha - 1)te^{-t} + (\alpha - 1)$$

$$u_{\alpha_r}(t, \alpha) = (2 - t)e^t - (\alpha - 1)te^{-t} - (\alpha - 1)$$

$$u_{\beta_l}(t, \beta) = \frac{1}{2}[(4 - \beta - 2t)e^t - (\beta + 2\beta t)e^{-t}] - \beta$$

$$u_{\beta_r}(t, \beta) = \frac{1}{2}[(4 - \beta - 2t)e^t + (\beta + 2\beta t)e^{-t}] + 2\beta$$

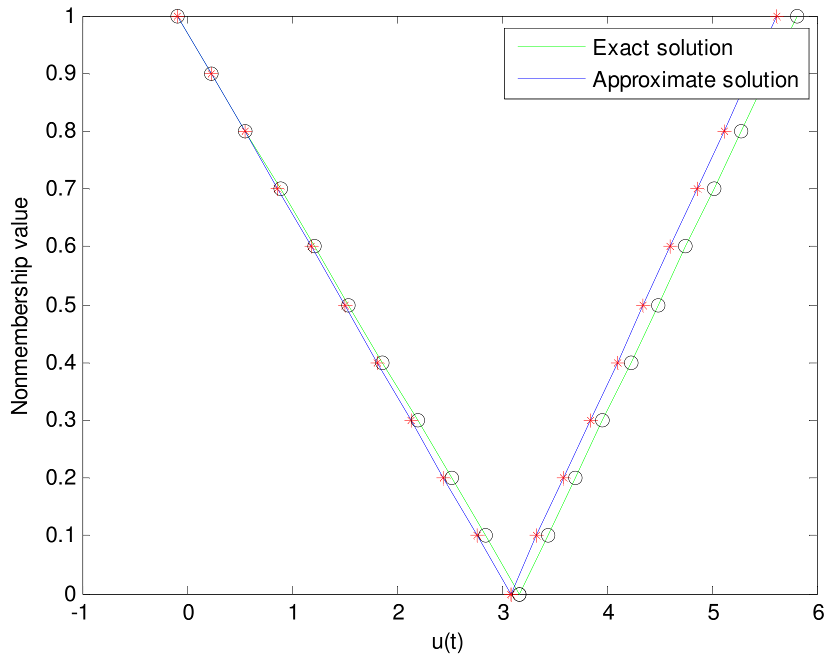

Figure 2. Exact solution and approximate solution for the non-membership function at $t = 1$ for Example 1.

Now we will use ADM to find the approximate solution. The equation becomes

$$\frac{d^2u_{\alpha_l}}{dt^2} = (\alpha - 1) + 2\frac{du_{\alpha_r}}{dt} - u_{\alpha_l}$$

$$\frac{d^2u_{\alpha_r}}{dt^2} = (1 - \alpha) + 2\frac{du_{\alpha_l}}{dt} - u_{\alpha_r}$$

with the initial conditions

$$u_{\alpha_l}(0,\alpha) = 1 + \alpha\ ,\ \ u_{\alpha_r}(0,\alpha) = 3 - \alpha$$

$$u'_{\alpha_l}(0,\alpha) = \alpha\ ,\ \ u'_{\alpha_r}(0,\alpha) = 2 - \alpha$$

and

$$\frac{d^2u_{\beta_l}}{dt^2} = -\beta + 2\frac{du_{\beta_r}}{dt} - u_{\beta_l}$$

$$\frac{d^2u_{\beta_r}}{dt^2} = 2\beta + 2\frac{du_{\beta_l}}{dt} - u_{\beta_r}$$

with the initial conditions

$$u_{\beta_l}(0,\beta) = 2 - 2\beta\ ,\ \ u_{\beta_r}(0,\beta) = 2 + 2\beta$$

$$u'_{\beta_l}(0,\beta) = 1 - 2\beta\ ,\ \ u'_{\beta_r}(0,\beta) = 1 + \beta$$

Here, $L \equiv \frac{d^2}{dt^2}$ and by operating the two sides of the above equation with the inverse operator (namely, $L^{-1}(*) \equiv \iint(*)dtdt$) and using the initial conditions, we get

$$u_{\alpha_l}(t,\alpha) = (1+\alpha) + \int_0^t[(3\alpha - 6) + (\alpha - 1)x]dx + 2\int_0^t u_{\alpha_r}(x,\alpha)dx - \int_0^t\int_0^x u_{\alpha_l}(s,\alpha)\,dsdx$$

