Notes on Intuitionistic Fuzzy Sets
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On intuitionistic fuzzy slightly β-continuous functions

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Abstract: In this paper the concept of intuitionistic fuzzy slightly β-continuous functions are introduced and studied. Intuitionistic fuzzy slightly β-continuity generalize intuitionistic fuzzy β-continuity. Besides giving characterizations and basic properties of this function, preservation theorems of intuitionistic fuzzy slightly β-continuous are also obtained. We also study relationships between intuitionistic fuzzy slightly β-continuity and separation axioms. Moreover, we investigate and the relationships among intuitionistic fuzzy slightly β-continuity and compactness and connectedness.
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1 Introduction

Ever since the introduction of fuzzy sets by L. A. Zadeh [15], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C. L. Chang [2]. K. Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Çoker [3] introduced the intuitionistic fuzzy topological spaces. T. Noiri [12] introduced slightly β-continuous functions. E. Ekici [6] introduced fuzzy slightly β-continuous functions. In this paper, we have introduced the concept of intuitionistic fuzzy slightly β-continuous functions and studied their properties. Also we have given preservation theorems of intuitionistic fuzzy slightly β-continuous functions. We also study relationships between this function and separation axioms. Moreover, we investigate the relationships among intuitionistic fuzzy slightly β-continuity and compactness and connectedness.

2 Preliminaries

Definition 2.1. [1] Let X be a non-empty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in \mathrm{X}\}$$

where the mappings $\mu_A(x) : X \to I$ and $\nu_A(x) : X \to I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non-membership (namely) $\nu_A(x)$ for each element x∈ X to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in \mathrm{X}\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in \mathrm{X}\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;

(ii) $\bar{A}$ (or A^c) = $\{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$;

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle | x \in X\}$;

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle | x \in X\}$.

We will use the notation $A = \{\langle x, \mu_A, \nu_A\rangle | x \in X\}$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$.

Definition 2.3. [3] $0_\sim = \{\langle x, 0, 1\rangle | x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle | x \in X\}$.

Let $\alpha, \beta \in$ [0,1] such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP) $p_{(\alpha,\beta)}$ is intuitionistic fuzzy set defined by $p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise} \end{cases}$

Definition 2.4. [3] An intuitionistic fuzzy topology (IFT) in Çoker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim$, $1_\sim \in \tau$;

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i; i \in J\} \subseteq \tau$.

In this paper by (X, τ), (Y, σ) or simply by X, Y, we will denote the intuitionistic fuzzy topological spaces (IFTS) . Each IFS which belongs to τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\bar{A}$ of an IFOS A in X is called an intuitionstic fuzzy closed set (IFCS) in X. A IFS X is called intuitionistic fuzzy clopen (IF clopen) iff it is both intuitionistic fuzzy open and intuitionistic fuzzy closed.

Let X and Y be two non-empty sets and f: (X, τ) $\to$ (Y, σ) be a function.

If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle | y \in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle | x \in X\}$. Since $\mu_B(x), \nu_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B x)(f(x))$, $f^{-1}(\nu_B(x)) = \nu_B(x)(f(x))$.

Definition 2.5. [10] Let $p_{(\alpha,\beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition 2.6. [3] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionstic fuzzy closure of A are defined by

(i) cl (A) = $\bigcap \{C \mid C$ is an IFCS in X and $C \supseteq A\}$;

(ii) int (A) = $\bigcup \{D \mid D$ is an IFOS in X and $D \subseteq A\}$;

It can be also shown that cl (A) is an IFCS, int (A) is an IFOS in X and A is an IFCS in X if and only if cl(A) = A ; A is an IFOS in X if and only int(A) = A.

Proposition 2.1. *[3] Let (X,τ) be an IFTS and A, B be IFSs in X. Then the following properties hold:*

(i) $cl\overline{(A)} = \overline{(int(A))},\ \ int\overline{(A)} = \overline{(cl(A))}$*;*

(ii) *int $(A) \subseteq$ A $\subseteq$ cl (A) .*

Definition 2.7. [8] An IFS A in an IFTS X is called an intuitionistic fuzzy β-open set (IFβOS) if $A \subseteq$ cl (int(cl(A)). The complement of an IFβOS A in IFTS X is called an intuitionistic fuzzy β-closed (IFβCS) in X.

