Eighth Int. Conf. on IFSs, Varna, 20-21 June 2004
NIFS Vol. 10 (2004), 3, 33-36

Symmetrical Difference over Intuitionistic Fuzzy Sets

Anton Antonov

Centre of Biomedical Engineering, Bulgarian Academy of Sciences
Bl. 105 Acad. G. Bontchev Str., 1113 Sofia, Bulgaria
E-mail: *a.antonov@clbme.bas.bg*

Abstract: In this article we will define mathematical operation symmetrical difference over intuitionistic fuzzy sets, and explore its properties.
Keywords: intuitionistic fuzzy sets, symmetrical difference, properties of symmetrical difference over intuitionistic fuzzy sets.

Symmetrical difference over ordinary sets is defined in Kuratowski [1] through basic operations (union, intersection, negation) over ordinary sets in this way.

$$A \div B = \left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right) \tag{1}$$

Over intuitionistic fuzzy sets (IFS) are defined different operations and relations, e.g.: $A \subset B, A = B, \overline{A}, A \cap B, A \cup B, A + B, A.B, A @ B, A\$B, A * B, A \mapsto B$. These operations and their properties are proved in Atanassov [2]. We will use some of them to proof the operation symmetrical difference.

Proposition 1: Let A and B are IFSs over E and operation symmetrical difference is defined on this way.

$$\begin{aligned} A \div B = \{\langle x, &\max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))), \\ &\min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x)))\rangle | x \in E\}. \end{aligned} \tag{2}$$

Proof: $A \div B = \{\langle x, \max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))),$

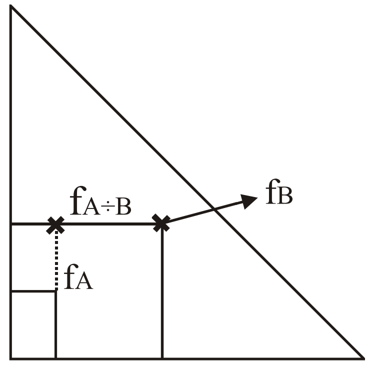
$$\begin{aligned} &\min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x)))\rangle | x \in E\} \\ &= \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle | x \in E\} \cup \\ &\{\langle x, \min(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x))\rangle | x \in E\} \\ &= \{\langle x, \mu_A(x), \nu_A(x) | x \in E\rangle\} \cap \{\langle x, \nu_B(x), \mu_B(x) | x \in E\rangle\} \cup \\ &\{\langle x, \nu_A(x), \mu_A(x) | x \in E\rangle\} \cap \{\langle x, \mu_B(x), \nu_B(x) | x \in E\rangle\} \\ &= \left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right). \end{aligned}$$

and $A, \overline{A}, B, \overline{B}$ are IFS.

The geometrical interpretation of the symmetrical difference is the following. If A and B are two IFSs over E, then a function $f_{A\div B}$ will assign to $x \in E$ a point $f_{A\div B}(x) \in F$ with coordinates:

$$\{\langle x, \max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))), \min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x)))\rangle | x \in E\}$$

Here is presented geometrical interpretation of symmetrical difference over IFS (Figure 1).



$(\nu_A < \mu_A < \nu_B = \mu_B)$

Figure 1.

Properties of symmetrical difference over IFS.

Property 1: $A \div A = A \cap \overline{A}$.

Proof: Let A is IFS then we can write down expression $A \div A$.

$$A \div A = \{\langle x, \max(\min(\mu_A(x), \nu_A(x)), \min(\nu_A(x), \mu_A(x))),$$
$$\min(\max(\nu_A(x), \mu_A(x)), \max(\mu_A(x), \nu_A(x)))\rangle | x \in E\}$$
$$= \{\langle x, \min(\mu_A(x), \nu_A(x)), \max(\nu_A(x), \mu_A(x)\rangle | x \in E\}$$
$$= \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} \cap \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}$$
$$= A \cap \overline{A}.$$

Property 2: $A \div B = B \div A$.

Proof: Let A and B are IFS then we can present $A \div B$ and $B \div A$ with operations union, intersection and negation and examine the equality $A \div B = B \div A$. Hence:

$$A \div B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$
$$B \div A = (B \cap \overline{A}) \cup (\overline{B} \cap A)$$
$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (B \cap \overline{A}) \cup (\overline{B} \cap A)$$
$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (\overline{A} \cap B) \cup (A \cap \overline{B}) \quad (A \cap B = B \cap A \text{ - see Atanassov [2]})$$
$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

Property 3: $A \div B = \overline{(A \mapsto B)} \cup \overline{(B \mapsto A)}$.

Proof: Here for our proof we will use implication over IFS $A \mapsto B = \overline{A} \cup B$.