$$u_{\alpha_r}(t,\alpha) = (3-\alpha) + \int_0^t[-3\alpha + (1 - \alpha)x]dx + 2\int_0^t u_{\alpha_l}(x,\alpha)dx - \int_0^t\int_0^x u_{\alpha_r}(s,\alpha)\,dsdx$$

$$u_{\beta_l}(t,\beta) = (2-2\beta) - (3+6\beta)t - \beta\frac{t^2}{2} + 2\int_0^t u_{\beta_r}(x,\beta)dx - \int_0^t\int_0^x u_{\beta_l}(s,\beta)\,dsdx$$

$$u_{\beta_r}(t,\beta) = (2+2\beta) - (3-5\beta)t + \beta t^2 + 2\int_0^t u_{\beta_l}(x,\beta)dx - \int_0^t\int_0^x u_{\beta_r}(s,\beta)\,dsdx$$

Now applying the ADM we get

$$u_{\alpha_l 0}(t,\alpha) = (1+\alpha) + \int_0^t [(3\alpha-6) + (\alpha-1)x]dx$$

$$u_{\alpha_r 0}(t,\alpha) = (3-\alpha) + \int_0^t [-3\alpha + (1-\alpha)x]dx$$

$$u_{\alpha_l k+1}(t,\alpha) = 2\int_0^t u_{\alpha_r k}(x,\alpha)dx - \int_0^t\int_0^x u_{\alpha_l k}(s,\alpha)\,dsdx,\ k\geq 0.$$

$$u_{\alpha_r k+1}(t,\alpha) = 2\int_0^t u_{\alpha_l k}(x,\alpha)dx - \int_0^t\int_0^x u_{\alpha_r k}(s,\alpha)\,dsdx,\ k\geq 0.$$

$$u_{\beta_l 0}(t,\beta) = (2-2\beta) - (3+6\beta)t - \beta\frac{t^2}{2}$$

$$u_{\beta_r 0}(t,\beta) = (2+2\beta) - (3-5\beta)t + \beta t^2$$

$$u_{\beta_l k+1}(t,\beta) = 2\int_0^t u_{\beta_r k}(x,\beta)dx - \int_0^t\int_0^x u_{\beta_l k}(s,\beta)\,dsdx,\ k\geq 0.$$

$$u_{\beta_r k+1}(t,\beta) = 2\int_0^t u_{\beta_l k}(x,\beta)dx - \int_0^t\int_0^x u_{\beta_r k}(s,\beta)\,dsdx,\ k\geq 0.$$

On substituting and solving the above equation, we obtain the approximate solution after six iterations as

$$u_{\alpha_l 0}(t,\alpha) + u_{\alpha_l 1}(t,\alpha) + u_{\alpha_l 2}(t,\alpha) + u_{\alpha_l 3}(t,\alpha) + u_{\alpha_l 4}(t,\alpha) + u_{\alpha_l 5}(t,\alpha)$$

$$= (1+\alpha)(1 + 3\frac{t^2}{2!} + 5\frac{t^4}{4!} - 61\frac{t^6}{6!} - 39\frac{t^8}{8!} - \frac{t^{10}}{10!}) + (3\alpha-6)(t + 3\frac{t^3}{3!} + 5\frac{t^5}{5!} - 61\frac{t^7}{7!} - 39\frac{t^9}{9!} - \frac{t^{11}}{11!})$$

$$+(\alpha-1)\left(\frac{t^2}{2!} + 3\frac{t^4}{4!} + 5\frac{t^6}{6!} - 61\frac{t^8}{8!} - 39\frac{t^{10}}{10!} - \frac{t^{12}}{12!}\right) + (3-\alpha)(2t + 4\frac{t^3}{3!} + 6\frac{t^5}{5!} - 56\frac{t^7}{7!} + 10\frac{t^9}{9!})$$

$$-3\alpha\left(2\frac{t^2}{2!} + 4\frac{t^4}{4!} + 6\frac{t^6}{6!} - 56\frac{t^8}{8!} + 10\frac{t^{10}}{10!}\right) + (1-\alpha)(2\frac{t^3}{3!} + 4\frac{t^5}{5!} + 6\frac{t^7}{7!} - 56\frac{t^9}{9!} + 10\frac{t^{11}}{11!})$$

$$u_{\alpha_r 0}(t,\alpha) + u_{\alpha_r 1}(t,\alpha) + u_{\alpha_r 2}(t,\alpha) + u_{\alpha_r 3}(t,\alpha) + u_{\alpha_r 4}(t,\alpha) + u_{\alpha_r 5}(t,\alpha)$$