Definition 2.8. Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

(i) intuitionistic fuzzy continuous, if and only if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y [8];

(ii) intuitionistic fuzzy β-continuous if and only if $f^{-1}(B)$ is an IFPOS in X, for each IFOS B in Y [8];

(iii) intuitionistic fuzzy totally continuous if and only if $f^{-1}(B)$ is an IF clopen sets in X, for each IFOS B in Y [11].

Definition 2.9. [7] A fuzzifying function $f : X \longrightarrow Y$ is called fuzzy β-irresolute if inverse image of each fuzzy β-open set is fuzzy β-open.

Definition 2.10. [13] A function $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ from a intuitionistic fuzzy topological space $(X,\ \tau)$ to another intuitionistic fuzzy topological space $(Y,\ \sigma)$ is said to be intuitionistic fuzzy β-irresolute if $f^{-1}(B)$ is an IFβOS in $(X,\ \tau)$ for each IFβOS B in $(Y,\ \sigma)$.

Definition 2.11. [3, 13] Let X be an IFTS. A family of $\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x)\rangle | i \in J\}$ intuitionistic fuzzy open sets (intuitionistic fuzzy β-open sets) in X satisfies the condition $1_\sim = \cup\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x)\rangle | i \in J\}$ is called a intuitionistic fuzzy open (intuitionistic fuzzy β-open) cover of X. A finite subfamily of a intuitionistic fuzzy open (intuitionistic fuzzy β-open) cover $\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x)\rangle | i \in J\}$ of X which is also a intuitionistic fuzzy open (intuitionistic fuzzy β-open) cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}(x), \nu_{G_i}(x)\rangle | i \in J\}$.

Definition 2.12. [4] An IFTS X is called intuitionistic fuzzy compact if each intuitionistic fuzzy open cover of X has a finite subcover for X.

3 Intuitionistic fuzzy slightly β-continuous functions

Definition 3.1. A function $f : (X,\ \tau) \to (Y,\ \sigma)$ from a intuitionistic fuzzy topological space $(X,\ \tau)$ to another intuitionistic fuzzy topological space $(Y,\ \sigma)$ is said to be intuitionistic fuzzy slightly β-continuous if for each intuitionistic fuzzy point $p_{(\alpha,\beta)} \in X$ and each intuitionistic fuzzy clopen set B in Y containing $f(p_{(\alpha,\beta)})$, there exists a fuzzy intuitionistic fuzzy β-open set A in X such that $f(A) \subseteq B$.

Theorem 3.1. *For a function $f : X \longrightarrow Y$, the following statements are equivalent:*

1. *f is intuitionistic fuzzy slightly β-continuous;*
2. *for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy β-open;*
3. *for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy β-closed;*
4. *for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy β-clopen.*

Proof. (1) $\Rightarrow$ (2) Let B be IF clopen set in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since $f(p_{(\alpha,\beta)}) \in B$, by (1) there exists a IFPOS $A_{p_{(\alpha,\beta)}}$ in X containing $p_{(\alpha,\beta)}$ such that $A_{p(\alpha,\beta)} \subseteq f^{-1}(B)$. We obtain that $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} A_{p_{(\alpha,\beta)}}$. Thus, $f^{-1}(B)$ is IFβ-open.
(2) $\Rightarrow$ (3) Let B be IF clopen set in Y. Then $\overline{B}$ is IF clopen. By (2) , $f^{-1}(\overline{B})$ = $\overline{f^{-1}(B)}$ is IF β-open. Thus $f^{-1}(B)$ is IFβ-closed.
(3) $\Rightarrow$ (4) Let B be IF clopen set in Y. Then by (3) $f^{-1}(B)$ is IFβ-closed. Also $\overline{B}$ is IF clopen and (3) implies $f^{-1}(\overline{B})$ = $\overline{f^{-1}(B)}$ is IF β-closed. Hence $f^{-1}(B)$ is IFβ-open. Thus $f^{-1}(B)$ is IFβ-clopen.
(4) $\Rightarrow$ (1) Let B be IF clopen set in Y containing $f(p_{(\alpha,\beta)})$. By (4), $f^{-1}(B)$ is IFβ-open. Let us take A = $f^{-1}(B)$. Then $f(A)\subseteq B$. Hence, f is IF slightly β-continuous. □

Lemma 3.1. *[9] Let g:X$\to$X$\times$Y be a graph of a mapping f: (X,τ) $\to$ (Y,σ) . If A and B are IFS's of X and Y respectivly, then g^{-1} ($1_\sim$ $\times$B) = ($1_\sim$ $\cap$ f^{-1} (B))*