Hence:

$$A \div B = \overline{(A \mapsto B)} \cup \overline{(B \mapsto A)}.$$

$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = \overline{(\overline{A} \cup B)} \cup \overline{(\overline{B} \cup A)} \quad (A \mapsto B = \overline{A} \cup B \text{ - see Atanassov [2]})$$

$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (A \cap \overline{B}) \cup (B \cap \overline{A}) \quad (\overline{\overline{A} \cup \overline{B}} = A \cap B \text{ -see Atanassov [2]})$$

$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (A \cap \overline{B}) \cup (\overline{A} \cap B) \quad (A \cap B = B \cap A \text{ -see Atanassov [2]})$$

Property 4: $A \div B = \overline{(A \mapsto B) \cap (B \mapsto A)}$

Proof: In order to examine the equality we will use implication over IFS $A \mapsto B = \overline{A} \cup B$. Hence:

$$A \div B = \overline{(A \mapsto B) \cap (B \mapsto A)}$$

$$\Rightarrow A \div B = \overline{(A \mapsto B)} \cup \overline{(B \mapsto A)} \quad (\overline{\overline{A} \cup \overline{B}} = A \cap B \text{ -see Atanassov [2]})$$

$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = \overline{(\overline{A} \cup B)} \cup \overline{(\overline{B} \cup A)} \quad (A \mapsto B = \overline{A} \cup B \text{ - see Atanassov [2]})$$

$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (A \cap \overline{B}) \cup (B \cap \overline{A}) \quad (\overline{\overline{A} \cup \overline{B}} = A \cap B \text{ -see Atanassov [2]})$$

$$\Rightarrow (A \cap \overline{B}) \cup (\overline{A} \cap B) = (A \cap \overline{B}) \cup (\overline{A} \cap B) \quad (A \cap B = B \cap A \text{ -see Atanassov [2]})$$

Following properties don't hold from definition of symmetrical difference over IFS.

Property 5. $(A \div B) \div C = A \div (B \div C)$.

Property 6. $(A \div C) \cap (B \div C) = (A \cap B) \div C$.

Property 7. $(A \cup B) \div C = (A \div C) \cup (B \div C)$.

Property 8. $(A + B) \div C = (A \div C) + (B \div C)$.

Property 9. $(A \div B) + C = (A + C) \div (B \div C)$.

We will check equality 5 and the other equalities from 6 to 9 can be verified on the same way.

$$\begin{aligned} A \div B = \{\langle x, &\max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))), \\ &\min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x)))\rangle | x \in E\}. \end{aligned}$$

$$\begin{aligned} (A \div B) \div C = \{\langle x, &\max(\min(\max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))), \nu_C(x)), \\ &\min(\min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x))), \mu_C(x))), \\ &\min(\max(\min(\max(\nu_A(x), \mu_B(x)), \max(\mu_A(x), \nu_B(x))), \mu_C(x))), \\ &\max(\max(\min(\mu_A(x), \nu_B(x)), \min(\nu_A(x), \mu_B(x))), \nu_C(x)))\rangle | x \in E\}. \end{aligned}$$

$$\begin{aligned} B \div C = \{\langle x, &\max(\min(\mu_B(x), \nu_C(x)), \min(\nu_B(x), \mu_C(x))), \\ &\min(\max(\nu_B(x), \mu_C(x)), \max(\mu_B(x), \nu_C(x)))\rangle | x \in E\} \end{aligned}$$

$$\begin{aligned} A \div (B \div C) = \{\langle x, &\max(\min(\mu_A(x), \min(\max(\nu_B(x), \mu_C(x)), \max(\mu_B(x), \nu_C(x)))), \\ &\min(\nu_A(x), \max(\min(\mu_B(x), \nu_C(x)), \min(\nu_B(x), \mu_C(x))))), \end{aligned}$$

$$\min(\max(\nu_A(x), \max(\min(\mu_B(x), \nu_C(x)), \min(\nu_B(x), \mu_C(x)))),$$
$$\max(\mu_A(x), \min(\max(\nu_B(x), \mu_C(x)), \max(\mu_B(x), \nu_C(x)))))\rangle x \in E\}.$$

If A, B and C have concrete values like these

$$A = \{\langle x, 0.2, 0.1\rangle | x \in E\},$$
$$B = \{\langle x, 0.4, 0.3\rangle | x \in E\},$$
$$C = \{\langle x, 0.42, 0.41\rangle | x \in E\}$$

then

$$(A \div B) \div C = \{\langle x, 0.3, 0.41\rangle | x \in E\}$$

and

$$A \div (B \div C) = \{\langle x, 0.2, 0.4\rangle | x \in E\}.$$

From these results we can see that these expression is not equation.

References

[1] Kuratowski K. (1976), Introduction to set theory and topology, PWN – Polish Scientific Publishers WARSZAWA.

[2] Atanassov K. (1999), Intuitionistic Fuzzy Sets: Theory and Applications, Springer-Verlag.