$$= (3-\alpha)(1 + 3\frac{t^2}{2!} + 5\frac{t^4}{4!} - 61\frac{t^6}{6!} - 39\frac{t^8}{8!} - \frac{t^{10}}{10!}) - 3\alpha(t + 3\frac{t^3}{3!} + 5\frac{t^5}{5!} - 61\frac{t^7}{7!} - 39\frac{t^9}{9!} - \frac{t^{11}}{11!})$$

$$+(1-\alpha)\left(\frac{t^2}{2!} + 3\frac{t^4}{4!} + 5\frac{t^6}{6!} - 61\frac{t^8}{8!} - 39\frac{t^{10}}{10!} - \frac{t^{12}}{12!}\right) + (1+\alpha)(2t + 4\frac{t^3}{3!} + 6\frac{t^5}{5!} - 56\frac{t^7}{7!} + 10\frac{t^9}{9!})$$

$$+(3\alpha-6)\left(2\frac{t^2}{2!} + 4\frac{t^4}{4!} + 6\frac{t^6}{6!} - 56\frac{t^8}{8!} + 10\frac{t^{10}}{10!}\right) + (\alpha-1)(2\frac{t^3}{3!} + 4\frac{t^5}{5!} + 6\frac{t^7}{7!} - 56\frac{t^9}{9!} + 10\frac{t^{11}}{11!})$$

$$u_{\beta_l 0}(t,\beta) + u_{\beta_l 1}(t,\beta) + u_{\beta_l 2}(t,\beta) + u_{\beta_l 3}(t,\beta) + u_{\beta_l 4}(t,\alpha) + u_{\alpha_l 5}(t,\beta)$$

$$= (2-2\beta)\left(1 + 3\frac{t^2}{2!} + 5\frac{t^4}{4!} - 57\frac{t^6}{6!} - 39\frac{t^8}{8!} - \frac{t^{10}}{10!}\right) + (3+6\beta)\left(-t - 3\frac{t^3}{3!} - 5\frac{t^5}{5!} + 57\frac{t^7}{7!} + 39\frac{t^9}{9!} + \frac{t^{11}}{11!}\right)$$
$$+\beta\left(-\frac{t^2}{2!} - 3\frac{t^4}{4!} - 5\frac{t^6}{6!} + 57\frac{t^8}{8!} + 39\frac{t^{10}}{10!} + \frac{t^{12}}{12!}\right) + (2+2\beta)(2t + 4\frac{t^3}{3!} + 6\frac{t^5}{5!} + 72\frac{t^7}{7!} + 10\frac{t^9}{9!})$$
$$+ (3-5\beta)\left(-2\frac{t^2}{2!} - 4\frac{t^4}{4!} - 6\frac{t^6}{6!} - 72\frac{t^8}{8!} - 10\frac{t^{10}}{10!}\right) + 2\beta(2\frac{t^3}{3!} + 4\frac{t^5}{5!} + 6\frac{t^7}{7!} + 72\frac{t^9}{9!} + 10\frac{t^{11}}{11!})$$

$$u_{\alpha_r 0}(t,\beta) + u_{\alpha_r 1}(t,\beta) + u_{\alpha_r 2}(t,\beta) + u_{\alpha_r 3}(t,\beta) + u_{\alpha_r 4}(t,\beta) + u_{\alpha_r 5}(t,\beta)$$

$$= (2-2\beta)\left(2t+4\frac{t^3}{3!}+6\frac{t^5}{5!}+72\frac{t^7}{7!}+10\frac{t^9}{9!}\right)+(3+6\beta)\left(-2\frac{t^2}{2!}-4\frac{t^4}{4!}-6\frac{t^6}{6!}-72\frac{t^8}{8!}-10\frac{t^{10}}{10!}\right)$$
$$+\beta\left(-2\frac{t^3}{3!}-4\frac{t^5}{5!}-6\frac{t^7}{7!}-72\frac{t^9}{9!}-10\frac{t^{11}}{11!}\right)+(2+2\beta)\left(1+3\frac{t^2}{2!}+5\frac{t^4}{4!}-57\frac{t^6}{6!}-39\frac{t^8}{8!}-\frac{t^{10}}{10!}\right)$$
$$+(3-5\beta)\left(-t-3\frac{t^3}{3!}-5\frac{t^5}{5!}+57\frac{t^7}{7!}+39\frac{t^9}{9!}+\frac{t^{11}}{11!}\right)+2\beta\left(\frac{t^2}{2!}+3\frac{t^4}{4!}+5\frac{t^6}{6!}-57\frac{t^8}{8!}-39\frac{t^{10}}{10!}-\frac{t^{12}}{12!}\right)$$

The comparison between the exact and approximate solutions at $t = 1$ has been shown at Figures 3 and 4. We have calculated all data by using MATLAB.