Lemma 3.2. *[9] Let X and Y be intuitionistic fuzzy topological spaces, then (X,τ) is product related to (Y,σ) if for any IFS C in X, D in Y whenever $\bar{A} \not\supseteq$ C, $\bar{B} \not\supseteq$ D implies $\bar{A} \times 1_\sim \bigcup 1_\sim \times \bar{B} \supseteq C \times D$ there exists $A_1 \in \tau$, $B_1 \in \sigma$ such that $\overline{A_1} \supseteq$ C and $\overline{B_1} \supseteq$ D and $\overline{A_1} \times 1_\sim \bigcup 1_\sim \times \overline{B_1} = \bar{A} \times 1_\sim \bigcup 1_\sim \times \bar{B}$.*

Theorem 3.2. *Let f:X$\to$Y be a function and assume that X is product related to Y.If the graph g:X$\to$ X$\times$Y of f is IF slightly β-continuous then so is f.*

Proof. Let B be IF clopen set in Y.Then by lemma 3.1, $f^{-1}(B)$ = $1_\sim \cap f^{-1}(B)$ = g^{-1} ($1_\sim$ $\times B$). Now $1_\sim \times$ B is a IF clopen set in X$\times$Y. Since g is IF slightly β-continuous then g^{-1} ($1_\sim \times B$) is IF β-open in X. Hence $f^{-1}(B)$ is IF β-open in X. Thus f is IF slightly β-continuous. □

Theorem 3.3. *A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is IF slightly β-continuous if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IF clopen set B in Y such that $f(p_{(\alpha,\beta)}) \in B$, cl($f^{-1}(B)$) is IFN of IFP $p_{(\alpha,\beta)}$ in X.*

Proof. Let f be any IF slightly β-continuous mapping, $p_{(\alpha,\beta)}$ be an IFP in X and B be any IF clopen set in Y such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) \subseteq$ cl(int(cl($f^{-1}(B)$))) $\subseteq$ cl ($f^{-1}(B)$). Hence cl ($f^{-1}(B)$) is IFN of $p_{(\alpha,\beta)}$ in X.
Conversely, let B be any IF clopen set in Y and $p_{(\alpha,\beta)}$ be IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. According to assumption cl ($f^{-1}(B)$) is IFN of IFP $p_{(\alpha,\beta)}$ in X. So $p_{(\alpha,\beta)} \in$ int (cl ($f^{-1}(B)$)) $\subseteq$ cl (int (cl ($f^{-1}(B)$))). So, $f^{-1}(B) \subseteq$ int (cl ($f^{-1}(B)$)). Hence $f^{-1}(B)$ is IFPOS in X. Therefore f is IF slightly β-continuous. □

Proposition 3.1. *Every intuitionistic fuzzy β-continuous function is intuitionistic fuzzy slightly β-continuous. But the converse need not be true, as shown by the following example.*

Example 3.1. Let $X = \{a, b, c\}$,, τ =$\{0_\sim, 1_\sim, A\}$, σ = $\{0_\sim, 1_\sim, B, C, B \cup C, B \cap C\}$ where

$$A = \{\langle x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{b}{0.4})\rangle | x \in X\},$$

$$B = \{\langle x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.5}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.5})\rangle | x \in X\},$$

$$C = \{\langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.3}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{b}{0.7})\rangle | x \in X\}.$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c$. Then f is IF slightly β-continuous. But f is not β-continuous, since $f^{-1}(B \cap C)$ is not IFβOS in X as $f^{-1}(B \cap C) \nsubseteq cl(int(clf^{-1}(B \cap C))) = 0_\sim$

Proposition 3.2. *Every intuitionistic fuzzy β-irresolute function is intuitionistic fuzzy slightly β-continuous. But the converse need not be true, as shown by the following example.*

Example 3.2. Let $X = \{a, b\}, Y = \{c, d\}$, τ =$\{0_\sim, 1_\sim, A\}$, σ = $\{0_\sim, 1_\sim, B\}$ where

$$A = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.6}, \frac{b}{0.5})\rangle | x \in X\},$$

$$B = \{\langle y, (\frac{c}{0.4}, \frac{d}{0.5}), (\frac{c}{0.5}, \frac{d}{0.5})\rangle | y \in Y\}.$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d, f(b) = c$. Then f is IF slightly β-continuous. But it is not IF β-irresolute, since $f^{-1}(B) \nsubseteq cl(int(clf^{-1}(B)))$.

