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Notes on Intuitionistic Fuzzy Sets
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Note on isohesitant intuitionistic fuzzy sets

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Abstract: In the present paper, the class of all intuitionistic fuzzy sets defined over a universe set X, with the same hesitancy distribution is considered. Some properties and notions are defined and studied.
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1 Basic definitions and preliminaries

Here we recall some basic definitions and properties:

Definition 1 (cf. [1]). Let $A \subset X$ and $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are mappings such that for any $x \in X$ the inequality

$$\mu_A(x) + \nu_A(x) \leq 1 \tag{1}$$

holds. The set $\tilde{A} = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in E\}$ is called intuitionistic fuzzy set (or Atanassov set) over E.

The mappings μ_A and ν_A are called membership and non-membership function, respectively. The mapping $\pi_A : X \to [0,1]$, given by:

$$\pi_A(x) \stackrel{\text{def}}{=} 1 - \mu_A(x) - \nu_A(x),$$

is called hesitancy function.

The class of all intuitionistic fuzzy sets over X is further denoted by IFS(X).

Definition 2. Let $A, B \in \text{IFS}(X)$. If we have

$$\min_{x\in X} \pi_A(x) - \pi_B(x) = \max_{x\in X} \pi_A(x) - \pi_B(x) = 0,$$

we say that the sets A and B are isohesitant. The class of all isohesitant IFSs defined over X for a fixed mapping $\tilde{\pi} : X \to [0,1]$ will be further denoted by IFS($X, \tilde{\pi}$).

Definition 3 (cf. [1])**.** Let $A, B \in$ IFS(X). We say that A is strictly included in B and we write $A \subset B$ iff for all $x \in X$

$$\begin{cases} \mu_A(x) \leq \mu_B(x) \\ 1 - \nu_A(x) \leq 1 - \nu_B(X) \\ 1 - \nu_A(x) + \mu_A(x) < 1 - \nu_B(x) + \mu_B(x) \end{cases} \tag{2}$$

Remark 1. We note that for $A, B \in$ IFS$(X, \tilde{\pi})$ this condition is reduced to

$$\mu_A(x) < \mu_B(x). \tag{3}$$

Definition 4 (cf. [1])**.** Let $A, B \in$ IFS(X). We say that A is included in B and we write $A \subseteq B$ iff for all $x \in X$

$$\begin{cases} \mu_A(x) \leq \mu_B(x) \\ 1 - \nu_A(x) \leq 1 - \nu_B(X) \end{cases} \tag{4}$$

Remark 2. We note that for $A, B \in$ IFS$(X, \tilde{\pi})$ this condition is reduced to

$$\mu_A(x) \leq \mu_B(x), \tag{5}$$

which coincides with the definition of inclusion for fuzzy sets (FS) [3]. In fact fuzzy sets are a special subclass of the isohesitant intuitionistic fuzzy sets with $\tilde{\pi} \equiv 0$.

2 Some properties of the Isohesitant Intuitionistic Fuzzy Sets

Let X be a universe set and m be a measure chosen such that $0 < m(X) < \infty$. When X is discrete this measure is taken as the counting measure. Further, without loss of generality we will assume that $m(X) = 1$ (i.e. we will use a modified measure $m^* = \frac{1}{m(X)}m$ but we will keep the denotation m for simplicity).

For any $A, B \in$ IFS$(X, \tilde{\pi})$ we will assign an Intuitionistic Fuzzy Pair (IFP) for the validity of the inclusion $A \subseteq B$. In order to do so, let us define the following two sets $X_{A\subseteq B}, X_{B\subset A}$.

$$X_{A\subseteq B} = \{x | \mu_A(x) \leq \mu_B(x)\} \tag{6}$$

$$X_{B\subset A} = \{x | \mu_A(x) > \mu_B(x)\} \tag{7}$$

It is obvious that these sets are disjoint (non-overlapping) and that their union is exactly X, i.e. we have

$$m(X_{A\subseteq B}) + m(X_{B\subset A}) = m(X) = 1.$$

Further, let us denote for $A, B \in$ IFS$(X, \tilde{\pi})$ by

$$A \subseteq_{u,v} B$$

the fact that $V(A \subseteq B) = \langle u, v \rangle$, with $u = m(X_{A\subseteq B})$, $v = m(X_{B\subset A})$ (cf. [2]).