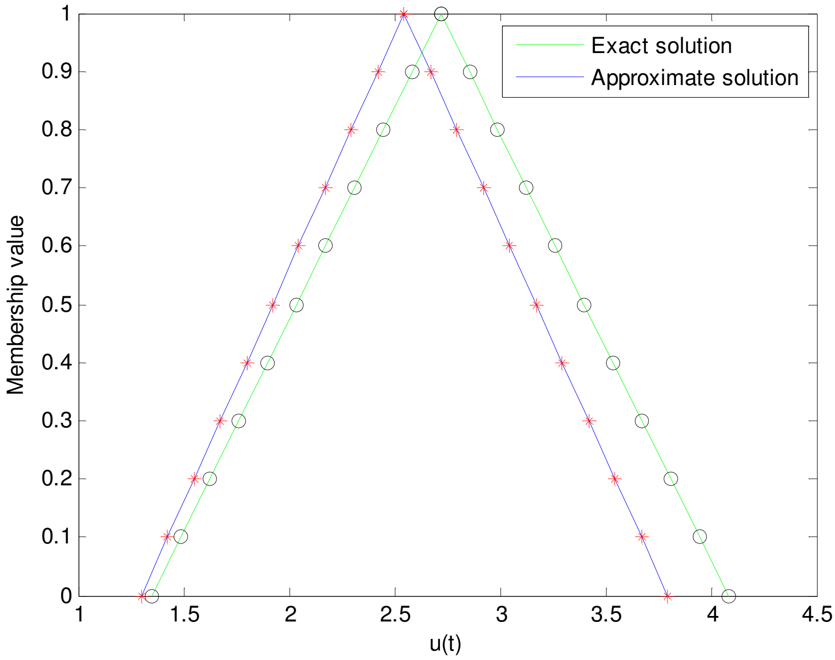

Figure 3. Exact solution and approximate solution for membership function at $t = 1$ for Example 2.

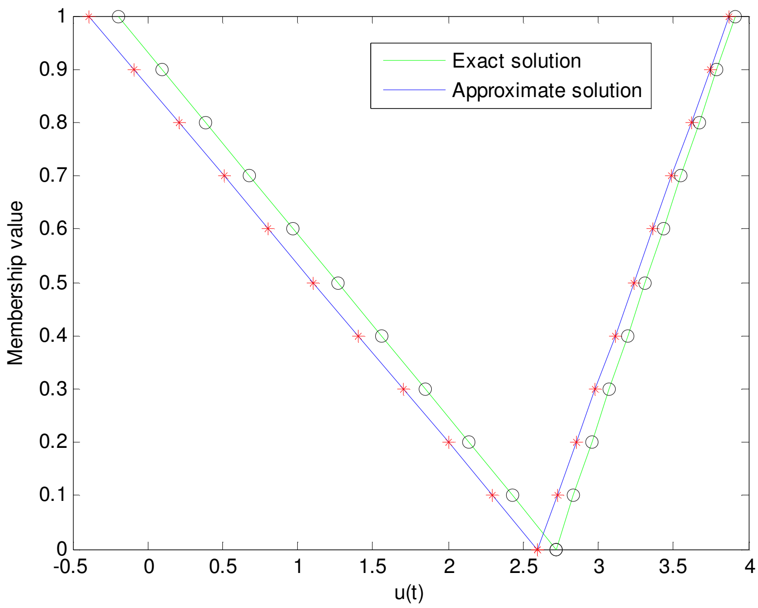

Figure 4. Exact solution and approximate solution for non-membership function at $t = 1$ for Example 2.

5 Conclusion

In this paper we presented intuitionistic fuzzy differential equation with linear differential operator which can be of any order and it also involves nonlinear functional. So our solution procedure gives the solutions of a large area of problems involving intuitionistic fuzzy differential equations. Note that we used ADM which gives solution even for some nonlinear problems that can't be solved by classical methods. Future research work will be try to solve any kind of fuzzy differential equations by improving and using ADM or any other method.

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