Theorem 3.4. *Suppose that Y has a base consisting of IF clopen sets. If $f : X \to Y$ is IF slightly β-continuous, then f is IF β-continuous.*

Proof. Let $p_{(\alpha,\beta)} \in X$ and let C be IFOS in Y containing $f(p_{(\alpha,\beta)})$. Since Y has a base consisting of IF clopen sets, there exists an IF clopen set B containing $f(p_{(\alpha,\beta)})$ such that $B \subseteq C$. Since F is iF slightly β-continuous, then there exists an IFβOS A in X containing $p_{(\alpha,\beta)}$ such that $f(A) \subseteq B \subseteq C$. Thus f is IF β-continuous. $\square$

Theorem 3.5. *If a function $f : X \to \Pi Y_i$ is a IF slightly fuzzy β-continuous, then $P_i \circ f : X \to Y_i$ is IF slightly β-continuous, where P_i is the projection of ΠY_i onto Y_i.*

Proof. Let B_i be any IF clopen sets of Y_i. Since, P_i is IF continuous and IF open mapping, and $P_i : \Pi Y_i \to Y_i$, ${P_i}^{-1}(B_i)$ is IF clopen sets in $\Pi\, Y_i$. Now $(P_i \circ f)^{-1}$ (B_i) = f^{-1} $({P_i}^{-1}(B_i))$. As f is IF slightly fuzzy β-continuous and ${P_i}^{-1}(B_i)$ is IF clopen sets, f^{-1} $({P_i}^{-1}(B_i))$ is IFβOS in X. Hence ($P_i \circ$f) is IF slightly fuzzy β-continuous. $\square$

Theorem 3.6. *The following hold for functions $f : X \to Y$ and $g : Y \to Z$*
(i) If f is IF slightly β-continuous and g is IF totally coninuous then $g \circ f$ is IF β-continuous.
(ii) If f is IF β-irresolute and g is IF slightly β-continuous then $g \circ f$ is IF slightly β-continuous.

Proof. (i) Let B be an IFOS in Z. Since g is IF totally continuous, $g^{-1}(B)$ is an IF clopen set in Y. Now $(g \circ f)^{-1}(B)$ = f^{-1} $(g^{-1}(B))$. Since f is IF slightly β-continuous, f^{-1} $(g^{-1}(B))$ is IFβOS in X. Hence $g \circ f$ is IF β-continuous.
(ii) Let B be IF clopen set in Z. Since g is IF slightly β-continuous, $g^{-1}(B)$ is an IFβOS in Y. Now $(g \circ f)^{-1}(B)$ = f^{-1} $(g^{-1}(B))$. Since f is IF β-irresolute, f^{-1} $(g^{-1}(B))$ is IFβOS in X which implies $g \circ f$ is IF slightly β-continuous. $\square$

4 Intuitionistic fuzzy β-separation axioms

In this section, we investigate the relationships between IF slightly β-continuous functions and IF β-separation axioms.

Definition 4.1. An IFTS (X, τ) is called $\beta - T_1$ ($co - T_1$, [14]) if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X there exits intuitionistic fuzzy β-open sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U$, $y_{(\nu,\delta)} \notin U$ and $y_{(\nu,\delta)} \in V$, and $x_{(\alpha,\beta)} \notin V$.

Theorem 4.1. *If $f : (X,\ \tau) \to (Y,\ \sigma)$ is IF slightly β-continuous injection and Y is $co - T_1$, then X is IF $\beta - T_1$.*

Proof. Suppose that Y is IF $co - T_1$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X, there exists IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A$, $f(y_{(\nu,\delta)}) \notin A$, $f(x_{(\alpha,\beta)}) \notin B$ and $f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly β-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF β-open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, $y_{(\nu,\delta)} \notin f^{-1}(A)$, $x_{(\alpha,\beta)} \notin f^{-1}(B)$, $y_{(\nu,\delta)} \in f^{-1}(B)$. This shows that X is IF $\beta - T_1$. $\square$

Definition 4.2. An IFTS X is said to be $\beta - T_2$ or β-Hausdorff($co - T_2$ or co-Hausdorff, [14]) if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X, there exits IF β-open sets (IF clopen sets) $U, V \in X$ such that $x_{(\alpha,\beta)} \in U$, $y_{(\nu,\delta)}$ in V and $U \cap V = 0_{\sim}$.