Let $A, B, C \in \text{IFS}(X, \tilde{\pi})$ and let us know that

$$A \subseteq_{u,v} B \subseteq_{u_1,v_1} C.$$

Does the above imply $A \subseteq_{\min(u,u_1),\max(v,v_1)} C$?

Unfortunately, the answer in general is no.

However, we can still provide some lower and upper bounds for the validity and non-validity of $A \subseteq C$ based on u, u_1, v and v_1.

Theorem 1. *Let $A, B, C \in$ IFS$(X, \tilde{\pi})$ and let*

$$A \subseteq_{u,v} B \subseteq_{u_1,v_1} C.$$

If we denote by $\langle u_2, v_2 \rangle$ the value of $V(A \subseteq C)$, we have that:

$$u_2 \in [\max(0, u + u_1 - 1), \min(1, 2 - v - v_1)] \tag{8}$$

$$v_2 \in [\max(0, v + v_1 - 1), \min(1, 2 - u - u_1)]. \tag{9}$$

Proof. We have

$$u = m(X_{A \subseteq B}),\; v = m(X_{B \subset A}),\; u_1 = m(X_{B \subseteq C}),\; v_1 = m(X_{C \subset B}).$$

Obviously

$$\begin{aligned} m(X \setminus (X_{B \subset A} \cap X_{C \subset B})) \geq u_2 \geq m(X_{A \subseteq B} \cap X_{B \subseteq C}) \\ m(X \setminus (X_{A \subseteq B} \cap X_{B \subseteq C})) \geq v_2 \geq m(X_{B \subset A} \cap X_{C \subset B}) \end{aligned} \tag{10}$$

But the left sides of (10) can be rewritten as (recall that we chose $m(X) = 1$)

$$m(X) - m(X_{B \subset A} \cap X_{C \subset B}) = 1 - m(X_{B \subset A} \cap X_{C \subset B}) \geq u_2$$

$$m(X) - m(X_{A \subseteq B} \cap X_{B \subseteq C}) = 1 - m(X_{A \subseteq B} \cap X_{B \subseteq C}) \geq v_2$$

But for any two sets $X_1, X_2 \subseteq X$ we have:

$$m(X) \geq m(X_1 \cup X_2) = m(X_1) + m(X_2) - m(X_1 \cap X_2),$$

which can be rewritten as:

$$m(X_1 \cap X_2) \geq m(X_1) + m(X_2) - m(X), \tag{11}$$

Hence,

$$-(m(X_1) + m(X_2) - 1) + 1 \geq -m(X_1 \cap X_2) + 1,$$

which yields:

$$2 - m(X_{B \subset A}) - m(X_{C \subset B}) \geq 1 - m(X_{B \subset A} \cap X_{C \subset B}) \geq u_2$$

$$2 - m(X_{A \subseteq B}) - M(X_{B \subseteq C}) \geq 1 - m(X_{A \subseteq B} \cap X_{B \subseteq C}) \geq v_2.$$

Now as to the right hand sides of (10), let us again consider (11). We have

$$u_2 \geq m(X_{A \subseteq B} \cap X_{B \subseteq C}) \geq m(X_{A \subseteq B}) + m(X_{B \subseteq C}) - 1,$$

$$v_2 \geq m(X_{B \subset A} \cap X_{C \subset B}) \geq m(X_{B \subset A}) + m(X_{C \subset B}) - 1.$$

This completes the proof. □

Remark 3. In the case of continuous universe X, it is possible that

$$A \subseteq_{\langle 1,0 \rangle} \not\equiv A \subset B,$$

i.e. when there is a subset of X with measure zero on which the two sets do not agree. In the case of discrete universes these two are equivalent.

3 Conclusion

In the present paper, we considered the class of isohesitant intuitionistic fuzzy sets and we studied a relation of inclusion with IFPs, which although not transitive in the general case, can sometimes yield sufficient inference, e.g. for a decision making process.

References

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