Theorem 4.2. *If $f : (X,\ \tau) \to (Y,\ \sigma)$ is IF slightly β-continuous injection and Y is $co - T_2$, then S is IF $\beta - T_2$.*

Proof. Suppose that Y is IF $co - T_2$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\nu,\delta)}$ in X, there exists IF clopen sets A, B in Y such that $f(x_{(\alpha,\beta)}) \in A$, and $f(y_{(\nu,\delta)}) \in B$. Since f is IF slightly β-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are IF β-open sets in X such that $x_{(\alpha,\beta)} \in f^{-1}(A)$, and $y_{(\nu,\delta)} \in f^{-1}(B)$. Also we have $f^{-1}(A) \cap f^{-1}(B) = 0_{\sim}$. Hence X is IF $\beta - T_2$. □

Definition 4.3. An IFTS X is said to be IF strongly β-regular (IF co-regular [14]) if for each IF β-closed (IF clopen) set C and each IFP $x_{(\alpha,\beta)} \notin C$, there exist intuitionistic fuzzy open sets A and B such that $C \subseteq A$, $x_{(\alpha,\beta)} \in B$ and $A \cap B = 0_{\sim}$

Definition 4.4. An IFTS X is said to be IF strongly β-normal (IF co-normal [14]) if for each IF β-closed (IF clopen) sets C_1 and C_2 in X such that $C_1 \cap C_2 = 0_{\sim}$, there exist intuitionistic fuzzy open sets A and B such that $C_1 \subseteq A$ and $C_2 \subseteq B$ and $A \cap B = 0_{\sim}$.

Theorem 4.3. *If F is IF slightly β-continuous injective IF open function from an IF strongly β-regular space X onto a IF space Y, then Y is IF co-regular.*

Proof. Let D be IF clopen set in Y and $y_{(\nu,\delta)} \notin D$. Take $y_{(\nu,\delta)} = f(x_{(\alpha,\beta)})$. Since f is IF slightly β-continuous, $f^{-1}(D)$ is a IF β-closed set in X. Let $C = f^{-1}(D)$. So $x_{(\alpha,\beta)} \notin C$. Since X is IF strongly β-regular, there exist intuitionistic fuzzy open sets A and B such that $C \subseteq A$, $x_{(\alpha,\beta)} \in B$ and $A \cap B = 0_{\sim}$. Hence, we have $D = f(C) \subseteq f(A)$ and $y_{(\nu,\delta)} = f(x_{(\alpha,\beta)}) \in f(B)$ such that $f(A)$ and $f(B)$ are disjoint IF open sets. Hence Y is IF co-regular. □

Theorem 4.4. *If f is IF slightly β-continuous, injective, IF open function from a IF strongly β-normal space X onto a IF space Y, then Y is IF co-normal.*

Proof. Let C_1 and C_2 be be disjoint IF clopen sets in Y. Since f is IF slightly β-continuous, $f^{-1}(C_1)$ and $f^{-1}(C_2)$ are IF β-closed sets in X. Let us take $C = f^{-1}(C_1)$ and $D = f^{-1}(C_2)$. We have $C \cap D = 0_{\sim}$. Since X is IF strongly β-normal, there exist disjoint IF open sets A and B such that $C \subseteq A$ and $D \subseteq$ B. Thus $C_1 = f(C) \subseteq f(A)$ and $C_2 = f(D) \subseteq f(B)$ such that $f(A)$ and $f(B)$ disjoint IF open sets. Hence Y is IF co-normal. □

5 Intuitionistic fuzzy covering properties and Intuitionistic fuzzy β-connectedness

In this section, we investigate the relationships between IF slightly β-continuous and IF compactness and between IF slightly β-continuous and IF connectedness.

Definition 5.1. An IFTS X is said to be

(1) IF β-compact if every IF β-open cover of X has a finite subcover, [13].

(2) IF countably β-compact if every β-open countably cover of X has a finite subcover.

(3) IF β-Lindelof if every cover of X by IF β-open sets has a countable subcover, [13].

(4) IF mildly compact if every IF clopen cover of X has a finite subcover.

(5) IF mildly countably compact if every IF clopen countably cover of X has a finite subcover.

(6) IF mildly Lindelof if every cover of X has IF clopen sets has a countable subcover.

Theorem 5.1. *Let $f : (X,\tau) \to (Y,\sigma)$ be a IF slightly β-continuous surjection. Then the following statements hold:*
(1) if X is IF β-compact, then Y is IF mildly compact.
(2) if X is IF β-Lindelof, then Y is IF mildly Lindelof.
(3) if X is IF countably β-compact, then Y is IF mildly countably compact.

Proof. (1) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly β-continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β-open cover of X. Since X is IF β-compact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.
(2) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly β-continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β-open cover of X. Since X is IF β-Lindelof, there exists a countable subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly Lindelof.
(3) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly β-continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β-clopen cover of X. Since X is IF countably β-compact, for countable β-clopen cover $\{f^{-1}(A_\alpha); \alpha \in I\}$ in X, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly countable compact. □

Definition 5.2. An IFTS X is said to be:
(1) IF β-closed compact if every β-closed of X has a finite subcover.
(2) IF β-closed Lindelof if every cover of X by β-closed sets has a countable subcover.
(3) IF countably β-closed compact if every countable cover of X by β-closed sets has a finite subcover.

Theorem 5.2. *Let $f : X \longrightarrow Y$ be a IF slightly β-continuous surjection. Then the following statements hold:*

(1) if X is IF β-closed compact, then Y is mildly compact.
(2) if X is IF β-closed Lindelof, then Y is mildly Lindelof.
(3) if X is IF countably β-closed compact, then Y is mildly countably compact.

Proof. (1) Let $\{A_\alpha; \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly β-continuous, then $\{f^{-1}(A_\alpha); \alpha \in I\}$ is IF β-closed cover of X. Since X is TF β-closed compact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus, we have $1_{\sim Y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.

Similarly, we can obtained the proof for (2) and (3) . □

Definition 5.3. [13] An IFTS (X, τ) is said to be intuitionistic fuzzy β-disconnected (IF β-disconnected) if there exists IFβOS A, B in X such that $A \neq 0_\sim$, $B \neq 0_\sim$ such that $A \cup B = 1_\sim$ and $A \cap B = 0_\sim$. If X is not IF β-disconnected then it is said to be intuitionistic fuzzy β-connected (IF β-connected) .

Theorem 5.3. *If Let f : $(X, \tau) \to (Y, \sigma)$ be a IF slightly β-continuous surjection, (X, τ) is an IF β-connected, then (Y,σ) is IF connected.*

Proof. Assume that (Y,σ) is not IF connected then there exists non-empty intuitionistic fuzzy open sets A and B in (Y,σ) such that $A \cup B = 1_\sim$ and $A \cap B = 0_\sim$. Therefore, A and B are intuitionistic fuzzy clopen sets in Y. Since f is IF slightly β-continuous, $C = f^{-1}(A) \neq 0_\sim$, $D = f^{-1}(B) \neq 0_\sim$, which are intuitionistic fuzzy β-open sets in X. And f^{-1} (A) $\cup f^{-1}(B) = f^{-1}$ $(1_\sim) = 1_\sim$, which implies $C \cup D = 1_\sim$. And $f^{-1}(A) \cap f^{-1}(B) = f^{-1}$ $(0_\sim) = 0_\sim$, which implies $C \cap D = 0_\sim$. Thus X is IF β-disconnected , which is a contradiction to our hypothesis. Hence Y is IF connected. □

Definition 5.4. [5] An intuitionistic fuzzy set A in intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy dense if there exists no intuitionistic fuzzy closed set B in (X, τ) such that $A \subseteq B \subseteq 1_\sim$.

Definition 5.5. [14] A IFTS X is called hyperconnected if every IF open set in X is dense.

Remark 5.1. The following example shows that IF slightly β-continuous surjection do not necessarily preserve IF hyperconnectedness.

Example 5.1. Let $X = \{a, b, c\}$, $\tau = \{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B, C, B \cup C, B \cap C\}$, where

$$A = \{\langle x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{b}{0.4})\rangle | x \in X\},$$

$$B = \{\langle x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.5}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.5})\rangle | x \in X\},$$

$$C = \{\langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.3}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{b}{0.7})\rangle | x \in X\}.$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a, f(b) = b$, $f(c) = c$. Then f is IF slightly β-continuous surjective. (X, τ) is hyperconnected. But (X, σ) is not hyperconnected.

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